

* حل كل أسئلة الفيزياء :-

① أهمية الكتاب

② أسئلة المحاضرات "اللايدان" [دكتورى //

③ أسئلة المحاضرات "اللايدان" [اللايدان كانت مشتركة
مع باقي الدكاترة

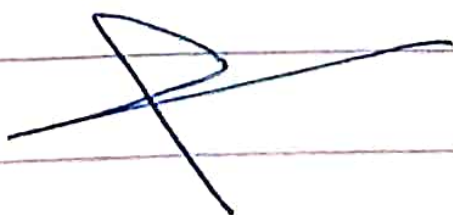
④ أسئلة كل شعبة في الكتاب

* "لأنه أحسنت فهمه إلا أنه أرى فيه نفي والديتان"

* تولى درسي الدكتور خالد رمضان وكنت متابع عفيفي
الدكتور ناجي العاهودي والام واللايدان مشتركة بيعة
الدكاترة.

المستوى الثالث

* زميلكم / فارسي الترتوري



Liquid water, ice, and the flickering cluster

Lecture (1)

الكميات والأبعاد Quantities and Dimensions

* علم الفيزياء مبني على الملاحظات

الكميات والأبعاد هي الأساس في الفيزياء

Type Quantities

- Three basic quantities (Fundamentals)

- LENGTH الطول

- MASS الكتلة

- TIME الوقت

- Other units in terms of these (Derivatives)

$$\text{Speed} = \text{Length} / \text{time}$$

$$\text{density} = \frac{\text{كتلة}}{\text{حجم}} = \frac{\text{Mass}}{(\text{Length})^3}$$

$$\text{force} = \text{كتلة} \times \text{تسارع} = \text{mass} \times \text{length} / (\text{time})^2$$

mass x acceleration

الوحدات (S)

الاختلافات

10^x	Prefix	Symbol
$x=18$	exa	E
$x=15$	peta	P
$x=12$	tera	T
$x=9$	giga	G
$x=6$	mega	M
$x=3$	kilo	k
$x=2$	hecto	h
$x=1$	deca	da

* Cgs system: cm, gram, second

Area [A] Volume [V] speed [v]

* الرمز يوحد بين قو

Work = force x distance

= Mass x (length)² / (time)²

* في عام 1970 هيئة القايير عرفت النظام الدولي باسم

(SI) أو (MKS)

الوحدات

• meter : الطول Mass: كغ Time: second

• speed = $\frac{m}{s}$

• Force = $kg \cdot m/s^2$

• density = kg/m^3

• work = Joule = $kg(m^2/s^2)$

Quantity | Description | Dimension | Units

Velocity $\frac{\text{Displacement}}{\text{Time}} = [V] = [LT^{-1}]$ m/sec

Acceleration $\frac{\text{Displacement}}{\text{Time}^2} = [a] = [LT^{-2}]$ m/sec²

Momentum $[P] = [MLT^{-1}]$ kg·m/sec

Force $[F] = [MLT^{-2}]$ Newton

Pressure $\frac{\text{Force}}{\text{Area}} = [P] = [ML^{-1}T^{-2}]$ Pa

Why Dimensional Analysis is important?

Help to understand Physics even if you not Physicist

Check your equation or Formula

Derive relation between physical quantities

Physical phenomenon

Lecture (2)

Dimensionless quantity

Is a quantity that has no unit and it is ratio between

two similar quantities to which no physical dimension

is assigned. It is also known as a pure number or pure

number or quantity of dimension one.

$\tan(\theta) \times \cos(\theta) \times \sin(\theta) \times \frac{2\pi r}{2r} = \pi$

$\frac{P}{V} \times \frac{V}{P} = 1$

$\frac{V}{V} = 1$

* Specific Gravity = $\frac{\text{density of the object}}{\text{density of water}}$

G (g)

* $W = \frac{1}{2}mv^2$ "1" "2" "8"

$W = F \times d = M \times L^2 T^{-2} \times L = ML^2 T^{-2}$

$\frac{1}{2}mv^2 = M \times L^2 T^{-2} = ML^2 T^{-2}$

$mgh = M \times \frac{L}{T^2} \times L = ML^2 T^{-2}$

$\frac{1}{2}mv^2 = C \times mgh$

* Derive relations between physical quantities involved in physical phenomena. استنتاج العلاقات بين الكميات الفيزيائية المتضمنة

تساعد طريقة تحليل الأبعاد أيضاً في تحديد العلاقات

* The method of dimension analysis can also help

اكتشاف

بعض الكميات المادية

in finding out relations between physical quantities

If we know how physical quantities depend on each

other, we can find the relation between them easily

مما يسهل العثور على كل الكميات

by equating dimension on both sides

المعادلات
Equations must be dimensionally consistent ???

Means that each term in the equation have the

same dimension $A+B=C+D$

لا يمكن أن تكون معادلة الأبعاد متساوية إذا كانت أبعاد المصطلحات متباينة في كلا الجانبين. تذكر أيضاً أنه إذا كانت المصطلحات متساوية الأبعاد فهذا لا يعني أنها متساوية رياضياً.

* $x_{\text{final}} = x_i + v_i t + \frac{1}{2} a t^2$

$x = \text{distance} = [L]$

$x_0 = \text{distance} = [L]$

$v_0 t = \text{velocity} \times \text{time} = [L T^{-1}] \times [T] = [L]$

$\frac{1}{2} a t^2 = \frac{1}{2} [L T^{-2}] \times [T]^2 = [L]$

in consistent and dimensionally incorrect

غير متسقة

غير متسقة الأبعاد

$\text{feet} = ft = 0.3048 \text{ m} / 1 \text{ in} = 0.0254 \text{ m}$

Suppose we are told that the acceleration (a) of a particle with uniform speed (v) in a circle of radius (r) is proportional to some power of v and some power of r . How we can determine the values of n and m ?

say $a \propto v^n r^m$

can determine the values of n and m by using the following steps:

$$v = \frac{L}{T} \quad / \quad r = L \quad / \quad a = L T^{-2} \quad \left| \begin{array}{l} a = r^{-1} v^2 \end{array} \right.$$

$$a = r^n v^m$$

$$L T^{-2} = L^n \cdot [L T^{-1}]^m$$

$$L T^{-2} = L^{n+m} T^{-m}$$

$$-m = -2$$

$$m = 2$$

$$n + m = 1 \rightarrow n = 1 - 2 \rightarrow n = -1$$

"g" / "m" / "L"
 $v \rightarrow$ speed

$$T = v L^x M^y g^z$$

$$[M^0 L^0] T = v L^x M^y g^z \quad \begin{array}{l} g = a \\ g = \frac{L}{T^2} \end{array}$$

$$M^0 L^0 T = v L^x M^y [L T^{-2}]^z$$

$$M^0 L^0 T = v L^{x+z} M^y T^{-2z}$$

$$y = 0 \quad / \quad x + z = 0 \quad / \quad -2z = 1$$

$$\frac{x+z=0}{2} \quad \frac{-2z=1}{2}$$

$$T = v L^{\frac{1}{2}} M^0 g^{\frac{1}{2}}$$

$$T = v \sqrt{\frac{L}{g}}$$

* Vector and Scalar Quantities *

■ Displacement: $\Delta x = x_f - x_i$ ■ Distance: $\Delta s = s_f - s_i$

- x To describe a vector we need more information than to describe a scalar. Therefore, vectors are more complex.

* The bold point is vector A is $\vec{A}$

* Or an arrow above the vector: $\vec{A}$

where m is the mass of the oscillating pendulum

length of the pendulum. Find α , β and γ .

$$F = \hbar \kappa M^{\alpha} L^{\beta} g^{\gamma}$$

$$f M^0 L^0 = V M^{\alpha} L^{y+B} T^{-2B}$$

$$\boxed{a=0} \left(\begin{array}{l} -2b = -1 \\ \boxed{b = \frac{1}{2}} \end{array} \right. \left| \begin{array}{l} y+b=0 \\ \boxed{y = -\frac{1}{2}} \end{array} \right.$$

$$f = k M^0 L^{-\frac{1}{2}} g^{\frac{1}{2}}$$

$$f = k L^{-\frac{1}{2}} g^{\frac{1}{2}}$$

* Negative Vectors *

- Two vectors are negative if they have the same magnitude but are 180° apart (opposite directions)

$$\vec{A} = -\vec{B} \quad \vec{A} + (-\vec{A}) = 0$$

الفردى المتضاد (النقط)

* Scalar Multiplication *

Scalar multiplication means multiplying a vector by a real number, such as 8, 6. The result is a parallel vector of a different length. If the scalar is positive, the direction doesn't change. If it's negative, the direction is exactly opposite.

عند إضافة المتجهات، اتجاهها يجب أن يؤخذ في الاعتبار.

When adding vectors, their direction must be taken into account.

units must be the same

Geometric Methods / use scale drawings
الطرق الهندسية

Algebraic Methods / more convenient
الطرق الجبرية

* in the pictures, we will always show vectors as arrows.

* To describe the magnitude of a vector we will use absolute value sign: $|\vec{A}|$ or just A .

* The vector can be represented by its magnitude A , and its direction θ , (A, θ) , where θ is the direction that the vector makes counterclockwise with the positive x-axis.

clockwise →

* Equality of two vectors *

- Two vectors are equal if they have the same magnitude and the same direction.

* Movement of vectors in a diagram

- Any vectors can be moved parallel to itself without being affected.



- المتجهات (Vector Analysis)
- A component is apart
 - it is useful to use rectangular

These are the projections of the vector along the x- and y-axes

A_x is the x-component

A_y is the y-component

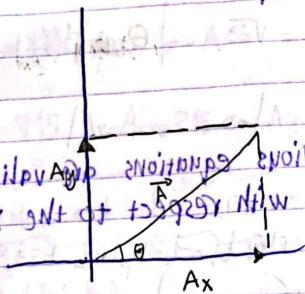
$$A_x = A \cos(\theta)$$

$$A_y = A \sin(\theta)$$

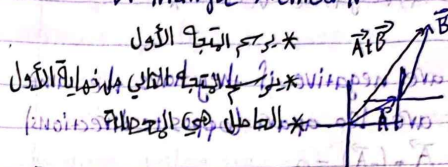
$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$\tan(\theta) = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



* Triangle Method *



The resultant is still drawn from the from the origin of the first vector to the end of the last vector

Special case of vector addition: Add the negative of the subtracted vector

$$A - B = A + (-B)$$

Continue with standard vector addition procedure

$A_x < 0$ $A_y > 0$	$A_x > 0$ $A_y > 0$	$\theta = 0^\circ / A_x = A \cos 0^\circ / A_y = A \sin 0^\circ = 0$
$A_x < 0$ $A_y < 0$	$A_x > 0$ $A_y < 0$	$\theta = 45^\circ / A_x = A \cos 45^\circ / A_y = A \sin 45^\circ$
		$\theta = 90^\circ / A_x = 0 / A_y = A \sin 90^\circ = A$
		$\theta = 135^\circ / A_x = A \cos 135^\circ / A_y = A \sin 135^\circ$
		$\theta = 180^\circ / A_x = -A \cos 0^\circ / A_y = 0$
		$\theta = 225^\circ / A_x = A \cos 225^\circ / A_y = A \sin 225^\circ$

$\theta = 270^\circ / A_x = 0 / A_y = -A \cos 0^\circ$

$\theta = 315^\circ / A_x = A \cos 315^\circ / A_y = A \sin 315^\circ$

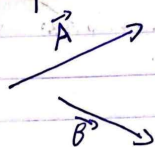
Addition and subtraction vectors

$\vec{A} = (A_x, A_y)$ and $\vec{B} = (B_x, B_y)$

$\vec{A} + \vec{B} = \vec{R}$, where $\vec{R} = (R_x, R_y)$

$R_x = A_x + B_x$ and $R_y = A_y + B_y$

$R = \sqrt{R_x^2 + R_y^2}$



If we have a vector $(4m, 60^\circ)$ find the x and y components

$A_x = A \cos \theta = 4 \cos (60^\circ) = 2m$

$A_y = A \sin \theta = 4 \sin (60^\circ) = 2\sqrt{3}m$

If we have a vector $(2m, -3m)$ find magnitude and direction

$A = \sqrt{(2)^2 + (-3)^2} = \sqrt{13}$

The previous equations are valid only if it is measured with respect to the x-axis

The components can be positive or negative and will have the same units as the original vector.

$$\vec{F} = (12, 60) = (12 \cos(60), 12 \sin(60))$$

$$5\vec{S} = (5, 37) = (5 \cos(37), 5 \sin(37))$$

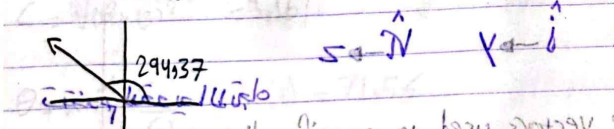
$$\vec{F} + 25\vec{S} = \vec{R} = (R_x, R_y)$$

$$\vec{R} = (12 \cos(60) + 10 \cos(37), 12 \sin(60) + 10 \sin(37))$$

$$\vec{R} = (1.98, 4.37)$$

$$R = \sqrt{(1.98)^2 + (4.37)^2} = 4.79$$

$$\theta = \tan^{-1}\left(\frac{4.37}{1.98}\right) = 294.37 = -65.6$$



Find the direction of the vector R.

Two vectors $\vec{A} = (4m, 30)$ and $\vec{B} = (5m, 217)$. Find $(\vec{A} - \vec{B})$

$$\vec{A} = (4m, 30) = (4 \cos(30), 4 \sin(30))$$

$$\vec{B} = (5m, 217) = (5 \cos(37), 5 \sin(37))$$

$$\vec{A} - \vec{B} = \vec{R} = (R_x, R_y)$$

$$\vec{R} = (4 \cos(30) - 5 \cos(37), 4 \sin(30) - 5 \sin(37))$$

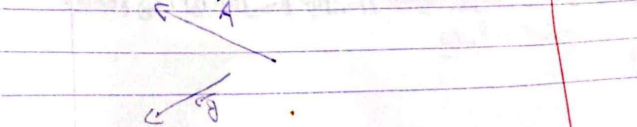
$$(R_x, R_y) = (-7.45, 5)$$

$$R = 8.97$$

$$\theta = \tan^{-1}\left(\frac{5}{-7.45}\right) = 33.86$$

Find $\vec{F} + 25\vec{S} = \vec{R}$

$$\vec{R} = (12 \cos(60) + 10 \cos(37), 12 \sin(60) + 10 \sin(37))$$



Then $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

* Vector A is described algebraically as $(-3, 5)$, while vector B is $(4, -2)$. Find the value of magnitude and direction of the sum (C) of the vectors A and B.

$$\vec{A} = -3\hat{i} + 5\hat{j} \quad \vec{B} = 4\hat{i} - 2\hat{j}$$

$$\vec{C} = \vec{A} + \vec{B} \quad \left| \begin{array}{l} \vec{C} = \hat{i} + 3\hat{j} \\ C_x = 1 \quad C_y = 3 \end{array} \right.$$

$$C = \sqrt{1^2 + 3^2} = 3.16$$

$$\theta = \tan^{-1} \frac{C_y}{C_x} = \tan^{-1}(3) = 71.56^\circ$$

(4) المتجه الوحدة

Unit vector / is a vector with a magnitude of one unit

any vector has associated unit vector in the same direction but having a magnitude of one

Any vector can be expressed as a scalar multiple of its unit

$$\hat{s} = \frac{\vec{s}}{s}$$

$$\text{and } |\hat{s}| = \left| \frac{\vec{s}}{s} \right| = 1$$

* Unit vector *

$$\hat{i} \rightarrow x \quad \hat{j} \rightarrow y \quad \hat{k} \rightarrow z$$

* Unit vectors used to specify direction.

* Unit vectors have a magnitude of 1

$$F = \sqrt{9+81+1} = \sqrt{91}$$

$$\hat{F} = \frac{1}{\sqrt{91}} (3\hat{i} - \hat{j} + 9\hat{k}) = \left(\frac{3}{\sqrt{91}}\hat{i} - \frac{1}{\sqrt{91}}\hat{j} + \frac{9}{\sqrt{91}}\hat{k} \right)$$

$$= \left(\frac{3}{\sqrt{91}}\hat{i} - \frac{1}{\sqrt{91}}\hat{j} + \frac{9}{\sqrt{91}}\hat{k} \right)$$

*Consider two vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

*The scalar product, or dot product, is a mathematical operation used to determine the component of a given force in the direction of the displacement.

The scalar product $\vec{A} \cdot \vec{B}$

$$A \cdot B \cos \theta$$

$$W = F \Delta r \cos \theta = \vec{F} \cdot \Delta \vec{r}$$

IF we have two vectors $\vec{C} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{D} = -\hat{i} + 5\hat{j} - \hat{k}$ find

① $\vec{C} + 2\vec{D}$ ② the unit vector in Direction of $\vec{C} + 2\vec{D}$

$$\vec{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k} = \vec{C} + 2\vec{D}$$

$$M_x = C_x + 2D_x = [2 + (2 \times -1)] = 0$$

$$M_y = C_y + 2D_y = [-3 + (2 \times 5)] = 7$$

$$M_z = C_z + 2D_z = [5 + (2 \times -1)] = 3$$

$$\vec{M} = 7\hat{j} + 3\hat{k}$$

$$\otimes \vec{D} + 2\vec{C} = \vec{F} \quad \vec{F} = (4+1)\hat{i} + (5-6)\hat{j} + (1+10)\hat{k}$$

$$\vec{F} = 5\hat{i} - \hat{j} + 11\hat{k}$$

*The dot product says something about how parallel two vectors are

*The dot product (scalar product) of two vectors can be thought of as the projection of one onto the direction of the other

يمكن اعتبار المنتج النقطي لمتجهين على أنه إسقاط أحدهما على اتجاه الآخر

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad \left| \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \right. \quad (S-1)$$

$$\vec{A} \cdot \hat{i} = A \cos \theta = A_x$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

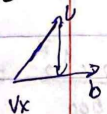
$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x \hat{i} (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) + A_y \hat{j} (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) + A_z \hat{k} (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\hat{i} \cdot \hat{j} = 0 \quad \hat{i} \cdot \hat{k} = 0 \quad \hat{j} \cdot \hat{k} = 0 \quad \hat{i} \cdot \hat{i} = 1 \quad \hat{j} \cdot \hat{j} = 1 \quad \hat{k} \cdot \hat{k} = 1$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

نتيجة قياسية



Properties of scalar Products

$$\textcircled{1} \vec{A} \text{ and } \vec{B} \text{ are } \perp \quad \vec{A} \cdot \vec{B} = 0$$

$$\textcircled{2} \vec{A} \text{ and } \vec{B} \text{ are parallel} \quad \vec{A} \cdot \vec{B} = AB$$

$$\textcircled{3} \vec{A} \cdot \vec{B} = 0 \quad \text{Either } \vec{A} = 0 \text{ or } \vec{B} = 0 \text{ or } \vec{A} \perp \vec{B}$$

$$\textcircled{4} \vec{A} \cdot \vec{A} = A^2 \quad \text{Because } \vec{A} \text{ is parallel to itself}$$

$$\textcircled{5} \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \quad \text{Commutative rule of multiplication}$$

$$\textcircled{6} (\vec{A} + \vec{B}) \cdot \vec{C} = \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{C} \quad \text{Distributive rule of multiplication}$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \quad \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0 \quad \hat{i} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0 \quad \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = 0$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

$$\vec{A} \cdot \vec{B} = 0 \quad \text{if } \vec{A} \perp \vec{B} \quad \text{if } \vec{A} \parallel \vec{B} \quad \vec{A} \cdot \vec{B} = AB$$

(1.4)

$$\text{Energy} = \text{Force} \times \text{length} = L \times MLT^{-2} = ML^2T^{-2}$$

$$\text{Mass} \times \text{speed}^2 = M \times \frac{L^2}{T^2} = ML^2T^{-2}$$

(1.5)

* displacement = $[L]$

$$\text{speed} \times \text{time}^2 = \frac{L}{T} \times T^2 = [L][T]$$

* $L = \frac{K}{T} \times T^n$ $T = T^n$
 $n=1$

(1.8)

$$\text{Area} = (100 \text{ ft} \times 0.3048 \frac{\text{m}}{\text{ft}}) \times (150 \text{ ft} \times 0.3048 \frac{\text{m}}{\text{ft}})$$

$$= 1393.54 \text{ m}^2$$

(1.9)

(1.1)

(1.1)

(1.1)

$V = \text{Area} \times \text{thickness}$

$$A = \frac{V}{T} = \frac{5000 \text{ cm}^3}{10^3} = \frac{5000 \times (10^{-2})^3}{10^3} = 50 \text{ m}^2$$

(1.2)

$$\frac{50 \times 10^3}{3600} = 13.89 \frac{\text{m}}{\text{sec}} = 14 \text{ m/sec}$$

(1.3)

$$\frac{220 \times 10^3}{0.1} = 2.2 \times 10^6$$

(1.4)

Wigap

$$(1-11) A_x = 2 / A_y = -1 / A_z = -2 \left\{ \hat{A} = \frac{(2i - j - 2k)}{3} \right\} \quad (1-1)$$

$$|A| = \sqrt{4+1+4} = 3$$

$$(1-12) A = 2i + 2j / B = 2i - 4j$$

$$\vec{A} + \vec{B} = \vec{R}$$

$$\vec{R} = (R_x, R_y)$$

$$R_x = A_x + B_x$$

$$= 2 + 2 = 4$$

$$R_y = A_y + B_y$$

$$= -2 - 4 = -6$$

$$\vec{R} = 4\hat{i} - 6\hat{j}$$

$$\theta = \tan^{-1}\left(\frac{-6}{4}\right) = -56.31^\circ$$

$$R = \sqrt{16+36} = \sqrt{52}$$

$$(1-13) d_3 = 13\hat{i} - 15\hat{j}$$

$$\vec{D} = (15+23+13)\hat{i} + (30-14-15)\hat{j} + (12-5)\hat{k}$$

$$\vec{D} = 51\hat{i} + \hat{j} + 7\hat{k}$$

$$D = \sqrt{(51)^2 + 1 + (7)^2} = 51.99$$

$$(1-9) 1 \text{ in} = 2.54 \text{ cm} /$$

$$\frac{1 \text{ in}}{32 \text{ day}} = \frac{2.54 \times 10^{-2} \times 10^9}{32 \times 24 \times 3600} = 9.19 \text{ nm/sec}$$

$$(1-10)$$

$$\theta = 30^\circ \rightarrow A = 125$$

$$A_x = 125 \cos(30)$$

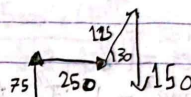
$$A_y = 125 \sin(30)$$

$$\Sigma X = 250 + 125 \cos(30) = 358.253$$

$$\Sigma Y = 75 + 125 \sin(30) - 150 = -12.5$$

$$R = 358.47$$

$$\theta = \tan^{-1}\left(\frac{-12.5}{358.253}\right) = -2^\circ \text{ south of East}$$



(1-16)

$$a \cdot b \cos \theta$$

$$4 \times 5 \times \cos(60) = 10$$

(1-17)

$$a \cdot b = (3 \times -5) + (-2 \times 4) + (7 \times -3) = -44$$

$$b \cdot a = (-5 \times 3) + (4 \times -2) + (-3 \times 7) = -44$$

(1-18) perpendicular

$$a \cdot b = (3 \times 1) + (2 \times -2) + (-1 \times -1) = 0$$

(1-19) $\vec{r}$ $|\vec{r}| = \sqrt{38}$

$$\vec{r} \cdot \vec{i} = (3\vec{i} + 2\vec{j} + 5\vec{k}) \cdot \vec{i} = 3$$

$$\vec{r} \cdot \vec{i} = |\vec{r}| |\vec{i}| \cos \theta$$

$$\cos \theta = \frac{3}{\sqrt{38}}$$

$$\cos^{-1}\left(\frac{3}{\sqrt{38}}\right) \rightarrow \theta = 60.88$$

$$\theta = 60.88$$

(1-14)

$$\vec{A} + \vec{B} = (3-1)\hat{i} + (-2-4)\hat{j} = 2\hat{i} - 6\hat{j}$$

$$\vec{A} - \vec{B} = (3+1)\hat{i} + (-2+4)\hat{j} = 4\hat{i} + 2\hat{j}$$

$$|\vec{A} + \vec{B}| = \sqrt{2^2 + (-6)^2} = \sqrt{40}$$

$$|\vec{A} - \vec{B}| = \sqrt{4^2 + 2^2} = \sqrt{20}$$

* Direction of $(\vec{A} + \vec{B})$

$$\theta = \tan^{-1}\left(\frac{-6}{2}\right) = -71.56$$

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.57$$

(1-15)

$$(A, \theta) = (50, 120)$$

$$A_x = A \cos(\theta) = 50 \cos(120) = -25 \text{ m}$$

$$A_y = A \sin(\theta) = 50 \sin(120) = 25\sqrt{3}$$

$$\vec{A} = -25\hat{i} + 25\sqrt{3}\hat{j} \text{ m}$$

Given the vectors $A = 3i + 2j - k$ and $B = 5i + 5j$ find

1) The dot product $A \cdot B$

$$A \cdot B = (3 \times 5) + (2 \times 5) + (-1 \times 0) = 25$$

2) The projection of A onto B $B \cos \theta$

$$\frac{A \cdot B}{|A|} \times \frac{B}{|B|} = \frac{25}{\sqrt{14}} \times \frac{B}{\sqrt{50}} = \frac{1}{5} (5i + 5j) = i + j$$

3) The angle between A and B

$$\cos \theta = \frac{A \cdot B}{|A| |B|}$$

$$|A| = \sqrt{9+1+4} = \sqrt{14}$$

$$|B| = \sqrt{50}$$

$$\theta = \cos^{-1} \left(\frac{25}{\sqrt{14} \times \sqrt{50}} \right) = 19.1^\circ$$

4) A vector of magnitude 2 in the xy plane perpendicular to B

$$C = 2k$$

*

$$u_y \cdot i = (3i + 2j + 5k) \cdot i = 3$$

$$u_y \cdot i = |u_y| \cdot |i| \cos \theta$$

$$\cos \theta = \frac{u_y \cdot i}{|u_y| |i|}$$

$$\cos \left(\frac{2}{\sqrt{38}} \right)^{-1} = 71.1^\circ = \theta$$

$$u_z \cdot i = (3i + 2j + 5k) \cdot i = 3$$

$$u_z \cdot i = |u_z| \cdot |i| \cos \theta$$

$$\cos \theta = \frac{u_z \cdot i}{|u_z| |i|}$$

$$\cos \left(\frac{5}{\sqrt{38}} \right)^{-1} = 35.8^\circ$$

الحافرة (٦)

Cross Product

The vector product or cross product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them. The direction of the product vector is perpendicular to the plane containing the two vectors in accordance with the right hand thumb rule.

على أن متجه آخر لا مقدار ما والحاصل ضرب مقدار متجهين وجيب الزاوية بينهما يكون اتجاه متجه الناتج عمودياً على المستوى الذي يحتوي على المتجهين وفقاً لقاعدة اليد اليمنى أو قاعدة كايهام اليد اليمنى.

$$\vec{C} = \vec{A} \times \vec{B}$$

$$|\vec{C}| = |\vec{A} \times \vec{B}| = AB \sin \theta$$

* Direction / C perpendicular to both A and B. First practice

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$|\vec{A} \cdot \vec{B}| = |\vec{A}| |\vec{B}| \cos \theta$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = 0 \Rightarrow \sqrt{(i_2)^2 + (j_2)^2} = 2$$

$$(5 \cdot i_2) + (5 \cdot j_2) = 0 \quad 4 = i_2^2 + j_2^2$$

$$5i_2 + 5j_2 = 0$$

$$i_2 + j_2 = 0$$

$$i_2 = -j_2$$

$$4 = 2j_2^2 \rightarrow j_2 = \sqrt{2}$$

$$i_2 = -\sqrt{2}$$

$$\vec{A} = -\frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j}$$

$$\hat{i} \times \hat{j} = \hat{k} / \hat{j} \times \hat{i} = -\hat{k} / \hat{j} \times \hat{k} = \hat{i} / \hat{k} \times \hat{j} = -\hat{i} / \hat{i} \times \hat{k} = \hat{j}$$

$$\hat{i} \times \hat{i} = 0 / \hat{j} \times \hat{j} = 0 / \hat{k} \times \hat{k} = 0$$

على عقارب الساعة (+) / مع عقارب الساعة (-)

في الضرب الاتجاهي حاصل ضرب المتجه في نفسه يساوي صفرًا لأن الزاوية = صفر

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} / \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

Then

$$\vec{A} \times \vec{B} = A_x \hat{i} (B_y \hat{j} + B_z \hat{k}) + A_y \hat{j} (B_z \hat{k} + B_x \hat{i}) + A_z \hat{k} (B_x \hat{i} + B_y \hat{j})$$

$$\vec{A} \times \vec{B} = (A_x B_y - A_y B_x) \hat{k} + (A_x B_z - A_z B_x) \hat{j} + (A_y B_z - A_z B_y) \hat{i}$$

* The cross product of two vectors results in a new vector

* θ is smaller angle between the vectors

* Cross product of any parallel vectors = zero

* Cross product is maximum for perpendicular vectors

* Cross product is a vector

* $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$$\vec{A} \times \vec{B} = C \Rightarrow C = AB \sin \theta; C \perp A; C \perp B$$

• Properties

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

• Parallel and orthogonal vectors

$$\vec{A} \parallel \vec{B} \Rightarrow \sin \theta = 0 \Rightarrow \vec{A} \times \vec{B} = 0$$

$$\vec{A} \perp \vec{B} \Rightarrow \sin \theta = 1 \Rightarrow \vec{A} \times \vec{B} = \vec{A} \times \vec{B}$$

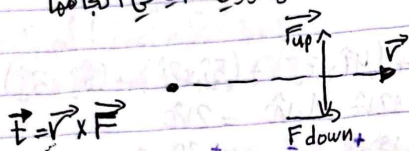
• Cross product of a vector with itself

$$\vec{A} \times \vec{A} = 0$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

• can have multiple forces applied at multiple points.
 - يمكن أن يكون لها قوى متعددة مطبقة في نقاط متعددة
 $\vec{v} = 2\hat{i} + 3\hat{j}$ $\vec{v} = 2\hat{i} + 3\hat{j}$

• Direction of $\vec{\tau}_{net}$ is angular acceleration axis
 اتجاه الناتج هو محور التسارع الزاوي أي في اتجاهها



• F_{up} : cause a counterclockwise rotation, then the torque is positive.
 - تسبب في دوران باتجاه عقارب الساعة
 - تكون العزم موجب

• F_{down} : cause clockwise rotation, then the torque is negative.
 - تسبب في دوران باتجاه عقارب الساعة
 - تكون العزم سالب

* Find $(\vec{A} \times \vec{B})$ where $\vec{A} = 2\hat{i} + 3\hat{j}$ $\vec{B} = \hat{i} + 2\hat{j}$
 $(2\hat{i} \times \hat{i}) + (2\hat{i} \times 2\hat{j}) + (3\hat{j} \times \hat{i}) + (3\hat{j} \times 2\hat{j})$
 $0 + 4\hat{k} - 3\hat{k} + 0 = \hat{k}$
 $4\hat{k} - 3\hat{k} = \hat{k}$

مع الدوران

* The torque is the cross product of a force vector with the position vector to its point of application.
 - هو الناتج المتقاطع للقوة مع متجه الموقع إلى نقطة التطبيق
 $\vec{\tau} = \vec{r} \times \vec{F}$

$|\tau| = rF \sin \theta = r_1 F_2 = r_2 F_1$

* The torque vector is perpendicular to the plane formed by the position vector and the force vector (e.g., imagine drawing them tail-to-tail)

متجه عزم الدوران عمودي على المستوى المتكون من متجه الموقع ومتجه القوة

* Right Hand Rule: curl fingers from $\vec{r}$ to $\vec{F}$, thumb points along torque

$\vec{\tau}_{net} = \sum_{all i} \vec{\tau}_i = \sum_{all i} \vec{r}_i \times \vec{F}_i$ (vector sum)

* Superposition *

Two vectors / $u = i + 2j + k$ and $v = j + 2k$ find

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(4-1) - \hat{j}(2-0) + \hat{k}(-1-0)$$

$$= 5\hat{i} - 2\hat{j} - \hat{k} = (5, -2, -1)T$$

Given the vector $A = \hat{i} - 2\hat{j} + 4\hat{k}$ and $B = 3\hat{i} + \hat{j} - 2\hat{k}$
find $A \times B$ and $|A \times B|$

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ 3 & 1 & -2 \end{vmatrix} = \hat{i}(-4-12) - \hat{j}(-2-12) + \hat{k}(1-6)$$

$$= -14\hat{i} + 14\hat{j} - 5\hat{k}$$

$$|A \times B| = \sqrt{(-14)^2 + (14)^2 + (-5)^2} = 19.2$$

Calculate torque, given a force and its location

$$\vec{F} = (2\hat{i} + 3\hat{j})N \quad \vec{r} = (4\hat{i} + 5\hat{j})m$$

$$\vec{\tau} = \vec{r} \times \vec{F} = (4\hat{i} + 5\hat{j}) \times (2\hat{i} + 3\hat{j})$$

$$(4\hat{i} \times 2\hat{i}) + (4\hat{i} \times 3\hat{j}) + (5\hat{j} \times 2\hat{i}) + (5\hat{j} \times 3\hat{j})$$

$$12\hat{k} - 10\hat{k} = 2\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

Find the vector $E = A \times C$, and $F = B \times C$, and the unit vector $\hat{E}$

$$\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 0 & -2 & 2 \end{vmatrix} = \hat{i}(4-0) + \hat{j}(-2-0) + \hat{k}(2-0) = 4\hat{i} + 2\hat{j} + 2\hat{k}$$

$$|\vec{E}| = \sqrt{16+4+4} = \sqrt{24}$$

$$\hat{E} = \frac{(4\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{24}}$$

$$\vec{F} = \vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -2 \\ 0 & -2 & 2 \end{vmatrix} = \hat{i}(0-4) - \hat{j}(2-0) + \hat{k}(-2-0) = -4\hat{i} - 2\hat{j} - 2\hat{k}$$

When two vectors A and B are drawn from a common point, the angle between them is θ . Show that the magnitude of their vector sum is given by $\sqrt{A^2 + B^2 + 2AB \cos \theta}$. If A and B have the same magnitude under what condition will the vector sum have the same magnitude of the vector A or B ?

For the following three vectors

$$A = -\hat{i} + 2\hat{j}, B = \hat{i} - 2\hat{k}, C = -2\hat{j} + 2\hat{k}$$

The angle between the vectors $D = A+B$ and $T = A-C$

$$\begin{aligned} \vec{D} &= \vec{A} + \vec{B} = -\hat{i} + 2\hat{j} + \hat{i} - 2\hat{k} = 2\hat{j} - 2\hat{k} \\ \vec{T} &= \vec{A} - \vec{C} = -\hat{i} + 2\hat{j} - (-2\hat{j} + 2\hat{k}) = -\hat{i} + 4\hat{j} - 2\hat{k} \end{aligned}$$

$$|\vec{D}| = \sqrt{4+4} = \sqrt{8}, |\vec{T}| = \sqrt{1+16+4} = \sqrt{21}$$

$$\vec{D} \times \vec{T} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -2 \\ -1 & 4 & -2 \end{vmatrix} = \hat{i}(-4-8) - \hat{j}(2-2) + \hat{k}(8-2) = -12\hat{i} + 6\hat{k}$$

$$|\vec{D} \times \vec{T}| = \sqrt{144+36} = \sqrt{180} = 6\sqrt{5}$$

$$|\vec{D} \times \vec{T}| = |\vec{D}| |\vec{T}| \sin \theta$$

$$\sin^{-1} \left(\frac{\sqrt{24}}{\sqrt{8} \times \sqrt{21}} \right) = 22.2^\circ$$

*

Two vectors $\vec{A}$ and $\vec{B}$ have the magnitudes of 2 and 5 respectively, and the magnitude of their ^{الضرب الاتجاهي} vector product is 6. Find the magnitude of their scalar product.

$$A \times B = |A| |B| \sin \theta$$

$$6 = 2 \times 5 \sin \theta$$

$$\theta = 37$$

$$A \cdot B = |A| |B| \cos \theta$$

$$= 2 \times 5 \times \cos(36.87)$$

$$= 8$$

* Two vectors $\vec{A} = \hat{i} - 3\hat{k}$, $\vec{B} = 3\hat{j} - \hat{k}$ find the unit vector in the direction of $\vec{B} \times \vec{A}$

$$\vec{F} = \vec{B} \times \vec{A}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 1 & 0 & -3 \end{vmatrix} = \hat{i}(-9-0) + \hat{j}(0-1) + \hat{k}(0-3)$$

$$= -9\hat{i} - \hat{j} - 3\hat{k}$$

$$|\vec{F}| = \sqrt{81+1+9} = \sqrt{91}$$

$$\hat{F} = \frac{1}{\sqrt{91}}(-9\hat{i} - \hat{j} - 3\hat{k})$$

third vector $\vec{C}$ has the magnitude of $\vec{A}$ and make an angle 90° to the vector $\vec{B}$

$$* \sqrt{i^2 + j^2} = 3$$

$$i^2 + j^2 = 9$$

$$2. C = 3\pi$$

$$\cos^{-1} \left(\frac{-9}{\sqrt{6} \times \sqrt{17}} \right) = 153$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 1 & 1 & 0 \end{vmatrix} = (0-2)\hat{i} - (0-2)\hat{j} + (2-1)\hat{k} = -2\hat{i} + 2\hat{j} + \hat{k}$$

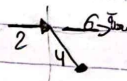
$$= \sqrt{4+4+9} = \sqrt{17}$$

$$\vec{A} - \vec{C} = 2\hat{i} - \hat{j} - \hat{k}$$

$$= \sqrt{4+1+1} = \sqrt{6}$$

* الـ Z محوري على المستوى (xy)
: يكون على المحور Z

A sailboat sails 2 km east, then 4 km southeast
 and then an additional unknown displacement. Its
 final position is 6 km directly west of the starting
 point. Find the third unknown displacement of the
 journey.



$$2 + 4 \cos(315) + A_x = 6$$

$$Ax = 4 - 2\sqrt{2}$$

$$(*) \quad y \sin(315) + A_H = 0$$

$$A_y = 2\sqrt{2} \text{ N}$$

$$\Theta = \frac{A_y}{A_x} = 67.5$$

$$\textcircled{*} A = \sqrt{(A_x)^2 + (A_y)^2}$$

$$A = 3$$

	X	y
2	2	0
4	$4\cos(315)$	$4\sin(315)$
A	A_x	A_y
	6	

السرعة المبدئية

$$1 \text{ A} = \frac{100 \times 1000}{3600} = 27.77 \text{ m/s}$$

$$1 \text{ km} = 0.6213$$

$$100 = 62.13$$

$$62.13 \text{ miles/hour}$$

$$v = \text{Area} \times \text{thickness}$$

$$t = \frac{v}{\text{Area}} = \frac{3.78 \times 10^3}{25} = 1.512 \times 10^4$$

$$\frac{\Delta v}{\Delta t} = \frac{30}{7 \times 60} = \frac{1}{14} \text{ gal/s}$$

$$1 \text{ gal} = 3.78 \text{ L}$$

$$\frac{\Delta v}{\Delta t} = \frac{1}{14} \times 3.78 \times 10^3 = 2.7 \times 10^2 \text{ L/s}$$

$$t = \frac{V}{\text{rate}} = \frac{1 \text{ m}^3}{0.87 \times 10^3} = 1.15 \text{ h}$$

$$= 1.15 \times 60 = 69 \text{ min}$$

$$\vec{A} + \vec{B} = \vec{R}$$

$$= (A_x + B_x, A_y + B_y)$$

$$R^2 = (A_x + B_x)^2 + (A_y + B_y)^2$$

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

$$A^2 = A_x^2 + A_y^2$$

$$R = \sqrt{A_x^2 + B_x^2 + A_y^2 + B_y^2 + 2(A_x B_x + A_y B_y)}$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$(5-3) \text{ km} = (2) \text{ km}$$

$$M^0 = M^y$$

$$\boxed{y=0}$$

$$* T^0 = T^{-2\alpha+B}$$

$$0 = -2\alpha + B \rightarrow \boxed{B=2}$$

$$\boxed{9} \quad v = k E^x \rho^y \quad \boxed{E = M L^{-1} T^{-2}} \quad \boxed{V = L T^{-1}}$$

$$L T^{-1} = [M L^{-1} T^{-2}]^x [M L^{-3}]^y$$

$$L T^{-1} = M^{x+y} L^{-x-3y} T^{-2x}$$

$$\otimes -2x = -1 \rightarrow \boxed{x = \frac{1}{2}}$$

$$\otimes L = L^{-x-3y}$$

$$-x-3y = 1 \rightarrow -\frac{1}{2}-3y = 1 \rightarrow -3y = \frac{3}{2} \rightarrow \boxed{y = -\frac{1}{2}}$$

$$\otimes x+y=0$$

$$V = \sqrt{\frac{E}{\rho}}$$

$$\boxed{6} \quad 36 \times (10^{-6})^3 = 36 \times 10^{-17} m^3$$

$$\boxed{7} \quad 36 \times 10^{-17} \times (10^3)^3 = 36 \times 10^{-8}$$

$$\boxed{5} \quad d = \frac{v}{f} = \frac{8.6g}{cm^3} = \frac{8.6 \times 10^{-3}}{(10^{-2})^3} = 8600 kg/m^3$$

$$\boxed{6} \quad G = \frac{F x r^2}{m M} = \frac{kg \cdot m \cdot m^2}{s^2 \times kg^2} = \frac{m^3}{s^2 \times kg}$$

$$\boxed{7} \quad \lambda = \sqrt{\frac{G h}{c^3}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6.63 \times 10^{-34}}{(3 \times 10^8)^3}} = 1.21 \times 10^{-26}$$

$$\sqrt{\frac{L^2 \times M \times L^2}{M \times L^3 \times T}} = \sqrt{\frac{L^2}{L}} = L$$

$$\boxed{8} \quad S = C g^a t^b m^c \quad \boxed{g = a = L x T^{-2}}$$

$$L = [L T^{-2}]^a \times [M]^c \times T^b$$

$$M^0 T^0 L = M^0 \times L^\alpha \times T^{-2\alpha+B}$$

$$L^\alpha = L \rightarrow \boxed{\alpha=1}$$

$$M^0 = M^{y+y}$$

$$y=0$$

$$* T^0 = T^{-2\alpha + B}$$

$$0 = -2\alpha + B \rightarrow B=2$$

$$[9] \quad v = k E^x p^y \quad E = M L^{-1} T^{-2} \quad v = L T^{-1}$$

$$L T^{-1} = [M L^{-1} T^{-2}]^x [M L^{-3}]^y \quad \rho = M L^{-3}$$

$$L T^{-1} = M^{x+y} L^{-x-3y} T^{-2x}$$

$$\otimes -2x = -1 \rightarrow x = \frac{1}{2}$$

$$\otimes L = L^{-x-3y}$$

$$-x-3y = 1 \rightarrow -\frac{1}{2}-3y = 1 \rightarrow -3y = \frac{3}{2} \rightarrow y = -\frac{1}{2}$$

$$\otimes x+y=0$$

$$V = \sqrt{\frac{E}{\rho}}$$

[6]

$$36 \times (10^{-6})^3 = 36 \times 10^{-17} m^3$$

$$36 \times 10^{-17} \times (10^3)^3 = 36 \times 10^{-8}$$

$$[5] \quad d = \frac{v}{f} = \frac{8.6 g}{cm^3} = \frac{8.6 \times 10^3}{(10^2)^3} = 8600 kg/m^3$$

$$[6] \quad G = \frac{F x r^2}{m M} = \frac{kg \cdot m \cdot m^2}{s^2 \times kg^2} = \frac{m^3}{s^2 \times kg}$$

$$[7] \quad \lambda = \sqrt{\frac{G h}{c^3}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6.63 \times 10^{-34}}{(3 \times 10^8)^3}} = 1.921 \times 10^{-26}$$

$$= \sqrt{\frac{L^3 \times M \times L^2}{M^2 \times L^3 \times T^2}} = \sqrt{\frac{L^3}{M \times T^2}} = L$$

$$[8] \quad S = C g^a t^b \quad S = g = a = L \times T^{-2}$$

$$L = [L T^{-2}]^a \times [M]^y \times T^b$$

$$M^0 T^0 L = M^y L^a T^{-2a+b}$$

$$L^a = L \rightarrow a=1$$

$$12) a) |A| = \sqrt{1+4+1} = \sqrt{6} \quad \left| \begin{array}{l} 2B_1 = -2j - 4k \\ 2B_2 = -2i + 2j + 2k \end{array} \right. \quad [N]$$

$$A+B = i-j+3k$$

$$A-2B = i+4j-3k$$

$$b) |A_2| = \sqrt{9+4} = \sqrt{13} \quad -2B_2 = -2i+2j+2k$$

$$A+B = 4i-j-k$$

$$A-2B = i+2j-4k$$

$$13) \vec{v} = (3, \frac{\pi}{3})$$

$$V_x = V \cos \theta = 3 \cos\left(\frac{\pi}{3}\right) = 1.5 \quad \vec{v} = 1.5i + 2.6j$$

$$V_y = V \sin \theta = 3 \sin\left(\frac{\pi}{3}\right) = 2.6$$

المعلمة في المثلث

$$10) \theta = 250^\circ \quad A = 7.3 \quad (A, \theta)$$

$$(7.3, 250^\circ)$$

$$A_x = A \cos \theta = -2.5$$

$$A_y = A \sin \theta = -6.86$$

$$\vec{A} = -2.5i - 6.86j$$

$$11) A+B = -C \rightarrow C = -A-B$$

$$A = (A_x, A_y) = (80 \cos(339^\circ), 80 \sin(339^\circ))$$

$$B = (B_x, B_y) = (105 \cos(101^\circ), 105 \sin(101^\circ))$$

$$C = -(A+B)$$

$$C_x = -(80 \cos(339^\circ) + 105 \sin(101^\circ)) = -110.5$$

$$C_y = -(80 \sin(339^\circ) + 105 \sin(101^\circ)) = -74.4$$

$$|C| = 133.2$$

$$\theta = 339^\circ$$

$$15) \textcircled{A} |A| = \sqrt{64+4+16} = \sqrt{84}$$

$$\hat{A} = \frac{1}{\sqrt{84}} \times (8i - 2j + 4k)$$

$$\textcircled{B} |B| = \sqrt{9+16+25} = \sqrt{50}$$

$$\hat{B} = \frac{1}{\sqrt{50}} \times (3i - 4j + 5k)$$

$$16) M \times N = \begin{vmatrix} i & j & k \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix} = (-6-1)i - (-18-2)j + (-6-4)k$$

$$= -7i + 16j - 10k$$

$$\vec{F} = 9\sqrt{5}$$

$$17) A \sin \theta = A \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$\tan \theta = 1$$

$$\theta = 45$$

$$14) a(6i-8j) + b(-8i+3j) + c = i$$

$$aA + bB = -C$$

$$a(6i-8j) + b(-8i+3j) = -26i - 19j$$

$$\textcircled{I} \begin{cases} 6a - 8b = -26 \\ -8a + 3b = -19 \end{cases} \rightarrow \begin{cases} 3a - 4b = -13 \times \frac{3}{4} \\ -8a + 3b = -19 \end{cases}$$

$$\begin{cases} 6a - 8b = -26 \\ -8a + 3b = -19 \end{cases}$$

$$\frac{9}{4}a - 3b = -\frac{39}{4}$$

$$-8a + 3b = -19$$

$$\rightarrow \frac{-23a}{4} = \frac{115}{4}$$

$$\boxed{a = 5}$$

$$-8 \times 5 + 3b = -19$$

$$\rightarrow \boxed{b = 7}$$

Suiiii *14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100*

$$\begin{aligned} \times A - B = \vec{R} &= (R_x, R_y) \\ \vec{R} &= (3\cos(30) - 0, 3\sin(30) - 3) \\ &= (2.96, -1.95) \quad \theta = 330 \end{aligned}$$

$$\begin{aligned} \times B - A = \vec{R} &= (R_x, R_y) \\ &= (B_x - A_x, B_y - A_y) \\ &= (0 - 3\cos(30), 3 - 3\sin(30)) \\ &= (-2.95, 1.95) \quad \theta = 329 \end{aligned}$$

$$\begin{aligned} \times A - 2B = \vec{R} &= (R_x, R_y) \\ &= (3\cos(30) - 0, 3\sin(30) - 2 \times 3) \\ &= (2.96, -4.95) \quad \theta = 300 \end{aligned}$$

$$\begin{aligned} [21] A \cdot B &= 0 \\ -6b + b^3 + 2b &= 0 \\ b^3 - 4b &= 0 \\ b(b^2 - 4) &= 0 \\ b(b-2)(b+2) &= 0 \\ b = 0 / b = \pm 2 \end{aligned}$$

$$[18] A + B + C = \vec{F}$$

$$4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - 3\hat{j} - \hat{i} = \hat{j} - 2\hat{j}$$

$$\begin{aligned} [19] \vec{F}_x &= A_x + B_x + C_x \\ &= 0 + 40\cos(45) + 30\cos(45) \\ &= 35\sqrt{2} \end{aligned}$$

$$\begin{aligned} \vec{F}_y &= A_y + B_y + C_y \\ &= 20 + 40\sin(45) - 30\sin(45) \\ &= 20 + 5\sqrt{2} \end{aligned}$$

$$\tan^{-1}\left(\frac{20+5\sqrt{2}}{35\sqrt{2}}\right) \rightarrow \theta = 28.67$$

$$\vec{F} = \sqrt{(35\sqrt{2})^2 + (20+5\sqrt{2})^2} = 56.4$$

$$[20] A + B = \vec{R} (R_x, R_y)$$

$$\begin{aligned} \vec{R} &= (A_x + B_x, A_y + B_y) \\ &= (3\cos(30) + 0, 3\sin(30) + 3) \\ &= (2.96, 4.95) \quad \theta = 60 \end{aligned}$$

$$\begin{aligned} A_y - b_y &= 8 \\ A_y + b_y &= 16 \end{aligned} \rightarrow 2A_y = 24 \quad \left| \begin{array}{l} 12 \cdot b_y = 8 \\ b_y = 4 \end{array} \right.$$

$$\vec{a} = 9\hat{i} + 12\hat{j} \quad \vec{b} = 3\hat{i} + 4\hat{j}$$

$$|a| = 15 \quad |b| = 5$$

$$\textcircled{1} a \cdot b = 15 \times 5 = 75$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 9 & 12 & 0 \\ 3 & 4 & 0 \end{vmatrix} = (0)\hat{i} - (0)\hat{j} + (36 - 36)\hat{k} = 0$$

24

$$\textcircled{2} |A \cdot b| = (2 \times 1) + (2 \times 1) = 4$$

$$|A| = \sqrt{5} \quad |b| = \sqrt{5}$$

$$\cos\left(\frac{4}{\sqrt{5} \times \sqrt{5}}\right) = 1 \Rightarrow \theta = 0^\circ$$

$$\textcircled{3} |A \cdot b| = (3) + (-2) + (6) = 7 \quad \left| \cos\left(\frac{7}{\sqrt{22} \times \sqrt{6}}\right) = \frac{1}{2} \right.$$

$$|A| = \sqrt{22} \quad |b| = \sqrt{6}$$

$$\textcircled{22} \sqrt{1+1+1} = \sqrt{3}$$

No Because his magnitude $\neq 1$

$$\textcircled{*} \text{No} \quad " \quad " \quad " = 1$$

$$\textcircled{*} \text{Yes} \rightarrow \vec{A} = -\hat{i}$$

$$A = \sqrt{(-1)^2} = \sqrt{1} = 1$$

$$\textcircled{*} \vec{A} = 3a\hat{i} + 4a\hat{j} \quad \left| \begin{array}{l} 25a^2 = 1 \\ a^2 = \frac{1}{25} \\ a = \frac{1}{5} \end{array} \right.$$

$$\sqrt{(3a)^2 + (4a)^2} = 1$$

$$9a^2 + 16a^2 = 1$$

$$\textcircled{23} 2\vec{c} = 6\hat{i} + 8\hat{j} \quad \left| \begin{array}{l} 4\vec{c} = 12\hat{i} + 16\hat{j} \end{array} \right.$$

$$\begin{aligned} A_x - b_x &= 6 \\ A_y - b_y &= 8 \end{aligned} \quad \left| \begin{array}{l} A_x + b_x = 12 \\ A_y + b_y = 16 \end{array} \right.$$

$$\begin{aligned} A_x - b_x &= 6 \\ A_x + b_x &= 12 \end{aligned} \rightarrow 2A_x = 18 \quad \left| \begin{array}{l} A_x = 9 \\ 9 - b_x = 6 \\ b_x = 3 \end{array} \right.$$

$$\text{E } A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 1 & -7 & 3 \end{vmatrix} = (-9-14)\hat{i} - (3-2)\hat{j} + (-7-3)\hat{k} \\ = -23\hat{i} - \hat{j} - 10\hat{k}$$

$$|A \times B| = \sqrt{42} \quad |A| = \sqrt{14} \quad |B| = \sqrt{59}$$

$$\sin\left(\frac{\sqrt{42}}{\sqrt{14}\sqrt{59}}\right)^{-1} = 13$$

$$\text{26 } A = -\hat{i} + 2\hat{j} - 5\hat{k} \quad B = 2\hat{i} + 3\hat{j} - 2\hat{k} \quad \text{①}$$

$$|A| = \sqrt{1+4+25} = \sqrt{30} \quad |B| = \sqrt{4+9+4} = \sqrt{17}$$

$$\vec{A} + \vec{B} = \vec{C} \\ \vec{C} = -\hat{i} + 5\hat{j} - 7\hat{k}$$

$$C = \sqrt{1+25+49} = \sqrt{75}$$

$$\vec{A} \cdot \vec{B} = (-1 \times 2) + (2 \times 3) + (-5 \times -2) = 14$$

$$|A| = \sqrt{30} \quad |B| = \sqrt{17}$$

$$\cos\left(\frac{14}{\sqrt{30}\sqrt{17}}\right)^{-1} = 51.7$$

$$\text{27 } A \cdot B = (3 \times 1) + (-1 \times -1) + (2 \times 1) = 4$$

$$|A| = \sqrt{14} \quad |B| = \sqrt{3}$$

$$\cos\left(\frac{4}{\sqrt{14}\sqrt{3}}\right)^{-1} \Rightarrow \Theta = 51.88$$

25

$$\text{a) } A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -1 \\ 2 & -1 & 3 \end{vmatrix} = (-6-1)\hat{i} - (3-2)\hat{j} + (-1-4)\hat{k} \\ = -7\hat{i} - \hat{j} - 5\hat{k}$$

$$|A \times B| = \sqrt{83} \quad |A| = \sqrt{6} \quad |B| = \sqrt{14}$$

$$\sin\left(\frac{\sqrt{83}}{\sqrt{6}\sqrt{14}}\right)^{-1} = 83.7$$

$$\text{b) } A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 11 \\ 5 & -1 & 2 \end{vmatrix} = (-14-11)\hat{i} - (4-55)\hat{j} + (-2+35)\hat{k} \\ = -25\hat{i} + 51\hat{j} + 33\hat{k}$$

$$|A \times B| = 3\sqrt{111} \quad |A| = \sqrt{174} \quad |B| = \sqrt{30}$$

$$\sin\left(\frac{3\sqrt{111}}{\sqrt{174}\sqrt{30}}\right)^{-1} = 57.33$$

$$[28] A = -1 + 2j - 5k \quad B = 2i + 3j - 2k$$

$$\otimes A \cdot B = (-1 \times 2) + (2 \times 3) + (-2 \times -5) = 14$$

$$\otimes A \times B = \begin{vmatrix} i & j & k \\ -1 & 2 & -5 \\ 2 & 3 & -2 \end{vmatrix} = (-4-15)i - (2-10)j + (-3-4)k = 11i - 12j - 7k$$

$$\otimes |A| = \sqrt{1+4+25} = \sqrt{30}$$

$$\hat{A} = \frac{1}{\sqrt{3}} \times (-i + 2j - 5k)$$

$$\otimes |A| = \sqrt{30} \quad |B| = \sqrt{4+9+4} = \sqrt{17}$$

$$\cos\left(\frac{14}{\sqrt{30} \times \sqrt{17}}\right)^{-1} = 51.7$$

$$\times \vec{A} + \vec{B} = \vec{C}$$

$$= i + 5j - 7k$$

$$|C| = \sqrt{1+25+49} = 5\sqrt{3}$$

$$A \times B = \begin{vmatrix} i & j & k \\ -1 & 2 & -5 \\ 2 & 3 & -2 \end{vmatrix} = (-4-15)i - (2-10)j + (-3-4)k = 11i - 12j - 7k$$

$$|A \times B| = \sqrt{409} \quad |A| = \sqrt{30} \quad B = \sqrt{17}$$

$$\sin\left(\frac{\sqrt{409}}{\sqrt{30} \times \sqrt{17}}\right)^{-1} = 63.57$$

$$[27] A = 2i - 5j \quad B = 5i + 2j$$

$$\otimes 2 \times 5 - 5 \times 2 = 0 \quad 2 \times 2 - 5 \times 5 = -21$$

$$A \cdot B = (2 \times 5) + (-5 \times 2) = 0$$

$$A \times B = \begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 5 & 2 & 0 \end{vmatrix} = (4-25)k = -21k$$

$$\otimes A = 2i - 5j \quad B = 4i - 10j$$

$$A \cdot B = (2 \times 4) + (-5 \times -10) = 58$$

$$A \times B = \begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 4 & -10 & 0 \end{vmatrix} = (-20-20)i = -40i$$

$$\boxed{31} \quad \vec{A} \times \vec{B} = \vec{C} \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 2 \\ 3 & 1 & 2 \end{vmatrix} = (-4)\hat{i} - (6)\hat{j} + (-9+8)\hat{k} \\ = -4\hat{i} - 6\hat{j} - \hat{k}$$

$$|\vec{C}| = \sqrt{16+36+1} = \sqrt{53}$$

$$\hat{C} = \frac{1}{\sqrt{53}} \times (-4\hat{i} - 6\hat{j} - \hat{k})$$

$$\boxed{32} \quad \vec{A} \cdot \vec{B} = 7i + 10j + 2k$$

$$\vec{A} \cdot \vec{B} = (1 \times 7) + (1 \times 10) + (1 \times 2) = 19$$

$$\boxed{29} \quad \vec{D} = \vec{A} + \vec{B} \quad \vec{A} = 1\hat{i} + 2\hat{j} \quad \vec{B} = 1 - 2\hat{k} \quad \vec{C} = -2\hat{j} + 2\hat{k} \\ \vec{D} = 2\hat{j} - 2\hat{k} \\ \vec{T} = \vec{A} - \vec{C} \\ = -1\hat{i} + 2\hat{j} + 2\hat{j} - 2\hat{k} = -1\hat{i} + 4\hat{j} - 2\hat{k}$$

$$D.T = (1 \times 0) + (4 \times 2) + (-2 \times -2) = 12$$

$$|\vec{T}| = \sqrt{21} \quad |\vec{D}| = 2\sqrt{2}$$

$$\cos\left(\frac{12}{\sqrt{21} \times 2\sqrt{2}}\right) = \frac{1}{2}$$

$$* \vec{E} = \vec{A} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ -1 & 2 & 2 \end{vmatrix} = (4-0)\hat{i} - (-2)\hat{j} + (2)\hat{k} \\ = 4\hat{i} + 2\hat{j} + 2\hat{k}$$

$$* \vec{F} = \vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -2 \\ 0 & -2 & 2 \end{vmatrix} = (0+4)\hat{i} - (2-0)\hat{j} + (-2)\hat{k} \\ = 4\hat{i} - 2\hat{j} - 2\hat{k}$$

$$E = \sqrt{16+4+4} = \sqrt{24} \quad \hat{E} = \frac{1}{\sqrt{24}} \times (4\hat{i} + 2\hat{j} + 2\hat{k})$$

2-6 $\Sigma t = \text{zero}$ $t_1 = t_2$
 $t_1 - t_2 = \text{zero}$ $v_1 F_1 = v_2 F_2$
 $1 \times 800 = v_2 \times 350$
 $v_2 = 2.29$

$\Sigma N = 40 + 350 + 800 = 1190 \text{ Newton}$

2-1 $\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 0 \\ 2 & 3 & 0 \end{vmatrix} = (0)\hat{i} - (0)\hat{j} + (12-10)\hat{k}$
 $= 2\hat{k} \text{ N.m}$

2-2 $\Sigma \tau = (10 \times 0.25) + (9 \times -0.25) + (12 \times 0.91)$
 $= -39.55 \text{ c.w}$

2-3 $\Sigma F_x = T_2 \cos(53) - T_1 \cos(37) = 0$
 $\Sigma F_y = T_2 \sin(53) + T_1 \sin(37) = 125$
 $0.96T_2 - 0.98T_1 = 0 \times \frac{3}{4}$
 $0.98T_2 + 0.96T_1 = 125$
 $0.945T_2 - 0.96T_1 = 0$
 $0.98T_2 + 0.96T_1 = 125$
 $1.25T_2 = 125$
 $T_2 = 100 \text{ N}$
 $T_1 = 75.93$

المفاصل (7)

The combination of bones, joints, muscles and related connective tissues are known as the musculoskeletal system.

يصف مزيج العظام والمفاصل والعضلات والأنسجة الضامة الربطية بالجهاز العضلي الهيكلي.

* Joints /

where the bones come together (and with power of the muscles) give a variety of range of motion (R.O.M) joints, classifications consist of 360 joints

Example

$\Sigma F_A = \Sigma F_B = \Sigma F_y = \text{zero}$

$\Sigma F_x = F_B + F_A \cos \theta = \text{zero}$

$\Sigma F_y = F_A \sin(120) - 2000 = \text{zero}$

$F_A = \frac{2000}{\sin(120)} = 2309.4$

$F_B = 1154.7$

2-7

$$\sum T_E = 0$$

$$(T \times 0,05) - (12 \times 0,15) - (12 \times 0,20) = 0$$

$$T = \frac{(12 \times 0,15) + (12 \times 0,35)}{0,05} = 120 \text{ N}$$

$$\sum T_T = 0$$

$$(E \times 0,05) - (12 \times 0,10) - (12 \times 0,30) = 0$$

$$E = \frac{(12 \times 0,10) + (12 \times 0,30)}{0,05} = 96 \text{ N}$$

الرمز دائماً كليتيك

$$F = Gm^2/r^2 \quad G = \frac{N \cdot m^2}{kg^{-2}}$$

2-4

$$\sum T = F_y \cos(\theta) - F_x \sin(\theta) + F_g \frac{1}{2} \cos(\theta)$$

2-5

$$F_x = A_x - T \cos \theta = \text{zero}$$

$$F_y = A_y + T \sin \theta - w_1 - w_2 = \text{zero}$$

$$\sum T = -400 \times 4 - 600 \times 2 + T \sin \theta \times 4 = \text{zero}$$

$$\sin \theta = \frac{3}{5} \quad \rightarrow \theta = 36,87^\circ$$

$$T = 1166,67$$

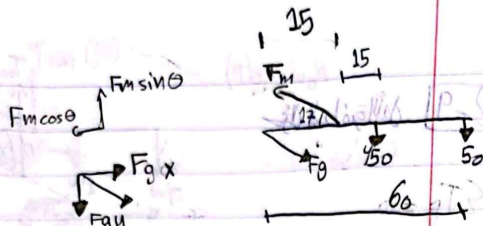
$$A_x = 1166,67 \times \frac{4}{5} = 933,336$$

$$A_y = w_1 + w_2 - T \sin \theta = 400 + 600 - 1166,67 \times \frac{3}{5} = 300 \text{ N}$$

$$A = \sqrt{(933,336)^2 + 300^2} = 1204,62 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{300}{933,336} \right) = 14,42^\circ$$

2-8



$$\sum F_y = 0$$

$$F_m \sin(17) - (50 \times 15) - (50 \times 45) = 0$$

$$F_m = 991.888$$

$$\sum F_x = 0$$

$$F_{gx} - (991.888) \cos(17) = 0$$

$$F_{gx} = 948.547 \text{ N}$$

$$\sum F_y = 0$$

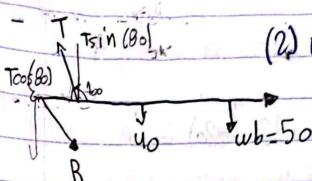
$$991.888 \sin(17) - F_y - 50 - 50 = 0$$

$$F_y = 194.999$$

$$F = \sqrt{(948.547)^2 + (194.999)^2} = 968.338$$

$$\theta = \tan^{-1}\left(\frac{194.999}{948.547}\right) = 11.96$$

Example 2 (1) عرّف



$$\sum \tau = 0$$

$$T \sin(80) \times 15 - (40 \times 15) - (50 \times 35) = 0$$

$$T = 795 \text{ N}$$

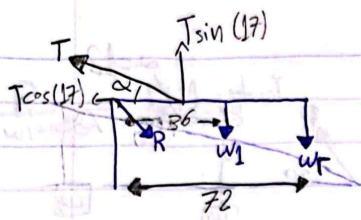
$$\sum F_x = 0$$

$$R_x - T \cos(80) = 0 \rightarrow R_x = 138$$

$$T \sin(80) - 40 - 50 = R_y \rightarrow R_y = 693$$

$$R = \sqrt{(138)^2 + (693)^2} = 706.16$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{693}{138}\right) = 78.5 \text{ degrees}$$



(9) الماكينة

$$\alpha = 17$$

ال

$$\sum \tau_R = 0$$

$$18T \sin(17) - (60 \times 36) - (70 \times 72) = 0$$

$$T = 1368$$

$$\sum F_x = 0$$

$$R_x - T \cos(17) = 0 \rightarrow R_x = 1308,22$$

$$\sum F_y = 0$$

$$T \sin(17) - 130 = R_y \rightarrow R_y = 270$$

$$R = \sqrt{(1308,22)^2 + (270)^2} = 1335,8$$

$$\theta = \tan^{-1}\left(\frac{270}{1308,22}\right) = 11,66$$

2-9 الماكينة

$$\sum \tau_R = 0$$

$$T_m = (30 \times 0,15) + (50 \times 0,35)$$

$$0,04$$

$$T_m = 550$$

$$\otimes T_m + F_j = w_1 + w_2$$

$$F_j = 30 + 50 - 550 = 470$$

الماكينة (2) الماكينة

$$\sum \tau_R = 0 \rightarrow 4T \sin(130) - 0,14 \times 40 \sin(50) - 0,2 \times 50 \sin(50)$$

$$130 \quad 130$$

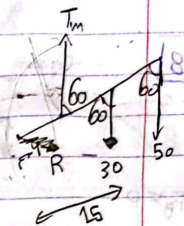
$$T = 515$$

$$\sum F_x = 0 \rightarrow R_x = 0$$

$$\sum F_y = 0$$

$$-R_y - 50 - 40 + T = 0$$

$$R_y = 425$$

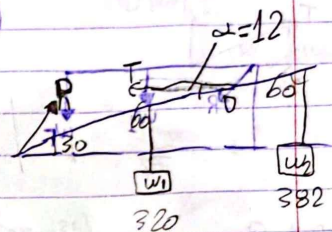


$$\frac{R}{w_1 + w_2} = 1,5, 1 \text{ gleich}$$

$$AB = 78 \text{ cm} \quad 1/6 \text{ (circled)}$$

$$AD = \frac{2}{3} AB = 52 \text{ cm}$$

$$AE = 39 \text{ cm}$$



$$\sum T_R = 0$$

$$0 = T \times 52 \times \sin(12) - 320 \times 39 \times \sin(60) - 382 \times 78 \times \sin(60)$$

$$T = 3386 \text{ N}$$

$$\sum F_x = 0$$

$$R_x - T \cos(12) = 0$$

$$R_x = 3312$$

$$R_y = T \sin(12) + 702$$

$$R_y = 1405$$

$$R = \sqrt{R_y^2 + R_x^2} = 3597,7$$

$$\theta = 22,9$$

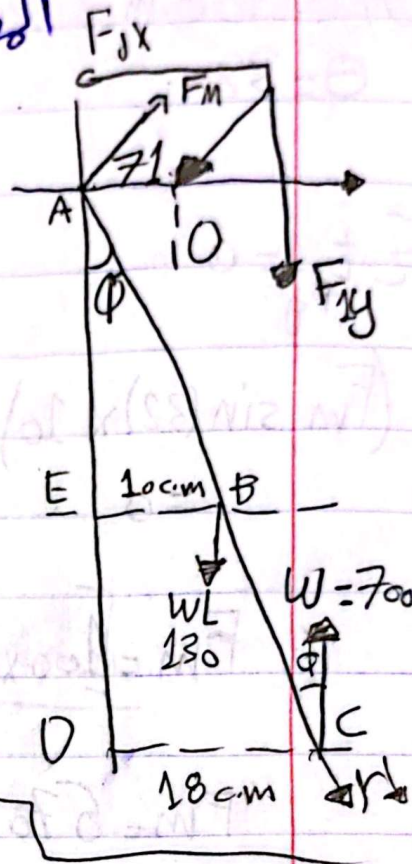
$$\frac{DC}{AC} = \sin \theta$$

$$AC = \frac{DC}{\sin \theta}$$

$$\frac{EB}{AB} = \sin \theta$$

$$F_m \sin(71) \quad F_m \cos(71)$$

$$\sum \tau_A = -F_{jy}(AO) \sin \theta - 130(AB) \sin \theta + 700(AC) \sin \theta = 0$$



$$F_{jy} = \frac{700(18) - (10)(130)}{7} = 1614 \text{ N}$$

$$\sum F_x = 0$$

$$-F_{jx} + F_m \cos(71)$$

$$F_{jx} = F_m \cos(71)$$

$$\sum F_y = 0$$

$$F_m \sin(71) + W - F_{jy} - WL = 0$$

$$F_{jy} = F_m \sin(71) + 570$$

$$F_m = 1104 \text{ N}$$

$$F_{jx} = 360 \text{ N}$$

$$F = \sqrt{F_{jx}^2 + F_{jy}^2}$$

$$\theta = 77.4^\circ$$

Example

$$\sum F_x = 0$$

$$W_2 \cos(\alpha) = 110 \cos(40) \quad \text{--- (1)}$$

$$\sum F_y = 0$$

$$W_2 \sin(\alpha) = 220 - 110 \sin(40)$$

$$W_x = 84.3 \text{ N} \quad W_y = 149.3$$

$$W = \sqrt{(149.3)^2 + (84.3)^2} = 171.45$$

$$\theta = 60.54$$

$$2-10 \quad \sum \tau = 0$$

$$-500 \times 0.18 \times \sin(70) + T \times 0.907 = 0$$

$$T = 1208 \text{ N}$$

$$\sum F_x = R \sin(15) - T \sin(20) = 0$$

$$R_x = 413.16$$

$$\sum F_y = 500 + T \cos(20) - R \cos(15) = 0$$

$$R_y = 1635$$

آخر سؤال في الالات

المثال الأول

$$OA = 10 \text{ cm}$$

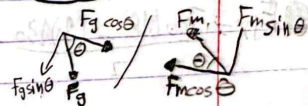
$$\theta = 32$$

$$\sum \tau_{F_g} = 0$$

$$(F_m \sin(32) \times 10) - 100 \times 17 \times \sin(120) - 75 \times 32 \times \sin(120) = 0$$

$$F_m = \frac{[100 \times 17 \sin(120)] + [75 \times 32 \sin(120)]}{10 \sin(32)}$$

$$F_m = 670 \text{ N}$$



$$\sum F_x = 0$$

$$F_{gx} = F_m \cos \theta$$

$$F_{gx} = 568.2$$

$$\sum F_y = 0$$

$$F_m \sin \theta - F_{gy} - 175 = 0$$

$$F_{gy} = 180$$

$$F = \sqrt{(180)^2 + (568.2)^2} = 596$$

$$\theta = 17.5$$

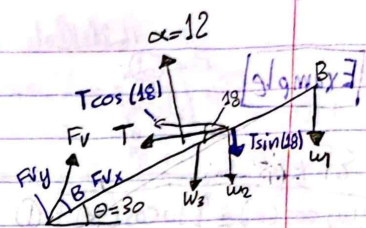
① $T = rF \sin \theta = 55 \times 0.850 = 46.75$

② $T_1 = T_2$
 $r_1 F_1 = r_2 F_2$
 $\frac{r_1}{r_2} = \frac{F_2}{F_1}$
 $\frac{0.96}{0.45} = \frac{F_2}{17.5}$
 $F = 23.33$

③ $F_1 = 200$ / $F_2 = 300$
 $r_1 = ??$ / $r_2 = 3$
 $200 r_1 = 300 r_2$
 $r_1 = 1.5 r_2$
 $2.5 r_1 = 3$
 $r_1 = 1.2$

④ $\Sigma F_x = 0$
 $20 \cos(30) = F_x \rightarrow F_x = 10\sqrt{3}$
 $\Sigma F_y = 0 \rightarrow 20 \sin(30) - 70 = F_y = 0$
 $F = 10\sqrt{39}$
 $\theta = -73.9$

2-11



$\Sigma F_x = 0$
 $F_{vx} - T \cos(12) = 0 \rightarrow F_{vx} = T \cos(12)$

$\Sigma F_y = 0$
 $F_{vy} = T \sin(12) + W_1 + W_2 + W_3$

$\Sigma \tau = 0$
 $(T \sin(12) \times 0.48) - \sin(60) \times [0.907 \times 0.72 + 0.48 \times 0.12 + 0.16 \times 0.36]$
 $T = 2.37 W$

$F_{vx} = 2.25$ / $F_{vy} = 1.38$
 $F = 2.6$ / $B = 31.5$

2-5

$$\boxed{7} \quad \Sigma \tau_F = 0$$

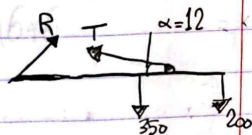
$$(T \sin(70) \times 0.95) - (12 \times 0.15 \times \sin(110)) - (100 \times 0.35 \times \sin(110))$$

$$T = 736$$

$$\boxed{8} \quad \Sigma \tau_F = 0$$

$$(T \sin(70) \times 0.97) - (12 \times 0.15 \times \sin(110)) - (100 \times 0.35 \times \sin(110))$$

$$T = 525.97$$



$\boxed{9}$

$$\boxed{10} \quad \Sigma \tau_R = 0$$

$$T \times 1.5 \times \sin(12) - 350 \times \frac{L}{2} - 200 \times L = 0$$

$$T = 1202$$

$$R_y / R_x$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\Theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

$\boxed{5}$

$$\Sigma F_x = 0$$

$$F_x - F_m \cos(45) = 0$$

$$F_x = F_m \cos(45) \quad \text{--- (1)}$$

$$\Sigma F_y = 0$$

$$F_h + F_y - F_m \sin(45) = 0$$

$$F_y = F_m \sin(45) - 375 \quad \text{--- (2)}$$

$$\Sigma \tau_F = 0$$

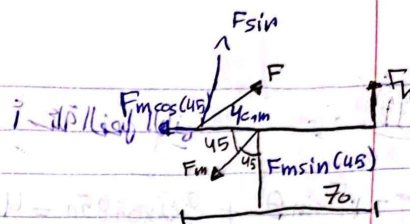
$$(375 \times 0.70) - (F_m \sin(45) \times 0.94) = 0$$

$$F_m = 9280$$

$$\boxed{6} \quad \Sigma \tau_F = 0$$

$$(0.05 \times 500 \times \sin 60) - (30 \times 0.15) \times (w_2 \times 0.35) = 0$$

$$w = 58.57$$



13 $\sum \tau_j = 0$

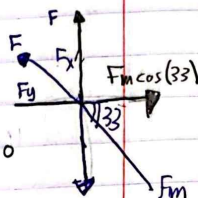
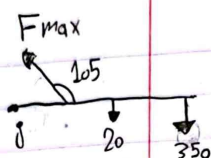
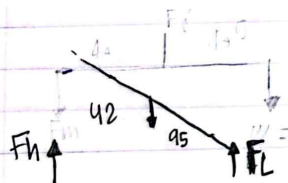
$$F \times 0.5 \sin(165) - (20 \times 0.15) - (350 \times 0.35) = 0$$

$$F = 2598.5$$

14 $\sum F_x = 0 \rightarrow F_x + F \cos(33) = 0$
 $\sum F_y = 0 \rightarrow -F \sin(33) - F_y - 50 = 0$
 $F_y = 82.67 / F_x = -50$

$$F = \sqrt{F_y^2 + F_x^2} = 96.6$$

$$\theta = 30.1516$$



11 $\sin \theta = \frac{20}{AC} \quad \left\{ \begin{array}{l} \sin \theta = \frac{38}{AE} \end{array} \right.$

$$\sum \tau_A = 0$$

$$r F \sin \theta - 40 \times (AC) \sin \theta - 100 \times (AE) \sin \theta = 0$$

$$r F \sin \theta - (800) - 3800 = 0$$

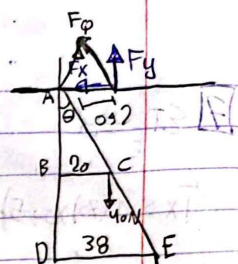
$$2 F_y = 4600$$

$$F_y = 2300$$

$$\sum F_x = 0$$

12 $\sum \tau_R = 0$

$$5P/10$$



P

Q

$$\boxed{3-5} \quad Y = 2 \times 10^6 / L = 3 / A = 0.5 \times 10^{-4} / \Delta L = 1 \times 10^{-3}$$

$$\epsilon = \frac{Y A \Delta L^2}{2L} = 0.0017$$

$$\boxed{3-6} \quad \text{قوة}$$

$$\boxed{3-7} \quad r = 0.04 \times 10^{-3} / L = 16 \times 10^{-2} / \Delta L = 4 \times 10^{-3} / F = 400 \text{ N}$$

$$\epsilon = \frac{1}{2} F \Delta L = 0.08 \text{ J}$$

$$\boxed{3-8} \quad r = 0.1 / mg = 9800 / \sigma = ?? / \epsilon = ??$$

$$\boxed{a} \quad \sigma = \frac{F}{A} = \frac{9800}{\pi (0.001)^2} = 3.12 \times 10^7$$

$$\boxed{b} \quad \epsilon = \frac{\sigma}{E} \rightarrow \epsilon = \frac{\sigma}{E} = 3.71 \times 10^7$$

$$\boxed{c} \quad \text{القوة المطبقة}$$

حل المسألة (3)

$$\boxed{3-1} \quad A = 0.015 / F = 15500$$

$$\boxed{a} \rightarrow \sigma = \frac{F}{A} = 1 \times 10^5 \text{ N/m}^2$$

$$\boxed{b} \rightarrow Y = \frac{\sigma}{\epsilon} \rightarrow \epsilon = \frac{\sigma}{Y} = \frac{1 \times 10^5}{200 \times 10^9} = 5 \times 10^{-7}$$

$$\boxed{c} \quad \epsilon = \frac{\Delta L}{L} \rightarrow \Delta L = L \epsilon = 0.05 \times 5 \times 10^{-7} = 4.75 \times 10^{-6}$$

$$\boxed{3-2} \quad L = 15 \times 10^{-2} / \Delta L = 3.7 \times 10^{-3} / F = 13.4 / \text{قوة}$$

$$\boxed{3-3} \quad F = 250 / r = 0.5 \times 10^{-3} / Y = 5 \times 10^9 \quad \Delta L = ?$$

$$Y = \frac{\sigma}{\epsilon} \rightarrow \epsilon = \frac{\sigma}{Y} = 0.0636$$

$$\boxed{3-4} \quad \epsilon = \frac{1}{2} F \Delta L$$

$$= 0.08 \text{ J}$$

$$r = 0.08 \times 10^{-3} / L = 16 \times 10^{-2} / F = 400 / \Delta L = 4 \times 10^{-3}$$

$$[3] \Delta L = ?? \quad L = 38 \times 10^{-2} \quad r = 2.1 \times 10^{-2} \quad Y = 9 \times 10^9$$

$$\Delta L = \frac{FL}{YA} = 4.5 \times 10^{-5}$$

$$[4] Y = 1.5 \times 10^{10} \quad r = 1.25 \times 10^{-2} \quad \sigma_B = 1.5 \times 10^8$$

$$[5] \sigma = \frac{F}{A} \rightarrow F = A \times \sigma = 7.36 \times 10^4$$

$$[6] Y = \frac{FL}{A \Delta L} \rightarrow \Delta L = \frac{FL}{YA} = 2.5 \text{ mm}$$

$$[5] F = 4630 \quad L = 610 \quad r = 0.150 \quad Y = 2.1 \times 10^{11}$$

$$\Delta L = \frac{FL}{AY} = 1.9 \times 10^{-4} \quad g = 9.8$$

$$[6] F = 6 \times A \quad F = 0.2$$

$$\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2} \quad \frac{0.92}{20} = \frac{(1 \times 10^{-3})^2}{r_2^2}$$

$$r = 0.01$$

$$[3-9] \frac{\Delta V}{V} = \frac{0.1}{100}$$

$$B = \frac{\Delta P}{\Delta V/V} \rightarrow \Delta P = 9 \times 10^6$$

3 الـ 10 في 1

$$[1] L = 4 \text{ m} \quad A = 0.2 \times 10^{-4} \quad Y = 8 \times 10^{10} \quad F = 2000 \quad \Delta L$$

$$Y = \frac{FL}{A \Delta L} \rightarrow \Delta L = \frac{FL}{YA} \quad \Delta L = 5 \times 10^{-3}$$

$$[2] L = 35 \times 10^{-2} \quad r = 1.8 \times 10^{-2} \quad m = 60 \quad F = 600 \text{ N} \quad 3F = 1764 \quad Y = 1.5 \times 10^{10}$$

$$\Delta L = ??$$

$$\frac{F}{P_{\text{old}}} = 882$$

$$Y = \frac{FL}{A \Delta L} \rightarrow \Delta L = \frac{FL}{YA} \rightarrow \Delta L = 1.9 \times 10^{-5}$$

$$[11] L = 1,2 \text{ m} \quad / \quad \epsilon = 1,3 \times 10^{-4} \quad \Delta L = ??$$

$$\epsilon = \frac{\Delta L}{L} \rightarrow \Delta L = \epsilon L = 1,56 \times 10^{-4}$$

$$[12] A = 10^{-3} \quad / \quad L = 0,4 \text{ m} \quad F = k \Delta L$$

$$Y = \frac{FL}{A \Delta L} \rightarrow Y = \frac{k \Delta L L}{A \Delta L} \rightarrow Y = \frac{kL}{A}$$

$$k = \frac{YA}{L} \rightarrow k = 3,75 \times 10^7$$

$$[13] Y = \frac{F/A}{\Delta L/L} \rightarrow F = \frac{Y \Delta L A}{L} = 706 \text{ N}$$

$$[14] A = 10^{-3} \quad / \quad L = 0,4 \quad / \quad F = 750 \quad / \quad \Delta L$$

$$Y = \frac{FL}{A \Delta L} \rightarrow \Delta L = \frac{FL}{AY} = 6,967 \times 10^{-6}$$

[b]

[c]

[7]

$$\sigma = 1,5 \times 10^8 \quad / \quad F = 100$$

$$A = \frac{F}{\sigma} = 6,667 \times 10^{-7}$$

$$A = \pi r^2$$

$$r = 4,6 \times 10^{-4}$$

$$d = 2r = 9,2 \times 10^{-4}$$

$$[8] L = 1,8 \quad / \quad \Delta L = 9 \times 10^{-3} \quad / \quad F = 3800$$

$$Y = 20 \times 10^{10}$$

$$Y = \frac{\sigma}{\epsilon} \rightarrow Y = \frac{FL}{A \Delta L} \rightarrow A = \frac{FL}{Y \Delta L}$$

$$A = 3,8 \times 10^{-5}$$

$$A = \pi r^2$$

$$r = 3,948$$

$$[9] L = 50 \text{ m} \quad / \quad r = 5 \times 10^{-3} \quad / \quad F = 900 \text{ N} \quad / \quad \Delta L = 1,6$$

$$Y = \frac{FL}{A \Delta L} \rightarrow Y = 3,58 \times 10^8$$

$$[10] r_{in} = 0,02 \quad / \quad r_{out} = 0,023 \quad / \quad \sigma = 5 \times 10^7 \quad / \quad F = ??$$

$$\sigma = \frac{F}{A} \rightarrow F = \sigma A = 2,9026 \times 10^4 \text{ N}$$

$$19) F = 500 / l = 1 / A = 4 \times 10^{-4} \quad \text{[189]}$$

$$\epsilon = \frac{A \epsilon B^2}{2V} = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{A \epsilon B^2}{m Y}} = 2939$$

$$20) \Delta P = 1 \times 10^7 \quad 1 - \uparrow = \epsilon \quad \rho_{\text{new}} = \frac{\text{mass}}{\text{new Volume}}$$

$$\Delta V = \frac{\Delta P V_0}{B} = 4.93 \times 10^{-3}$$

$$B = 2.34 \times 10^9$$

$$21) \sigma = 4 \times 10^8 / r = 0.95 \times 10^{-2}$$

$$a) \Rightarrow F = \pi \left(\frac{10^{-2}}{2} \text{ m} \right)^2 \times 4 \times 10^8 = \pi \times 10^4 \text{ N}$$

$$b) \Rightarrow F = \sigma B A$$

$$0.95 \times 10^{-2} \times 2\pi \times \frac{10^{-2}}{2} \times 4 \times 10^8 = 2\pi \times 10^4$$

$$15) L = 10 / r = 2 \times 10^{-2} / F = 900 / s = 2.5 \times 10^{-10} \quad \text{[11]}$$

$$S = \frac{F A}{\Delta x} \rightarrow \frac{\Delta x}{L} S = \frac{F}{A} \rightarrow \Delta x = \frac{F L}{A S} = 5.93 \times 10^{-5}$$

$$16) F = 500 / r = 2 \times 10^{-2} / h = 3 \times 10^{-2} / s = 8 \times 10^{-10} \quad \text{[51]}$$

$$S = \frac{F h}{A \Delta x} \rightarrow \Delta x = \frac{F h}{S A} = 1.5 \times 10^{-7}$$

$$17) F = 600 / \epsilon = 1 \times 10^9 / h = 0.97 \times 10^{-2} / r = 2 \times 10^{-2}$$

$$\Delta x = ?? \quad s = 1 \times 10^9 \quad \text{[51]}$$

$$\Delta x = \frac{F h}{S A} = 3.94 \times 10^{-6} \quad \text{[11]}$$

$$18) \frac{\Delta V}{V_0} = 2 \times 10^{-3} / B = 1.8 \times 10^9$$

$$B = \frac{F A}{\Delta V} \rightarrow \frac{F}{A} = B \frac{\Delta V}{V}$$

$$\frac{F}{A} = 3.46 \times 10^6$$

$$\boxed{22} B = 2 \times 10^9$$

$$\Delta P = -2 \times 10^9 \times \frac{0.004}{1.009} = -0.165 \times 10^9$$

$$\frac{220 \text{ mm}}{\text{mm}} = 220 \text{ mm}$$

$$\frac{0.004}{1.009}$$

شايتر (3) مضطرة (2)

3.6

$$L = 90 \times 10^{-2} \quad / \quad A = 6 \times 10^{-4} \quad / \quad m = 70 \quad / \quad h = 60 \times 10^{-2}$$

$$Y = 14 \times 10^9 \quad \sigma_B = 10^8$$

أقصى طاقة

$$\epsilon = \frac{AL \sigma_B^2}{2Y}$$

$$\epsilon = 192,86$$

الطاقة الناتجة
عن القفز

كلما القدميه

$$\epsilon = mgh$$

$$= 420 \text{ J}$$

الطاقة أكبر بكثير من أقصى حد وبالتالي احتمالية الكسر كبيرة جداً

القدم الواحدة = 210 J

Example

$$m = 50 \quad / \quad L = 1 \quad / \quad A = 4 \times 10^{-4} \quad / \quad \sigma_B = 10^8$$

$$VC = \text{kinetic energy} = \frac{1}{2} m v^2 = \frac{1}{2} \times 50 \times v^2$$

الطاقة
الاحتمالية

$$\epsilon = \frac{AL \sigma_B^2}{2Y} = 192,85 \text{ J}$$

$$25 v^2 = 192,85$$

$$v = 2,79$$

Example $F = 20$ / $A = 14 \times 10^{-4}$ / $h = 5 \times 10^{-3}$ / $S = 3 \times 10^6$ 2.5

$$S = \frac{F/A}{\Delta x/h} \rightarrow \Delta x = \frac{Fh}{As} = 294 \times 10^{-5} \text{ m}$$

Example

* حل المسألة الرابع *

$$\boxed{4-1} \quad T_c = \frac{5}{9} (T_c - 32) \rightarrow 9T_c = 5T_c - 160$$

$$4T_c = -160$$

$$\boxed{T_c = -40}$$

$$\boxed{4-2} \quad n = 0.1 \quad / \quad V = 1 \quad / \quad T = 273 \text{ K} \quad / \quad R = 8.314$$

$$\boxed{a} \quad PV = nRT$$

$$P = 2.27 \times 10^5$$

$$PV = nRT \Rightarrow nRT = \frac{Pm}{\rho} \quad / \quad n = \frac{m}{M_w}$$

$$\frac{m}{M_w} RT = \frac{Pm}{\rho} \quad \rho = \frac{P M_w}{RT} = 3.92 \text{ Kg/m}^3$$

$$\boxed{4-3} \quad V_1 = 15 \quad / \quad P_1 = 2 \quad / \quad V_2 = 12 \quad / \quad P_2 = 3.95$$

$$T_1 = 300$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \frac{P_2 V_2 T_1}{P_1 V_1} = T_2$$

$$T_2 = 420 \text{ K}$$

$$\boxed{4-8} \quad \alpha = 130 \times 10^{-6} / T = 20 / r_1 = 2.2 \times 10^{-2} \quad (S-N)$$

$$r_2 = 2.21 \times 10^{-2}$$

$$\frac{\Delta A}{A_0} = 2 \alpha \Delta T \quad \left\{ \begin{array}{l} A_0 = 15.21 \\ A = 16.35 \end{array} \right. \quad (S-N)$$

$$\Delta T = \frac{\Delta A}{2 \alpha A_0}$$

$$\boxed{4-9} \quad T_p = 20 + \frac{0.014}{2 \times (130 \times 10^{-6}) \times (15.21)} \quad (S-N)$$

$$T_p = 55.4$$

$$\boxed{4-9} \quad L_0 = 30 \quad r = \frac{1.5}{2} \quad \Delta T = 65$$

$$\alpha = 9 \times 10^{-6}$$

$$\Delta L = L \alpha \Delta T = 0.01755 \text{ cm}$$

$$\Delta A = 2 \alpha A_0 \Delta T = 2.406 \times 10^{-6}$$

$$\Delta V = 3 \alpha (A_0 L_0) \Delta T = 0.00904 \text{ cm}^3$$

$$\boxed{4-9} \quad P_1 = 6 \text{ bar} / T_1 = 17.4 \text{ bar} \quad \left(\frac{P_1}{T_1} = \frac{P_2}{T_2} \right)$$

$$T_2 = 280 \text{ K}$$

$$P_2 = 3 \text{ bar} \quad \rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$T_2 = 840 \text{ K} \quad (S-N)$$

$$\boxed{4-5} \quad V = 0.3 / n = 2 / T = 20 \quad (S-N)$$

$$V_{\text{av}} = \frac{3}{2} V_{\text{BT}}$$

$$\boxed{4-6} \quad \Delta L = L \alpha \Delta T = 0.00327$$

$$\boxed{4-7} \quad \Delta T = 40 \quad \alpha = 11 \times 10^{-6}$$

$$\Delta L = L_0 \alpha \Delta T \rightarrow \Delta L = 0.0013$$

$$\text{a) } \rightarrow 30 + 0.0013 = 30.0013$$

$$\text{b) } \sigma = \frac{Y \Delta L}{L_0} = \frac{200 \times 10^9 \times 0.0013}{30} \quad (S-N)$$

$$= 8.97 \times 10^7$$

Problem

$$1 \quad C = -195,81 \quad \times -320,458 \quad F \quad \times 77,34$$

$$2 \quad \Delta T_F = \frac{q}{\dot{m}} \Delta T_C \quad \left(\times 450 \right)$$

$$= 810$$

$$3 \quad \bar{T}$$

$$4 \quad V_1 = 100 \quad R = P_2$$

$$T_1 = 273 \quad V_2 = 87,2$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \rightarrow T_2 = \frac{T_1 V_2}{V_1} = 238,05 \text{ K}$$

$$5 \quad PV = nRT$$

$$\rho = \frac{PM}{RT}$$

$$\rho = \frac{PM}{RT}$$

$$4-10 \quad C_X = \frac{m_w c_w (T_F - T_i)}{m_X (T_i - T_F)} =$$

$$4-11 \quad C_h = 91 / C_c = 12 \quad \bar{T}$$

$$4-12 \quad L = 12 \times 10^{-3} / A = 1,5 \times 10^{-2} / \Delta T = 2$$

$$4-13 \quad \bar{T}$$

$$4-14$$

$$4-15$$

$$P-N$$

$$9) \quad r = 2 \times 10^{-2} / \Delta T = 70 / Y = 20,6 \times 10^{-10}$$

$$\alpha = 11 \times 10^{-6}$$

$$Y = \frac{\sigma}{\epsilon}$$

$$\sigma = Y \epsilon$$

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

$$\frac{F}{A} = Y \times \alpha \Delta T \rightarrow F = Y A \times \alpha \Delta T$$

$$= 2 \times 10^5$$

$$11) \quad l = 12 \quad T_0 = -5 \quad T = 35 \quad \alpha = 1 \times 10^{-5}$$

$$\Delta L = L_0 \times \Delta T \times \alpha \rightarrow 12 \times (35 - (-5)) \times 1 \times 10^{-5}$$

$$4,8 \times 10^{-3}$$

$$12) \quad L = 2m \quad A = 10 \times 10^{-4} \quad \Delta T = 100$$

$$Y = 2 \times 10^{11} \quad \alpha = 1,72 \times 10^{-5}$$

$$13) \quad \Delta L = L_0 \times \alpha \times \Delta T \quad \left| \quad \epsilon = Y \rightarrow \sigma = Y \epsilon \right.$$

$$= 3,944 \times 10^{-3} \quad \left| \quad \frac{F}{A} = Y \frac{\Delta L}{L} \right.$$

$$F = 3,944 \times 10^5$$

$$6) \quad V_1 = 3 \quad \Delta P = 25 \times 10^9$$

$$B = - \frac{\Delta P}{\Delta V} \times V \quad B = 250$$

$$7) \quad r = 15 \times 10^{-2} \quad T = 20 \quad P = 1$$

$$b) \quad V_{AV} = \frac{3}{2} \times 1,38 \times 10^{-23} \times 293 = 6,0651 \times 10^{-22}$$

$$V_B =$$

$$10) \quad PV = N k_B T$$

$$N = \frac{PV}{k_B T} = \text{Wagen}$$

$$8) \quad M_w = 20,9 \quad V = 10 \quad T = 300$$

$$V = L^3 \quad A = L^2$$

$$a) \quad PV = nRT$$

$$n = \frac{PV}{RT} = \frac{1,013 \times 10^5 \times (0,1)^3 \times 20,9 \times 1}{8,314 \times 300}$$

$$= 1,17$$

$$b) \quad F = mg = 11,7$$

$$c) \quad P = \frac{F}{A} \quad \left| \quad F = PA \right.$$

$$1,013 \times 10^5 \times 10$$

$$=$$

$$14) V_0 = 2 \times 10^{-7} / r = 5 \times 10^{-5} / \Delta T = 100$$

$$\alpha = 1.82 \times 10^{-4} \quad h = ??$$

$$V = V_0 [1 + 3\alpha \Delta T] \quad h = \frac{V_0 \Delta T (1 - 3\alpha)}{\pi r^2}$$

$$h \times \pi r^2 = V_0 [1 + 3\alpha \Delta T]$$

$$h = \frac{V_0 [1 + 3\alpha \Delta T]}{\pi r^2}$$

$$15) m = 0.05 / \Delta T = 60$$

$$Q = cm \Delta T \rightarrow C = \frac{Q}{m \Delta T} = 5.17 \times 10^3$$

$$16) m_x = \frac{m_w c_w (T_i - T_w)}{c (T_i - T_f)}$$

$$m_x = \frac{5 \times 4200 (340 - 300)}{490 (900 - 340)} \quad Q = m_i c_i \Delta T$$

$$m_x = 3906 \quad 894 \times 13$$

$$13) r = 1 \text{ at } 20^\circ \text{C}$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$1 = 2\pi \sqrt{\frac{L}{9.8}} \rightarrow L_0 = 0.24824 \text{ m}$$

$$\alpha = 3 \times 10^{-5}$$

$$L = L_0 (1 + \alpha \Delta T)$$

$$L = 0.24835$$

$$T = 2\pi \sqrt{\frac{L}{g}} = 1.00022$$

$$\Delta T = 0.0001499$$

$$T_{\text{delay}} = 30 \times 24 \times 60 \times 60 \times 0.0001499$$

$$= 388.77$$

$$T_{\text{delay}} = 6.5 \text{ min/month}$$

$$[21] A=3 / L = 0.96 \times 10^{-2} / \Delta F = 75 F$$

$$H = \kappa A \frac{\Delta T}{L}$$

$$0.98 \times 3 \times \frac{41.67}{0.96 \times 10^{-2}}$$

$$= 16,668 \times 10^3$$

$$[22] \Delta C = \frac{5}{9} \Delta F \rightarrow \Delta C = \frac{5}{9} \times 75 = 41.67$$

$$H = \kappa A \frac{\Delta T}{L} =$$

$$[17] A=1.2 / L=4 \times 10^{-2} /$$

$$H = \kappa A \frac{\Delta T}{L}$$

$$[18] T = 5800 K / r = 6.96 \times 10^8 / e = 0.96$$

$$P = e \sigma A T^4$$

$$0.96 \times \frac{4\pi}{3} (6.96 \times 10^8)^2 \times 5.67 \times 10^{-8} \times (5800)^4$$

$$= 3.749 \times 10^{26}$$

$$[19] H = 280 W / A = 1.5 / d = 0.003$$

$$\kappa = 0.2$$

$$H = \kappa A \frac{\Delta T}{L} \rightarrow \Delta T = 28 K$$

$$[20] \frac{1}{T}$$

حل أسئلة المراجعة

5.2 $P_1 = 3 \text{ atm} / y_1 = 0 / v_1 = ?$
 $v_1 = 2 \text{ cm}$

السؤال
 $P_2 = 1 \text{ atm} / y_2 = 10 / v_2 = ?$
 $v_2 = 0.5$

$$v_1 v_1^2 = v_2 v_2^2$$

$$v_1 = \frac{v_2^2}{v_1^2} \times v_2$$

$$v_1 = \frac{1}{16} v_2$$

$$v_2^2 - v_1^2 = \frac{255}{256} v_2^2$$

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g (y_2 - y_1)$$

$$3 - 1 = \frac{1}{2} \times 1000 (v_2^2 - v_1^2) + 10 \times 1000 \times 10$$

$$2 \times 10^5 = \frac{1}{2} \times 1000 \times \frac{255}{256} v_2^2 + 1000 \times 10 \times 10$$

$$\downarrow = 498 v_2^2 + 100000$$

$$102600 = 498 v_2^2$$

$$V = 14.35$$

$w = \text{wide}$ $n = \text{narrow}$
E 5-5 $V_w = 6 \times 10^3$ / $V_n = 4 \times 10^3$ | $\frac{V_w}{V_n} = \frac{V_n^2}{V_w^2} = \frac{1}{9}$
 $P_w = 8$ / $P_n = 4$ | $V_n = 9 V_w$

$$P_w - P_n = \frac{1}{2} \rho (V_n^2 - V_w^2)$$

$$4 \times 10^3 = \frac{1}{2} \times 1060 \times (81 V_w^2 - V_w^2)$$

$$4 \times 10^3 = 530 \times 80 V_w^2$$

$$V_w = 0.9307 \rightarrow V_n = 9 \times 0.9307$$

$$Q = A_w V_w = \pi V_n^2 \times V_w = 3.14 \times (3 \times 10^3)^2 \times 9 \times 0.9307 = 3.95 \times 10^5$$

E 5-6 $V_2 = \frac{A_1}{A_2} V_1 \rightarrow \frac{0.908}{0.904} \times 0.125 = 0.125$

pressure drop is

$$P_1 - P_2 = \frac{1}{2} \times 1060 \times (0.125^2 - 0.125^2) = 24.8 \approx 25$$

E 5-3 $P_5 = 3$ / $h_s = 2m$ / $V_s = ?$ | $\frac{V_s}{V_n} = \frac{1}{10000}$
 $P_2 = 14mm$ / $h_n = 0$ / $V_n = ?$

$$P_s - P_n = \frac{1}{2} \rho (V_n^2 - V_s^2) + \rho g (y_n - y_s)$$

$$(3-1) \times 1013 \times 15 = \frac{1}{2} \times 1000 (V_n^2 - V_s^2) + 1000 \times 10 \times -2$$

$$222600 = 500 (V_n^2 - V_s^2) \quad \left| \begin{array}{l} V_n = 10000 V_s \\ V_s = \frac{V_n}{10000} \end{array} \right.$$

$$222600 = 500 (V_n^2 - \frac{V_n^2}{(10000)^2})$$

$$222600 = 500 V_n^2$$

$$V_n = 21908$$

$$\Delta t = \frac{\Delta V}{Q} = \frac{\frac{\pi \times R^2 \times H}{\pi \times V^2 \times V_n}}{10^4 \times 21908} = \frac{1 \times 2}{10^4 \times 21908} = 9.48 \times 10^{-9}$$

$$= 15.81 \mu s$$

Q₁ → $\dot{V} = 4 \text{ mm}^3 / \text{s} = 4 \times 10^{-6} / \text{s} = 2.5 \times 10^{-4} \text{ m}^3 / \text{s}$

II $Q = A \bar{v} \rightarrow \bar{v} = \frac{Q}{A} = \frac{1 \times 10^{-6}}{\pi (4 \times 10^{-3})^2} = 0.00198 \text{ m/s}$
 $v_{\text{max}} = 0.004$

III $\Delta P = \frac{8 \eta L \bar{v}}{R^2} = \frac{8 \times 2.5 \times 10^{-4} \times 0.004}{(4 \times 10^{-3})^2} = 5$

IV $R_p = \frac{\Delta P}{Q} = \frac{5}{1 \times 10^{-6}} = 5 \times 10^6$

Q₂ → $R_p = \frac{\Delta P}{Q} = \frac{4 \eta L \bar{v}}{R^2 \times \pi R^2 \times \bar{v}}$

$R = \sqrt[4]{\frac{8 \eta L}{R_p \times \pi}} = \sqrt[4]{\frac{8 \times 3.14 \times 10^{-3} \times 30 \times 10^{-2}}{4 \times 10^8 \times \pi}}$

$R = 2.44 \times 10^{-6} \text{ m}$
 $d = 4.88 \times 10^{-6} \text{ m}$

E 5-7 $P_B + \rho g h_B = P_H + \rho g h_H$

* standing $h_H - h_B = -0.55$
 * Bending $h_B - h_H = 0.54$

$(P_B)_{\text{bending}} - (P_B)_{\text{standing}} = \rho g [0.54 - (-0.55)]$

$\Delta P = 1060 \times 10 \times 0.99 = 9540 = 9.54 \times 10^3 \text{ Pa}$

5-14 $r = 100 \times 10^{-6} / L = 2 \times 10^{-2}$

$R_p = \frac{8 \eta L}{\pi R^4} = \frac{8 \times 2.5 \times 10^{-4} \times 0.002}{3.14 \times (100 \times 10^{-6})^4} = 1.07 \times 10^{12}$

$Q = \frac{\Delta P}{R_p} = \frac{2.33 \times 10^3}{1.07 \times 10^{12}} = 2.17 \times 10^{-9} \text{ m}^3 / \text{s}$

$v_{\text{max}} = \frac{2Q}{\pi R^2} = \frac{2 [2.17 \times 10^{-9}]}{3.14 \times 10^{-8}} = 0.236 \text{ m/s}$

$\phi = Q \Delta P = [2.17 \times 10^{-9}] \times 2.33 \times 10^3 = 5.05$

$N_R = \frac{2P \bar{v} R}{\eta} = \frac{2 \times (1060) \times 0.136 / 2 \times (100 \times 10^{-6})}{2.1 \times 10^{-3}} = 6$

5-10 $V_L = 4 \times 10^{-2} / V_{in} = 0.94 \times 10^{-3} / P = 2400$
 $Q = \frac{4 \times 10^{-6}}{60} / 8 = 1060 / \eta = 0.0028 \times 10^{-1}$

$$Q = \frac{\pi \Delta P R^4}{8 \eta L} \rightarrow \Delta P = \frac{8 Q L \eta}{\pi R^4}$$

$$P_2 = P_1 + \frac{8 Q L \eta}{\pi R^4} \quad P_2 = P_{blood} \rho g h$$

$$h = \frac{P_1 + \frac{8 Q L \eta}{\pi R^4}}{\rho_{blood} \times g} =$$

5-14 $V = 1 \times 10^{-2} \quad Q = \frac{10^{-3}}{60} / \rho = 1060$

$$\eta = 2.91 \times 10^{-3} \times 10^{-1}$$

$$\textcircled{*} Q = A \bar{V} \rightarrow \bar{V} = \frac{Q}{A} = \frac{10^{-3}}{60 \times \pi (1 \times 10^{-2})^2}$$

$$= 0.0053$$

$$N_R = \frac{2 \rho V R}{\eta} = \frac{2 \times 1060 \times 0.0053 \times (1 \times 10^{-2})}{2.91 \times 10^{-3}} = 535$$

5-1 $r = 1 \times 10^{-2} / Q = 3 \times 10^{-5}$

$$Q = \frac{\Delta V}{\Delta t} = \frac{3 \times 10^{-3}}{60} = 5 \times 10^{-5}$$

$$\textcircled{1} Q = A \times V \rightarrow V = \frac{Q}{A} = \frac{5 \times 10^{-5}}{\pi \times (1 \times 10^{-2})^2} = 0.159$$

$$\textcircled{2} V_2 = V_1 \times \frac{V_1^2}{V_2^2} = 0.159 \times \frac{1}{0.925} = 0.172$$

5-4 $P = P_0 + \rho g h$
 $= 101300 + (1000) \times 10 \times 5 = 1.513 \times 10^5$

5-8 $L = 1.8 / a = 12 / g = 10$

$$P_B = P_H + \rho (g + a) (h_H - h_B)$$

$$= 13300 + 1060(??) \times 0.935 = 5.14 \times 10^4$$

$$P_F = P_H + \rho (g + a) (h_H)$$

$$= 13300 + 1060(22) \times (1.45) = 47.114 \times 10^3$$

$$[6] P = \frac{F}{A} \rightarrow F = PA = 1,103 \times 10^5 \times 8,5 \times 10^{-2} \times 11 \times 10^{-2}$$

$$[7] F = PA = (120 \text{ mm Hg}) \times \left(\frac{1,103 \times 10^5}{760 \text{ mmHg}} \right) \times 15 \times \frac{1}{10^4}$$

$$= 23,992 \text{ N}$$

$$[8] P_0 = h \rho g = 0,120 \times 13,6 \times 10^3 \times 9,80 = 1,56 \times 10^4$$

$$P_0 = h \rho g = 0,080 \times 13,6 \times 10^3 \times 9,80 = 1,07 \times 10^4$$

$$P_w h_w \rho = \rho_b h_b \rho$$

$$h_w = \frac{300 \times 13,6}{1} = 4080 \text{ mm} = 4,08$$

$$P = 60 / A = 600 \times 10^{-4}$$

$$P = \frac{F}{A} \rightarrow F = 60 \times \frac{133}{1} \times 600 \times 10^{-4}$$

حل المسألة الخامسة

$$[1] A = 1,50 \times 10^{-4} / F = 550 /$$

$$P = \frac{F}{A} = \frac{550}{1,50 \times 10^{-4}} = 3,54 \times 10^6$$

$$[2] P = \frac{F}{A} = \frac{1 \times 10^{-3} \times 9,8}{\pi \times (0,2 \times 10^{-2})^2} = 0,0779$$

$$[3] P = \rho g h \rightarrow h = \frac{P}{\rho g} = \frac{1,013 \times 10^5}{9,8 \times 13,6 \times 10^3} = 0,76$$

$$[4] P = \rho g h = 9,8 \times 11 \times 10^3 \times 10,25 \times 10^3$$

$$= 1,10495 \times 10^8$$

$$[5] P = \frac{F}{A} = \frac{0,300}{(1,10 \times 10^{-2})^2} = 2,9727 \times 10^3$$

$$15) \rho = 1.3 \times 10^3 \quad \Delta h = 0.67$$

$$\Delta P = \rho g h \rightarrow = 1.3 \times 10^3 \times 0.67 \times 9.8 = 8535.8$$

$$16) v_1 = 1 \times 10^{-2} / v_2 = 0.5 \times 10^{-2} / \Delta P = 6660$$

$$\begin{aligned} A_2 v_2 &= A_1 v_1 \\ v_1^2 v_2 &= v_2^2 v_1 \\ v_1 &= \frac{v_2^2}{v_1^2} v_2 \\ v_1 &= \frac{1}{4} v_2 \\ v_2 &= 4 v_1 \end{aligned} \quad \left\{ \begin{aligned} \Delta P &= \frac{1}{2} \rho (v_2^2 - v_1^2) \\ 6660 &= \frac{1}{2} \times 1000 \times (15 v_1^2 - v_1^2) \\ v_1 &= 0.9423 \\ v_2 &= 3.7692 \end{aligned} \right.$$

$$Q = 2.96 \times 10^{-4}$$

$$7) v_{in} = 1 \times 10^{-2} / v_{out} = 1.5 \times 10^{-3} / v_1 = 25 \times 10^{-2} / v_2 = ?$$

$$v_1^2 = v_2^2 \quad 2 = \frac{v_1^2}{v_2^2} \times v_1 \rightarrow v_1 = 11.11$$

$$11) \rho = \frac{F}{A} = \frac{500}{1 \times 10^{-5}} = 5 \times 10^8$$

$$12) \Delta P = 20 = 2 = 22 \quad \text{الفوق - التحت}$$

$$\Delta P = \rho g h \rightarrow h = \frac{\Delta P}{\rho g} = \frac{22 \times 133}{9.8 \times 10^3} = 0.271$$

$$13) a - \Delta h = 16 / Q = 2.5 \times 10^3$$

$$\rho g (h_1 - h_2) = \frac{1}{2} \rho v_2^2$$

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 9.8 \times 16} = 17.7$$

$$14) \Delta P = \rho g h$$

$$h = \frac{2 \times 10^5}{10 \times 10^3} = 20$$

$$[20] R = 3 \times 10^{-4} / L = 3 \times 10^{-2} / \Delta h = 1 \text{ m}$$

$$Q = \frac{\pi \Delta P R^4}{8 \eta L} \quad \left\{ \begin{array}{l} \Delta P = \rho g \Delta h \\ = 1060 \times 10 \times 1 = 1060 \times 10^4 \end{array} \right.$$

$$Q = \frac{\pi \times 1060 \times 10^4 \times [3 \times 10^{-4}]^4}{8 \times 3 \times 10^{-2} \times 2908 \times 10^{-3}} = 5.403 \times 10^{-7}$$

$$\Delta t = \frac{\Delta V}{Q} = \frac{500 \times 10^{-6}}{5.403 \times 10^{-7}} = 15.42 \text{ min}$$

$$[21] a = 3g \quad \text{Upward acceleration}$$

$$\begin{aligned} B_b &= B_h + \rho(g+a)(h_h - h_b) \\ &= 1393 \times 10^3 + 1060(40) \times (-0.3) \\ &= 580 \text{ Pa} \end{aligned}$$

$$[7] Q = \frac{\Delta V}{\Delta t} \rightarrow \Delta t = \frac{\Delta V}{Q} = \frac{4 \times 10^{-3}}{\pi \times (10^{-2})^2 \times 0.2} = 50.93$$

$$[18] \quad \begin{array}{l} v_1 = 30 \times 10^2 \\ v_2 = 15 \times 10^2 \end{array} \quad \begin{array}{l} Q = 0.2 \\ h = 60 \times 10^2 \end{array} \quad \begin{array}{l} P_1 = 1 \text{ atm} \end{array}$$

$$* Q_1 = A_1 v_1 \rightarrow v_1 = \frac{Q_1}{A_1} = \frac{0.2}{\pi \times (30 \times 10^2)^2} = 0.707$$

$$v_2 = \frac{v_1^2}{v_2^2} v_1 \rightarrow v_2 = 2.828$$

$$P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (h_1 - h_2)$$

$$= 1.013 \times 10^5 + \frac{1}{2} \times 1000 (0.707^2 - 2.828^2) + 1000 \times 9.8 \times (-0.3) = 1.034 \times 10^5$$

$$[19] A = 2 \times 10^{-4} / v_1 = 40 \times 10^{-2}$$

$$Q = A \bar{v} = 2 \times 10^{-4} \times [40 \times 10^2] = 8 \times 10^{-5}$$

$$[21] \quad r = 1 \times 10^{-2} \quad \bar{v} = 55 \times 10^{-2} \quad L = 30 \times 10^{-2}$$

$$N_R = \frac{2R\bar{v}\rho}{\eta} = \frac{2 \times 1 \times 10^{-2} \times 55 \times 10^{-2} \times 1060}{2.08 \times 10^{-3}} = 5605.7$$

$$* R_F = \frac{Q\eta L}{\pi R^4} = 159 \text{ KPa}$$

$$* P = R_F Q^2 = [159 \times 10^3] \times [\pi \times [0.101]^2 \times 0.55]^2 = 4.97 \times 10^{-3}$$

$$[25] \quad a = 1.5 \times 10 = 15 \text{ m/s}^2 \quad \downarrow$$

$$\Delta h = 0.94$$

$$P_B = P_H + \rho(g-a)(h_H - h_B) = 13.3 \times 10^3 + 1060(-5) \times 0.94 = 15.42 \times 10^3$$

$$P_F = P_H + \rho(g-a)h_H = 13.3 + 1060(10-15) \times 1.94 = 5.880 \text{ KPa}$$

$$[22] \quad a = 10 \quad g = 10$$

⊗ Up

$$P_b = P_H + \rho(g+a)(h_H - h_b) = 13.3 \times 10^3 + 1060 \times 20 \times (-0.3) = 6940$$

$$P_F = P_H + \rho(g+a)h_H = 13.3 \times 10^3 + 1060 \times 20 \times 1.4 = 43 \text{ KPa}$$

⊗ Down $g-a=0$

$$P_B = P_H = 13.3 \text{ KPa}$$

$$P_F = P_H = 13.3 \text{ KPa}$$

$$[23] \quad r = 2.6 \times 10^{-3} \quad L = 8.4 \times 10^{-2} \quad \Delta P = 400 \quad \bar{v} = ??$$

$$\bar{v} = \frac{\Delta P R^2}{8\eta L} = 1.93$$

$$[20] v_2 = 1.5 v_1 \quad \frac{A_1}{A_2} = \frac{v_2^2}{v_1^2} = \frac{v_1^2}{2.25 v_1^2}$$

$$A_2 = 2.25 A_1$$

$$\text{*a*} \quad \frac{Q_2}{Q_1} = \frac{R_2^4}{R_1^4} \rightarrow \frac{1.5^4 R_1^4}{R_1^4}$$

$$Q_2 = 5.0625 Q$$

$$\text{*} \quad \frac{\Delta P_2}{\Delta P_1} = \frac{\frac{8 L \eta Q_2}{\pi R_2^4}}{\frac{8 L \eta Q}{\pi R_1^4}} = \left(\frac{R_1}{R_2}\right)^4 = \left(\frac{R_1}{1.5 R_1}\right)^4 = 0.197$$

$$[24] v_1 = 0.1 / v = 0.2$$

$$N_R = \frac{28 \bar{v} R}{\eta} = \frac{2 \times 1000 \times 0.2 \times 0.1}{10^{-3}} = 40000$$

is turbulent

$$\text{*} Q = A \bar{v} = \pi (0.1)^2 \times 0.2 = 6.28 \times 10^{-3}$$

$$[26] v_{in} = 2 \times 10^{-3} / \frac{V}{2.25} = 0.003 \quad L = 0.005$$

$$\text{*} V_{max} = 2 \times 0.003 = 0.006$$

$$\text{*} Q = A \bar{v} = \pi \times (2 \times 10^{-3})^2 \times 0.003$$

$$= 3.77 \times 10^{-7}$$

$$\text{*} \bar{v} = \frac{\Delta P R^2}{8 \eta L} \rightarrow \Delta P = \frac{8 \eta L \bar{v}}{R^2} = 693 \text{ Pa}$$

$$[27] \Delta P = 100 / v_1 = 10 \times 10^{-2}$$

$$\text{*} \Delta P = \frac{F}{A} \rightarrow F = \Delta P \times A$$

$$= 100 \times \pi (10 \times 10^{-2})^2$$

$$= \pi$$

$$[29] \bar{v} = 0.015 \quad P = Q \Delta P$$

$$= \pi \times 100 \times (10 \times 10^{-2})^2 \times 0.015$$

$$= 4.712 \times 10^3$$

$$\boxed{33} \quad r = 2 \times 10^{-3} / v = 40 \times 10^{-2}$$

$$* Q = A \bar{v} = \pi (2 \times 10^{-3})^2 \times 40 \times 10^{-2} = 5.02 \times 10^{-6}$$

$$* Q = \frac{\Delta V}{\Delta t} \rightarrow \Delta V = 5.02 \times 10^{-6} \times 30 = 1.507 \times 10^{-4}$$

$$\boxed{34} \quad A_1 = 1 \times 10^{-4} / A_2 = 0.4 \times 10^{-4}$$

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{1 \times 10^{-4} \times \bar{v}_1}{0.4 \times 10^{-4}} = 0.139 \bar{v}_1$$

$$\boxed{35} \quad A_1 V_1 = A_2 V_2 \quad A_2 = \frac{V_1}{V_2} \times A_1 = 4 \times 10 \times 10^{-4} = 40 \text{ cm}^2$$

$$A_2 = N A \rightarrow N = \frac{A_2}{A} = \frac{40}{\pi (5 \times 10^{-4})^2} = 5.09 \times 10^7$$

$$\boxed{30} \quad L = 0.11 \times 10^{-2} / r = 25 \times 10^{-6}$$

$$* R_f = \frac{8 \eta L}{\pi R^4} = 1.49 \times 10^{13}$$

$$* \Delta P = 1.3 \text{ kPa}$$

$$Q = \frac{\Delta P}{R_f} = \frac{1.3 \times 10^3}{1.49 \times 10^{13}} = 8.72 \times 10^{-11}$$

$$* \boxed{31} \quad Q = \frac{5 \times 10^{-3}}{60} = 8.33 \times 10^{-5}$$

b)

$$\boxed{32} \quad Q = \frac{5 \times 10^{-3}}{60} / v = 1 \times 10^{-2}$$

$$Q = A v \quad v = \frac{Q}{A} = \frac{5 \times 10^{-3}}{60 \pi (1 \times 10^{-2})^2} = 0.927$$