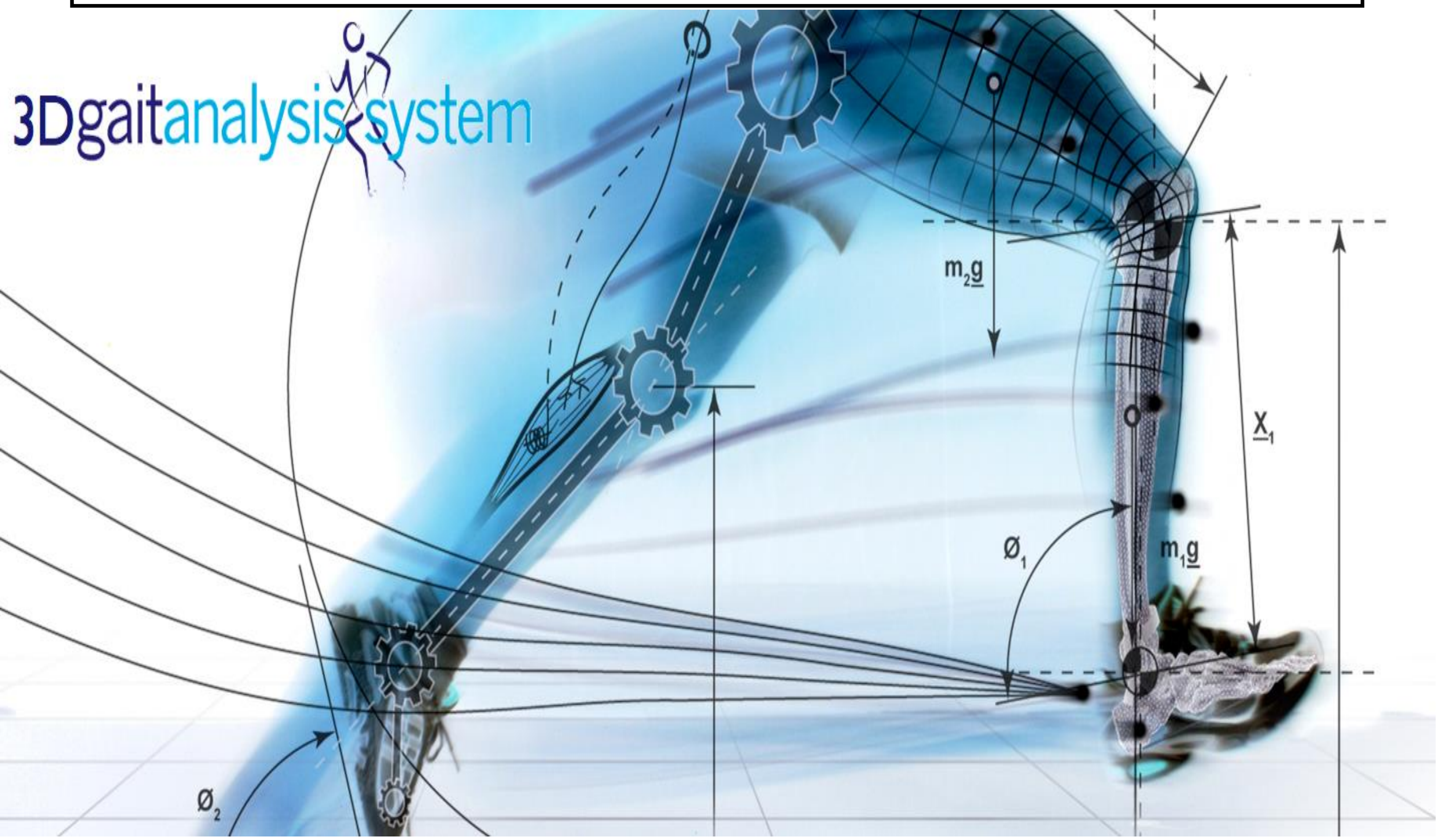
Physics For Medical Sciences

Student Solution Manual



Hassan Salem Ashour

Professor of Physics and Electro-Optics

Al-Azhar university- Gaza

Table of Content

Preface

For the second edition of Physics to Medical Sciences, all end of the chapter problems have been solves. We tries hard to elucidate solution steps as much we can. Multiple choice questions (MCQs) are added to this solution manual to help student focus on the main concepts in each chapter. Furthermore, we added several previous years solved sample midterms and final exams to help student in exam preparation.

Mistakes and transcription errors may be reduced but are unlikely to be eliminated. All problems have been rechecked from the first edition, and many (too many, of course) errors have been corrected.

Any corrections or suggestions for improvements will be appreciated. You can contact me on:

hashour@alazhar.edu.ps

Hassan Salem Ashour

Professor of Physics and Electro-Optics

Jan 2019

Chapter 1

Solutions

Part I: MCQs

Answered Multiple Choice Questions with some explanations

1. Which of the following is a derived quantity:

a) Force b) Mass

c) Length d) Time

2. Which is not a base unit in SI units?

a) Kilogram **b) Joule**

c) Meter d) Kelvin

3. The unit of force is________ and its symbol is _______ which is the correct pair?

a) Newton, n b) Newton, N

c) newton, n **d) newton, N**

4. SI unit of pressure is

a) Nm^2 b) N^2m

c) $\boldsymbol{Nm^{-2}}$ d) Nm

5. An alternate unit to $kgms^{-1}$ is

a) Js **b) $\boldsymbol{Ns}$**

c) Nm d) N

6. the SI units of pressure in terms of base units are

a) $\boldsymbol{kg\ m^{-1}\ s^{-2}}$ b) $kg\ m^{-1}\ s^{-3}$

c) $kg\ m\ s^{-2}$ d) $kg\ m^3\ s^{-2}$

7. Which of the following is least multiple:

a) Pico b) deci

c) Nano d) milli

8. Which one is the highest power multiple?

a) giga b) mega

c) kilo d) deca (it is the prefix of 10: Not used prefix)

9. The prefix pico is equal to

a) 10^{-3} b) 10^{-6}

c) 10^{-9} **d) $\mathbf{10^{-12}}$**

10. the number 0.0023 can be expressed in scientific notation as

a) 23×10^{-4} b) 0.23×10^{-2}

c) $\mathbf{2.3 \times 10^{-3}}$ d) None of the above

11. Unit of G is ?

a) $Nm^2\ kg^2$ b) $N\ m^2\ kg$

c) $\boldsymbol{N\ m^2\ kg^{-2}}$ d) None

12. The dimension of force is

a) MLT^{-1} **b) $\boldsymbol{MLT^{-2}}$**

c) $ML^{-1}T$ d) ML^2T^{-2}

13. The dimension formula for the quantity light year is

a) L^2T^{-2} b) T

c) ML^2T^{-2} **d) $\boldsymbol{L}$**

Explanation: light year is a distance; the distance that light travels in one year.

14. the dimension of torque are

a) $ML^{-1}T^{-2}$

b) $ML^{2}T^{-1}$

c) $\boldsymbol{ML^{2}T^{-2}}$

d) $ML^{-2}T^{2}$

15. Dimensions for acceleration due to gravity is

a) MLT^{-2}

b) MLT

c) LT^{2}

d) $\boldsymbol{M^{0}LT^{-2}}$

16. $M^{0}L^{0}T^{-1}$ refers to quantity

a) Velocity

b) Time period

c) **Frequency**

d) Force

17. Dimensions of force is

a) MLT^{-1}

b) $\boldsymbol{MLT^{-2}}$

c) $ML^{-1}T$

d) $ML^{-1}T^{2}$

18. $ML^{-1}T^{-2}$ refers to quantity

a) Force

b) Pressure

c) Momentum

d) Energy

19. Which one is a vector:

a) Length

b) Volume

c) Velocity

d) Work

20. An example of scalar quantity is

a) Displacement

b) Speed

c) Velocity

d) Torque

21. Name the quantity which is vector:

a) Density

b) Power

c) Charge

d) Moment of Force

22. The direction of a vector in **space** is specified by:

a) One angle

b) Two angle

c) Three angle

d) No angle

23. If both components of a vector are negative, then resultant lies in:

a) 1st quadrant

b) 2nd quadrant

c) 3rd quadrant

d) 4th quadrant

24. In which quadrant the two rectangular components of a vector have same sign?

a) 1st

b) 2nd

c) both 1st and 3rd

d) 4^{th}

25. If the x-component of a vector is positive and y-component is negative, then resultant vector lies in what quadrant:

a) 1st quadrant

b) 2nd quadrant

c) 3rd quadrant

d) 4th quadrant

26. If vector A lies in the third quadrant, its direction will be:

a) $180^{\circ} - \varphi$

b) $360^{\circ} - \varphi$

c) $\boldsymbol{180^{\circ} + \varphi}$

d) None

27. A single vector having the same effect as all the original vectors taken together, is called

a) Resultant vector

b) Equal vector

c) Position vector

d) Unit vector

28. The unit vector in the direction of vector $\vec{A} = 2i - 2i + k$ is

a) $\vec{A} = 2i - 2i + k$

b) $\vec{A} = (2i - 2i + k)/9$

c) $\boldsymbol{\vec{A} = (2i - 2i + k)/3}$

d) $\vec{A} = (2i - 2i + k)/5$

29. Mathematically, unit vector is described as:

a) $\widehat{A} = \vec{A}|A|$

b) $\hat{A} = \frac{|A|}{\vec{A}}$

c) $\boldsymbol{\widehat{A} = \frac{\vec{A}}{|A|}}$

d) $\hat{A} = \vec{A} \cdot \vec{A}$

30. The unit vector along y-axis is

a) i

b) j

c) k

d) None

31. The magnitude of the vector $\frac{2}{3}i - \frac{1}{3}j + \frac{2}{3}k$ is:

a) $Zero$

b) One

c) $Three$

d) $\frac{1}{9}$

32. If $\vec{A} = A_x i + A_y j + A_z k$ and $\vec{A} = B_x i + B_y j + B_z k$ then:

a) $\boldsymbol{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$

b) $\vec{A} \cdot \vec{B} = A_x B_y + A_y B_z + A_z B_x$

c) $\vec{A} \cdot \vec{B} = A_y B_z + A_z B_y + A_z B_x$

d) $\vec{A} \cdot \vec{B} = A_x B_z + A_y B_y + A_z B_x$

33. The projection of a vector $\vec{B}$ over $\vec{A}$ is:

a) $A \cos \theta$

b) $\boldsymbol{B \cos \theta}$

c) $A \sin \theta$

d) $B \sin \theta$

34. The cross product $i \times j$ is equal to

a) $Zero$

b) One

c) $-k$

d) $\boldsymbol{k}$

35. The magnitude of vector product is given by:

a) $AB \sin\theta \ \hat{n}$

b) $\boldsymbol{AB \sin\theta}$

c) $AB \cos\theta$

d) $AB \tan\theta$

Explanation: here $\hat{n}$ is a unit vector points in the direction of the vector $\vec{C} = \vec{A} \times \vec{B}$

36. Two vectors A and B are making angle θ with each other. The projection of vector B on vector A is written as.

a) $\frac{\vec{A}\cdot\vec{B}}{|A|}$

b) $\frac{\vec{A}\cdot\vec{B}}{|B|}$

c) cos θ

d) Both a and b are correct.

Explanation: the projection of the vector $\vec{B}$ on $\vec{A}$ is $B\cos\theta$, applying rule:

$$\frac{\vec{A}\cdot\vec{B}}{|A|} = \frac{AB\cos\theta}{|A|} = B\cos\theta$$

37. When two vectors are anti-parallel, the angle between them is:

a) Zero

b) 180°

c) 90°

d) 270°

38. A unit vector is obtained by dividing a vector with:

a) Its direction

b) Its magnitude

c) Its magnitude and direction

d) None

39. The magnitude of a vector $\vec{A} = A_x i - A_y j$

a) $A_x^2 + A_y^2$

b) $A_x^2 - A_y^2$

c) $\sqrt{A_x^2 + A_y^2}$

d) $\sqrt{A_x^2 - A_y^2}$

40. Vectors A is along y axis, its component along x axis is:

a) A

b) A/2

c) Zero

d) 2A

41. The angle between rectangular components of vector is:

a) 45° b) 60° **c) 90°** d) 180°

42. A force of 10N is acting along x-axis, its component along y-axis is

a) 10N b) 5N c) 8.66N **d) Zero N**

43. If vector $\vec{A}$ is acting along y-axis, its y-component is:

a) $\boldsymbol{A}$ b) $A\ cos\ \theta$ c) $A\ sin\ \theta$ d) $Zero$

44. If $\vec{A} = 2i - j + 3k$, then the magnitude of vector $\vec{A}$ is:

a) 4 b) 14 **c)** $\sqrt{\mathbf{14}}$ d) None

45. $i \cdot (j \times k)$ is equal to

a) 1 b) 0 c) -1 d) $-k$

Explanation: $i \cdot (j \times k) = i \cdot (i) = 1$

46. The **cross** product of vectors will be **minimum** when the angle between vectors is

a) $35°$ b) $90°$ **c)** $\mathbf{0}°$ d) $45°$

47. Dot product of two non-zero vectors is **zero,** when angle between them is:

a) $0°$ b)) $30°$ c) $45°$ **d)** $\mathbf{90}°$

48. The cross product $i \times i = j \times j = k \times k$ is equal to

a) 1 b) -1 **c) Zero** d) None

49. The scalar product of two vectors is maximum when they are:

a) Parallel b) Perpendicular

c) Anti-parallel d) None

50. The magnitude of cross-product and dot-product of two vectors are equal, the angle between them is

a) Zero

b) 45°

c) 90°

d) 180°

51. The magnitude of dot and cross product of two vectors are $6\sqrt{3}$ and 6 respectively. The angle between them will be

a) 0°

b) 30°

c) 45°

d) 60°

52. If $A_x = A_y$, then the angle between the vector $\vec{A}$ with x-axis will be:

a) 0°

b) 30°

c) 45°

d) 90°

53. $\vec{A} \times \vec{A}$ is

a) A

b) A^2

c) $2A$

d) $\boldsymbol{Zero}$

Part II: End of Chapter Problems Solutions

1. The speed limit on some interstate highways is roughly $100\ km/h$. (a) What is this in meters per second? (b) How many miles per hour is this?

Solution

(a) $\frac{100\text{ km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = \underline{27.77\text{ m/s} = 27.8\text{ m/s}}$

(b) $\frac{100\text{ km}}{\text{h}} \times \frac{1\text{ mi}}{1.609\text{ km}} = \underline{62\text{ mi/h}}$

2. One gallon of paint $(volume = 3.78 \times 10^{-3} m^3)$ covers an area of $25.0\ m^2$. What is the thickness of the paint on the wall?

Solution:

$$Volume = Area \times thickness$$

$$3.78 \times 10^{-3} m^3 = 25.0\ m^2 \times l$$

$$l = \frac{3.78 \times 10^{-3} m^3}{25.0\ m^2} = 0.1512 \times 10^{-3} m = 0.1512\ mm$$

3. Assume that it takes $7.00\ min$ to fill a $30.0 - gal$ gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time, in hours, required to fill a 1-cubic-meter volume at the same rate. $(1\ U.S.gal = 231\ in^3)$

Solution:

a. The rate is volume per second

$$\frac{\Delta V}{\Delta t} = \frac{30.0}{7 \times 60} = \frac{1}{14}\ gallon/second$$

b. 1 gallon=$3.78 \times 10^{-3}\ m^3$

$$\frac{\Delta V}{\Delta t} = \frac{30.0}{7 \times 60} = \frac{1}{14} \frac{gallon}{second} = \frac{1}{14} \frac{3.78 \times 10^{-3}\ m^3}{second} = 0.27 \times 10^{-3} m^3/second$$

$$\frac{\Delta V}{\Delta t} = 0.27\ litre/second$$

c. The time needed at this rate

$$t = \frac{Volume}{rate} = \frac{1m^3}{0.27 \times 10^{-3} m^3/second} \cong 3704\ second \cong 1.028\ hour$$

4. A micro-laser volume is about $36 \mu m^3$ express the laser volume in:

a. mm^3 b. m^3

Solution:

$$V = 36\ (\mu m)^3 = 36(10^{-6} m)^3$$

$$V = 36(10^{-3} 10^{-3} m)^3 = 36(10^{-3} mm)^3$$

$$V = 36 \times 10^{-9} (mm)^3$$

$$V = 36(10^{-6} m)^3 = 36(10^{-6})^3 m^3 = 36 \times 10^{-18} m^3$$

5. Iron density is about $8.6\ g/cm^3$ express the density in kg/m^3.

Solution:

$$\rho = 8.6 \frac{g}{cm^3} = 8.6 \frac{10^{-3} kg}{(10^{-2} m)^3} = 8.6 \frac{10^{-3} kg}{10^{-6}\ m^3} = 8.6 \times 10^3 kg/m^3$$

6. Newton's law of universal gravitation is represented by

$$F = G\frac{mM}{r^2}$$

Here $\boldsymbol{F}$ is the gravitational force, M and m are masses, and r is a length. Force has the SI units $kg\ m/s^2$. What are the SI units of the proportionality constant G?

Solution:

We can rewrite the gravitational force equation,

$$G = \frac{Fr^2}{Mm} = \frac{Nm^2}{kg^2} = \frac{kg.m.s^{-2}.m^2}{kg^2} = \frac{m^3.s^{-2}}{kg}$$

7. The smallest meaningful measure of length is called the "Plank length", and is defined in terms of the three fundamental constants of nature, the speed of light $c = 3 \times 10^8\ m/s$, the gravitational constant $G = 6.67 \times 10^{-11}\frac{m^3}{kg\cdot s^2}$, and Plank's constant$h = 6.63 \times 10^{-34} kg.m^2/s$. The Plank length λ_p is given by the following combination of these three constants: $\lambda_p = \sqrt{\frac{Gh}{c^3}}$

Show that the dimensions of λ_p are length $[L]$, and find the order of the magnitude of λ_p.

Solution:

a. First the dimensions for each constant:

$$[c] = \frac{[L]}{[T]}, [G] = \frac{[L^3]}{[M][T^2]}, \text{ and } [h] = \frac{[M][L^2]}{[T]}$$

The substitute these constant in your equation, that is

$$[\lambda_p] = \sqrt{\frac{\frac{[L^3]}{[M][T^2]}\frac{[M][L^2]}{[T]}}{\left(\frac{[L]}{[T]}\right)^3}} = \sqrt{\frac{\frac{[L^3]}{[T^2]}\frac{[L^2]}{[T]}}{\left(\frac{[L]}{[T]}\right)^3}} = \sqrt{\frac{\frac{[L^5]}{[T^3]}}{\frac{[L^3]}{[T^3]}}} = \sqrt{\frac{[L^5]}{[L^3]}} = \sqrt{[L^2]} = [L]$$

b. The order of the magnitude of λ_p

$$\lambda_p = \sqrt{\frac{Gh}{c^3}} = \sqrt{\frac{6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2} h \times 6.63 \times 10^{-34} kg.m^2/s}{(3 \times 10^8\ m/s)^3}} \approx 4 \times 10^{-35} m$$

Which is on the order of the magnitude between $10^{-35}\ or\ 10^{-34}$

8. We express the distance through which the person falls, before springing back, by the equation

$$S = Cg^{\alpha} t^{\beta} m^{\gamma}$$

Where $\alpha, \beta, and\ \gamma$ are unknown parameters to be determined, C is an overall constant that cannot be determined using this method, m is person's mass, and g is the acceleration due gravity. Find the $\alpha, \beta, and\ \gamma$ using dimension analysis.

Solution:

First of all list the dimensions of all the parameters,

$$[S] = L, [g] = LT^{-2}, [m] = M, [t] = T$$

Apply in the equation

$$L = (LT^{-2})^{\alpha} \times T^{\beta} \times M^{\gamma}$$

Rearrange and equate the similar dimensions, that is

$$\alpha = 1, -2\alpha + \beta = 0, \gamma = 0$$

Solve for β we have $\beta = 2$

$$S = Cgt^2$$

9. The velocity of sound waves through any material depends on its density, ρ, and its modulus of elasticity, E. Thus the velocity v has the following form

$$v = kE^x \rho^y$$

Where k is a dimensionless constant and x, y are indices need to find using Dimension analysis. Hint: E dimensions are $ML^{-1}T^{-2}$.

Solution:

First of all list the dimensions of all the parameters,

$$[v] = LT^{-1}, [E] = ML^{-1}T^{-2}, [\rho] = ML^{-3}$$

Then apply them in the velocity equation

$$LT^{-1} = (ML^{-1}T^{-2})^x \times (ML^{-3})^y$$

Rearrange and equate the similar dimensions, that is

$$M^0LT^{-1} = M^{x+y} \times L^{-x-3y} \times T^{-2x}$$

Matching the dimensions on the left and right hand sides of the equations, gives

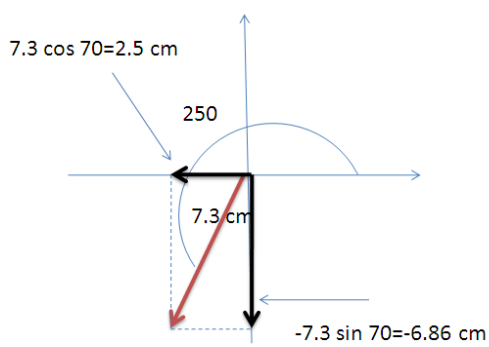
From M we have $x + y = 0 \rightarrow x = -y$, from T we have $-2x = -1$

$$x = \frac{1}{2} \rightarrow y = -\frac{1}{2}$$

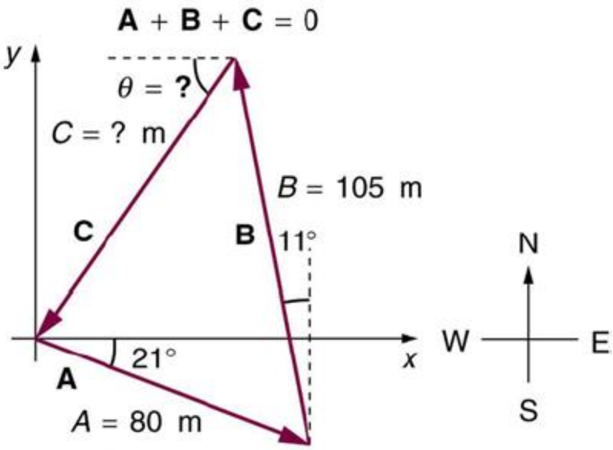
$$v = kE^{1/2}\rho^{-1/2} = k\sqrt{\frac{E}{\rho}}$$

10. What are the components of a vector $\boldsymbol{A}$ in the xy plane if its direction is 250° counterclockwise from the positive x-axis and its magnitude is 7.3 units?

Solution:



11. A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be $80.0\ m$ long and the next to be $105\ m$. These sides are represented as displacement vectors **A** from **B** in Figure below. She then correctly calculates the length and orientation of the third side **C**. What is her result?



Solution $\mathbf{C} = -\mathbf{A} - \mathbf{B}$

$$C_y = -A_y - B_y = -(80.0\,\text{m})\sin(-21°) - (105\,\text{m})\sin 101° = -74.40\,\text{m}$$

$$C_x = -A_x - B_x$$

$$C_x = -A\cos(\text{-}21°) - B\cos 101° = -(80.0\ \text{m})\cos(-21°) - (105\,\text{m})\cos 101° = -54.65\,\text{m}$$

$$C = \sqrt{{C_x}^2 + {C_y}^2} = \sqrt{(-54.65\,\text{m})^2 + (-74.40\,\text{m})^2} = \underline{92.3\,\text{m}}$$

$$\theta = \tan^{-1}\frac{|C_y|}{|C_x|} = \tan^{-1}\left(\frac{74.40\,\text{m}}{54.65\,\text{m}}\right) = \underline{53.7°\ \text{S of W}}$$

12. Find $|\mathbf{A}|$, $\mathbf{A}+\mathbf{B},$ *and* $\mathbf{A}\text{ - }2\mathbf{B}$ for the flowing sets of vectors:

a. $\mathbf{A} = i - 2j + k,\ \ \mathbf{B} = j + 2k$

b. $\mathbf{A} = 3i - 2k,\ \ \ \ \mathbf{B} = i - j + k$

Solution:

a. $|A| = \sqrt{1+4+1} = \sqrt{6}$

$A + B = i + 0j + 3k$

$A - 2B = A + (-2B),\ -2B = -2j - 4k$

$A + (-2B) = i - 4j - 3k$

b. Is similar to **a**

13. If $\mathbf{v}$ lies in the first quadrant and makes an angle $\pi/3$ with the positive-axis and $|\mathbf{v}| = 3.0\ units$, find in component form.

Solution:

$$v_x = v\cos\frac{\pi}{3} = 3\ \times\frac{1}{2} = 1.5, \qquad v_y = v\sin\frac{\pi}{3} = 3\times\frac{\sqrt{3}}{2} = 1.5\sqrt{3}, = 2.6$$

14. If $\mathbf{A} = 6i - 8j\ units,\ \mathbf{B} = -8i + 3j\ units,\ and\ \mathbf{C} = 26i + 19j\ units$, determine **a** and **b** such that $a\mathbf{A} + b\mathbf{B} + \mathbf{C} = 0$

Solution:

$$a\mathbf{A} + bB + C =$$
$$a(6i - 8j) + b(-8i + 3j) + 26i + 19j = 0$$
$$(6a - 8b + 26)i + (-8a + 3b + 19)j = 0i + 0j$$

$$(6a - 8b + 26) = 0$$
$$(-8a + 3b + 19) = 0$$

Solving these equations for a and b, gives $a = 5$ and $b = 7$.

15. Find a unit vector that has the same direction as the given vectors

$\mathbf{A} = 8i - 2j + 4k$

$\mathbf{B} = 3i - 4j + 5k$

Solution:

$$\hat{A} = \frac{A}{|A|} = \frac{8i - 2j + 4k}{\sqrt{64 + 4 + 16}} = \frac{1}{\sqrt{84}}(8i - 2j + 4k)$$
$$\hat{B} = \frac{B}{|B|} = \frac{3i - 4j + 5k}{\sqrt{9 + 16 + 25}} = \frac{1}{\sqrt{50}}(3i - 4j + 5k)$$

16. Given $M = 6i + 2j - k$ and $N = 2i - j - 3k$, calculate the vector product $\mathbf{M}\times\mathbf{N}$.

Solution:

$$M \times N = \begin{vmatrix} i & j & k \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix} = -7i + 16j - 10k$$

17. If $|\boldsymbol{A} \times \boldsymbol{B}| = \boldsymbol{A} \cdot \boldsymbol{B}$ what is the angle between $\boldsymbol{A}$ and $\boldsymbol{B}$?

Solution:

Since $|A \times B| = |A||B|\sin\theta$, *and* $A \cdot B = |A||B|\cos\theta$

Equate

$$|A||B|\sin\theta = |A||B|\cos\theta \Rightarrow \sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^\circ$$

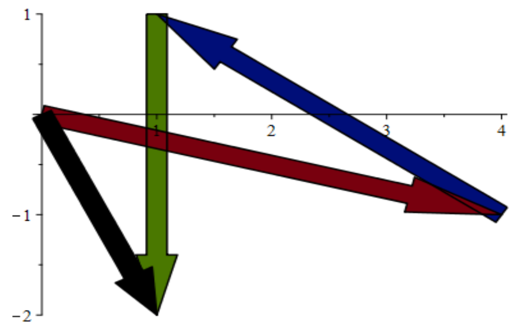
18. Three coplanar vectors $\mathbf{A} = 4i - j$; $\mathbf{B} = -3i + 2j$; *and* $\mathbf{C} = -3j$

Find the resultant vector

Solution:

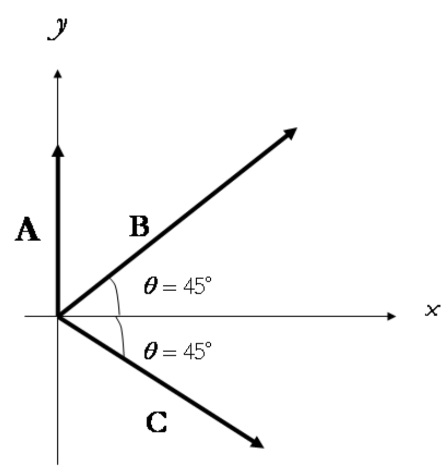
Simply the resultant vector is their sum, that is

$$\mathbf{A} + B + C = (4i - j) + (-3i + 2j) + (-3j)$$
$$A + B + C = (4 - 3 + 0)i + (-1 + 2 - 3)j$$
$$A + B + C = i - 2j$$



The sum of 3 vectors, showing the resultant in black.

19. Three vectors are oriented as shown in figure, where $|\boldsymbol{A}| = 20.0\ units$, $|\boldsymbol{B}| = 40.0\ units$, and $|\boldsymbol{C}| = 30.0\ units$. Find (a) the x and y components of the resultant vector (expressed in unit vector notation) and (b) the magnitude and direction of the resultant vector.



Solution:

The components of each vector can be determined with the figure help.

The vector A is aligned to the positive y-axis, thus

$$A = 0i + 20j$$

The vector B is inclined to the positive x-axis by 45 degrees, resolving the vector to its components

$$B_x = 40\cos(45) = \frac{40}{\sqrt{2}},\ B_y = 40\sin(45) = \frac{40}{\sqrt{2}}$$

$$B = 28.28i + 28.28j$$

Similarly for the vector C

$$C_x = 30\cos(45) = \frac{30}{\sqrt{2}},\ C_y = 30\sin(45) = \frac{30}{\sqrt{2}}$$

$$C = 21.21i - 21.21j$$

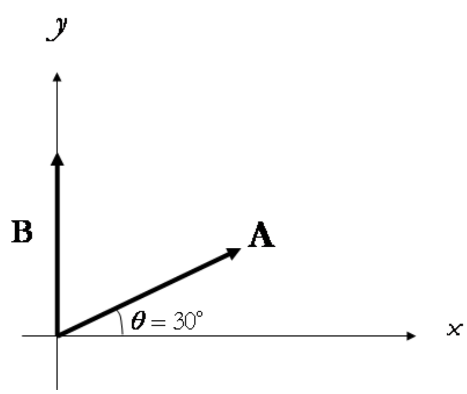
The magnitude and direction of the vector sum

$$A + B + C = (0i + 20j) + (28.28i + 28.28j) + (21.21i - 21.21j)$$
$$A + B + C = (0 + 28.28 + 21.21)i + (20 + 28.28 - 21.21)j$$
$$A + B + C = 49.49i + 27.07j$$

$$|A + B + C| = \sqrt{(49.49)^2 + (27.07)^2} = 56.41$$

$$\theta = \tan^{-1}\left(\frac{27.07}{49.49}\right) = 28.67^\circ$$

20. Each of the displacement vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ as in figure has a magnitude of $3.0\ m$. Find a. $\boldsymbol{A}+\boldsymbol{B}$, b. $\boldsymbol{A}-\boldsymbol{B}$, c. $\boldsymbol{B}-\boldsymbol{A}$, d. $\boldsymbol{A}-2\boldsymbol{B}$. Report all angles counterclockwise from the positive x axis.



Solution:

$$A_x = 30\cos(30) = 26,\ A_y = 30\sin(30) = 15$$
$$A = 26i + 15j$$

$$B = 0i + 30j$$

$$A + B = 26i + 45j,\ \theta = \tan^{-1}\left(\frac{45}{26}\right) = 60^\circ$$, lies in the first quadrant

$$A - B = 26i - 15j,\ \theta = \tan^{-1}\left(\frac{-15}{26}\right) = 330^\circ$$, lies in the fourth quadrant

$$B - A = -26i + 15j,\ \theta = \tan^{-1}\left(\frac{15}{-26}\right) = 150^\circ$$, lies in the second quadrant

$$A - 2B = 26i - 45j,\ \theta = \tan^{-1}\left(\frac{-45}{26}\right) = 300^\circ$$, lies in the fourth quadrant

21. For what values of b are the vectors $\mathbf{A} = -6i + bj + 2k,\ and\ \mathbf{B} = bi + b^2 j + bk$ perpendicular?

Solution:

When the dot product vanishes that means the two vectors are perpendicular on each other, that is

$$\mathbf{A} \cdot B = 0 = \left(-6i + bj + 2k\right) \cdot \left(bi + b^2 j + bk\right) = -6b + b^3 + 2b$$
$$b^3 - 4b = 0,$$
$$b\left(b^2 - 4\right) = 0,$$
$$b\left(b - 2\right)\left(b + 2\right) = 0,$$
$$b = 0, 2, -2.$$

22. Is the vector $(i + j + k)$ a unit vector? Justify your answer. (b) Can a unit vector have any components with magnitude greater than unity? Can it have any negative components? In each case justify your answer. (c) If $\vec{A} = a(3i + 4j)$, where a is a constant, determine the value of a that makes a unit vector.

Solution:

a. $(i + j + k)$ is not a unit vector since $\sqrt{1 + 1 + 1} = \sqrt{3}$,

b. No, this would leads to vector magnitude greater than unity,
c. Yes, unit vector can have a negative components,

d. $\hat{A} = \frac{(3i+4j)}{\sqrt{3^2+4^2}} = \frac{1}{5}(3i + 4j) \rightarrow a = \frac{1}{5}$

23. If $\vec{a} - \vec{b} = 2\vec{c}, \vec{a} + \vec{b} = 4\vec{c}, and\ \vec{c} = 3i + 4j$, then what are the vectors $\vec{a}$ and $\vec{b}$?

Then Find $\vec{a} \cdot \vec{b}\ \ and\ \vec{a} \times \vec{b}$

Solution:

a. In simple way:

$$\vec{a} - \vec{b} = 2\vec{c},$$
$$\vec{a} + \vec{b} = 4\vec{c},$$
$$adding$$
$$2\vec{a} = 6\vec{c} \rightarrow \vec{a} = 3\ \vec{c} = 9i + 12j$$
$$\vec{a} + \vec{b} = 4\vec{c} \rightarrow 3\ \vec{c} + \vec{b} = 4\vec{c} \rightarrow \vec{b} = \vec{c} = 3i + 4j$$

In a long way:

$$\vec{a} - \vec{b} = (a_x - b_x)i + (a_y - b_y) = 2(3i + 4j) = 6i + 8j$$

Comparing

$$(a_x - b_x) = 6 \qquad (1)$$

$$(a_y - b_y) = 8 \qquad (2)$$

$$\vec{a} + \vec{b} = (a_x + b_x)i + (a_y + b_y) = 4(3i + 4j) = 12i + 16j$$

Comparing

$$(a_x + b_x) = 12 \qquad (3)$$
$$(a_y + b_y) = 16 \qquad (4)$$

Solving equations 1 and 3 for $a_x and\ b_x$, and equations 2 and 4 for $a_y\ and\ b_y$
That is,

$$\vec{a} = 9i + 12j, and\ \vec{b} = 3i + 4j$$

b. $\vec{a} \cdot \vec{b} = 9 \times 3 + 12 \times 4 = 27 + 48 = 75,\ \ and\ \ \vec{a} \times \vec{b} = 0$

24. Find the angle between the vectors using the dot product.

a. $\mathbf{A} = i + 2j, \qquad \mathbf{B} = 2i - j$

b. $\mathbf{A} = 3i + 2j + 3k, \quad \mathbf{B} = i - j + 2k$

c. $\mathbf{A} = 3i - j + 2k, \quad \mathbf{B} = i - j + k$

Solution:

$a.\quad \mathbf{A} = i + 2j, \qquad \mathbf{B} = 2i - j \Rightarrow$

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right) = \cos^{-1}\left(\frac{0}{|A||B|}\right) = 90^\circ$$

$b.\quad \mathbf{A} = 3i + 2j + 3k, \quad \mathbf{B} = i - j + 2k \Rightarrow$

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right) = \cos^{-1}\left(\frac{7}{\sqrt{20}\sqrt{6}}\right) = 52.46^\circ$$

$c.\quad \mathbf{A} = 3i - j + 2k, \quad \mathbf{B} = i - j + k \Rightarrow$

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right) = \cos^{-1}\left(\frac{4}{\sqrt{14}\sqrt{3}}\right) = 22.2^\circ$$

25. Find the angle between the vectors using the cross product.

$a.\quad \mathbf{A} = i - 2j - k, \qquad \mathbf{B} = 2i - j + 3k$

$b.\quad \mathbf{A} = 2i - 7j + 11k, \qquad \mathbf{B} = 5i - j + 2k$

$c.\quad \mathbf{A} = i - 3j + 2k, \qquad \mathbf{B} = i - 7j + 3k$

Solution:

$a.\quad \mathbf{A} = i - 2j - k, \qquad \mathbf{B} = 2i - j + 3k$

$$\theta = \sin^{-1}\left(\frac{|A \times B|}{|A||B|}\right) = \sin^{-1}\left(\frac{|-5i - j + 3k|}{\sqrt{6}\sqrt{14}}\right) = 40.2^\circ$$

$b.\quad \mathbf{A} = 2i - 7j + 11k, \qquad \mathbf{B} = 5i - j + 2k$

$$\theta = \sin^{-1}\left(\frac{|A \times B|}{|A||B|}\right) = \sin^{-1}\left(\frac{|-3i + 51j + 33k|}{\sqrt{174}\sqrt{30}}\right) = 57.33^\circ$$

$c.\quad \mathbf{A} = i - 3j + 2k, \qquad \mathbf{B} = i - 7j + 3k$

$$\theta = \sin^{-1}\left(\frac{|A \times B|}{|A||B|}\right) = \sin^{-1}\left(\frac{|5i - j - 4k|}{\sqrt{14}\sqrt{59}}\right) = 13.03^\circ$$

26. Given tow vectors $\mathbf{A} = -i + 2j - 5k, \; and \; \mathbf{B} = 2i + 3j - 2k$, obtain the following Find the magnitude of each vector, write an expression for the vector sum, using the unit vector, find the magnitude of the vector sum, and find the angle between the vector ***A*** and ***B*** using: the scalar product and the vector product

Solution:

1. $|\mathbf{A}| = \sqrt{1+4+25} = \sqrt{30}, \quad |\mathbf{B}| = \sqrt{4+9+4} = \sqrt{17}$

2. $\mathbf{C} = \mathbf{A} + \mathbf{B} = (-1+2)i + (2+3)j + (-5-2)k = i + 5j - 7k$,

$$|\mathbf{C}| = \sqrt{1+25+49} = \sqrt{75} \quad \text{, so } \hat{C} = \frac{\mathbf{C}}{|\mathbf{C}|} = \frac{i+5j-7k}{\sqrt{75}}$$

3. $|\mathbf{C}| = \sqrt{75}$,

4. $\mathbf{A}\cdot\mathbf{B} = -2+6+10 = 14, \ \ \theta = \cos^{-1}\left(\frac{14}{\sqrt{30}\sqrt{17}}\right) = 51.7^\circ$

5. $\mathbf{A}\times\mathbf{B} = 11 \ \ i - 12j - 7k$, $\theta = \sin^{-1}\left(\frac{|\mathbf{A}\times\mathbf{B}|}{|\mathbf{A}||\mathbf{B}|}\right) = \sin^{-1}\left(\frac{\sqrt{121+144+49}}{\sqrt{30}\sqrt{17}}\right)$,

27. Find the scalar product and the vector product for the two vectors $\mathbf{A} = 2i - 5j \quad and \quad \mathbf{B} = 5i + 2j$ Find the scalar product and the vector product for the two vectors $\mathbf{A} = 2i - 5j \quad and \quad \mathbf{B} = 4i - 10j$

28. Solution

The dot product is: $\mathbf{A}\cdot\mathbf{B} = 10 - 10 = 0$, and $\mathbf{A}\cdot\mathbf{B} = 8 + 50 = 58$,

The vector product is :

$$\begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 5 & 2 & 0 \end{vmatrix} = i\begin{vmatrix} -5 & 0 \\ 2 & 0 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 5 & 0 \end{vmatrix} + k\begin{vmatrix} 2 & -5 \\ 5 & 2 \end{vmatrix} = k(4-(-25)) = 29k$$

$$\begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 4 & -10 & 0 \end{vmatrix} = i\begin{vmatrix} -5 & 0 \\ -10 & 0 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix} + k\begin{vmatrix} 2 & -5 \\ 4 & -10 \end{vmatrix} = k(-20-(-20)) = 0k$$

29. Given two vectors $\mathbf{A} = -i + 2j - 5k, \quad and \quad \mathbf{B} = 2i + 3j - 2k$, obtain the following

1. $\mathbf{A}\cdot\mathbf{B}$, 2. $\mathbf{A}\times\mathbf{B}$, 3. $\hat{A}$, 4. The angle between the two vectors ϑ , 5. The magnitude of the vector sum $\mathbf{A}+3\mathbf{B}$

Solution

The scalar product (dot) $\mathbf{A}\cdot\mathbf{B}$,

$\mathbf{A} = -i + 2j - 5k, \quad and \quad \mathbf{B} = 2i + 3j - 2k$
$\mathbf{A}\cdot\mathbf{B} = -2 + 6 + 10 = 14$

The cross product (vector) $\mathbf{A}\times\mathbf{B}$,

$$\mathbf{A}=-i+2j-5k,\quad and \qquad \mathbf{B}=2i+3j-2k$$

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix} i & j & k \\ -1 & 2 & -5 \\ 2 & 3 & -2 \end{vmatrix}=11i-12j-7k$$

The unit vector $\hat{A}$,

$$\mathbf{A}=-i+2j-5k,$$
$$|\mathbf{A}|=\sqrt{1+4+25}=\sqrt{30}$$
$$\hat{A}=\frac{-i+2j-5k}{\sqrt{30}}$$

The angle between the two vectors ϑ ,

$$\mathbf{A}=-i+2j-5k,\quad and \qquad \mathbf{B}=2i+3j-2k$$
$$|\mathbf{A}|=\sqrt{30},\quad |\mathbf{B}|=\sqrt{4+9+4}=\sqrt{17}$$
$$\cos(\vartheta)=\frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}||\mathbf{B}|}=\frac{14}{\sqrt{30}\sqrt{17}}\Rightarrow\vartheta=\cos^{-1}\left(\frac{14}{\sqrt{30}\sqrt{17}}\right)$$

The magnitude of the vector sum $\mathbf{A}+3\mathbf{B}$

$$\mathbf{A}=-i+2j-5k,\quad and \quad \mathbf{B}=2i+3j-2k$$
$$\mathbf{3B}=\mathbf{C}=3(2i+3j-2k)=6i+9j-6k$$
$$\mathbf{A}+\mathbf{C}=-i+2j-5k\ \ +6i+9j-6k$$
$$\mathbf{A}+\mathbf{3B}\ \ =5i+11j-11k$$
$$|\mathbf{A}+\mathbf{3B}|=\sqrt{25+121+121}=\sqrt{267}=16.34$$

30. For the following three vectors $\mathbf{A}=-i+2j$, $\mathbf{B}=i-2k$, *and* $\mathbf{C}=-2j+2k$ find,

1. The angle between the vectors $\mathbf{D}=\mathbf{A}+\mathbf{B}$, *and* $\mathbf{T}=\mathbf{A}-\mathbf{C}$

2. find the vectors $\mathbf{E}=\mathbf{A}\times\mathbf{C}$, *and* $\mathbf{F}=\mathbf{B}\times\mathbf{C}$, and the unit vectors $\hat{E}$

Solution

The vector $\mathbf{D}=(-1+1)i+(2+0)j+(0+(-2))k=0i+2j-2k$, and the vector $\mathbf{T}=(-1-0)i+(2-(-2))j+(0-2)k=-i+4j-2k$.

The magnitude of $|\mathbf{D}|=\sqrt{0+4+4}=\sqrt{8}$, and the magnitude of $|\mathbf{T}|=\sqrt{1+16+4}=\sqrt{21}$, the angle between the vectors D *and* T *is*,

$$\theta=\cos^{-1}\left(\frac{\mathbf{D}\cdot\mathbf{T}}{|\mathbf{D}||\mathbf{T}|}\right)=\cos^{-1}\left(\frac{8+4}{\sqrt{8}\sqrt{21}}\right)=22.2^{\circ}$$

$$\mathbf{E}=\begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ 0 & -2 & 2 \end{vmatrix}=i\,(4)-j\,(-2)+k\,(2)=4i+2j+2k$$

$$\hat{E}=\frac{\mathbf{E}}{|\mathbf{E}|}=\frac{4i+2j+2k}{\sqrt{16+4+4}}=\frac{4i+2j+2k}{\sqrt{24}}$$

$$\mathbf{F}=\begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & -2 & 2 \end{vmatrix}=i\,(-4)-j\,(2)+k\,(-2)=-4i-2j-2k$$

31. When two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ are drawn from a common point, the angle between them is θ . Show that the magnitude of their vector sum is given by $\sqrt{A^2+B^2+2AB\cos\theta}$. If $\boldsymbol{A}$ and $\boldsymbol{B}$ have the same magnitude, under what condition will their vector sum have the same have the magnitude of the vector $\boldsymbol{A}$ or $\boldsymbol{B}$?

Solution

a. The vector sum $\mathbf{A}+\mathbf{B}=(A_x+B_x)\ i+(A_y+B_y)j$, the magnitude of this vector sum is

$$|\mathbf{A}+\mathbf{B}|=\sqrt{(A_x+B_x)^2+(A_y+B_y)^2}=\sqrt{A_x^2+B_x^2+2A_xB_x+A_y^2+B_y^2+2A_yB_y}$$

Rearrange,

$$\sqrt{(A_x^2+A_y^2)+(B_x^2+B_y^2)+2A_xB_x+2A_yB_y}=\sqrt{A^2+B^2+2(A_xB_x+A_yB_y)}$$

But,

$$A_xB_x+A_yB_y=A\cdot B=|A||B|\cos(\theta)$$, so

$$|A+B| = \sqrt{A^2 + B^2 + 2AB\cos(\theta)}$$

b. if $|\mathbf{A}| = |\mathbf{B}|$, then $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}| = |\mathbf{B}|$

so we need to solve this,

$$|\mathbf{A}+\mathbf{B}| = \sqrt{A^2 + A^2 + 2A^2\cos(\theta)} = A\sqrt{2+2\cos(\theta)}$$, this means

$|\mathbf{A}+\mathbf{B}| = |\mathbf{A}|$, *when* $\sqrt{2+2\cos(\theta)} = 1$, solve for the angle θ which makes this true!

I think you can solve it! Yeah, you are right it is 120 degree.

By now, I think you can do the same steps for **A-B**, think about it!

32. Find the two unit vectors that are perpendicular to the plane formed by the two vectors

$$\boldsymbol{A} = 3i - 2j \text{ and } \boldsymbol{B} = 4i - 3j + 2k$$

Solution:

To find the vectors which perpendicular on the plane, find the cross product

$\mathbf{A} = 3i - 2j$, *and* $\mathbf{B} = 4i - 3j + 2k$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} i & j & k \\ 3 & -2 & 0 \\ 4 & -3 & 2 \end{vmatrix} = -4i - 6j - k$$

Thus the two vectors are $C = -4i - 6j - k$ *and* $-C$

33. Find a vector in the $x - y$ plane that is perpendicular to the vector $A = 10i - 7j$

Solution

Using the dot product property, that is

$$\boldsymbol{A}\cdot\boldsymbol{B} = AB\ cos\theta$$

$\boldsymbol{A}$ and $\boldsymbol{B}$ are perpendicular then $\boldsymbol{A}\cdot\boldsymbol{B} = 0$

Let

$$B = B_x i + B_y j$$

Thus,

$$\boldsymbol{A}\cdot\boldsymbol{B} = (10i - 7j)\cdot(B_x i + B_y j) = 0$$
$$10B_x - 7\,B_y = 0$$

Or,

$$10B_x = 7\,B_y \rightarrow B_x = \frac{7}{10}\,B_y$$

You have infinitely many solution, just pick any value or B_y, you have B_x that makes the vector $\boldsymbol{B}$ perpendicular on the vector $\boldsymbol{A}$!

Take $B_y = 10$

Then, $B_x = 7$

And your vector is

$$B = 7i + 10j$$

I urge to try another values for B_y and check your answer by performing the dot product $\boldsymbol{A} \cdot \boldsymbol{B} = 0$.

Chapter 2

Solutions

Part I: MCQs

Answered Multiple Choice Questions with some explanations

1. A body is in **dynamic** (means there is a motion) equilibrium only when it is

a) At rest

b) Moving with a variable velocity

c) Moving with uniform acceleration

d) Moving with uniform velocity

2. Torque has zero value, if the angle between $\vec{r}$ and $\vec{F}$ is

a) $\mathbf{0°}$

b) $90°$

c) $270°$

d) $180°$

3**.** The direction of torque is

a) Along position vector $\vec{r}$

b) Parallel to the plane containing $\vec{r}$and $\vec{F}$

c) Along force $\vec{F}$

d) Perpendicular to the plane containing $\vec{r}$ and $\vec{F}$

4. If a body is at rest, then it will be in

a) Static equilibrium

b) Dynamic equilibrium

c) Translational equilibrium

d) Unstable equilibrium

5. For a body to be in **complete** equilibrium,

a) $\sum \boldsymbol{F} = \mathbf{0}$ *and* $\sum \boldsymbol{\tau} = \mathbf{0}$

b) $\sum \tau = 0$

c) $\sum F = 0$

d) $None$

6. A body will be in **complete** equilibrium when it is satisfying:

a) 1st condition of equilibrium

b) 2nd condition of equilibrium

c) Both 1st and 2nd condition of equilibrium

d) Impossible

7. SI unit of torque is:

a) Nm^{-1}

b) $\boldsymbol{Nm}$

c) Nm^{-2}

d) $None$

8. Torque is defined as.

a) Turning effect of force

b) Cross product of force and position vector

c) Product of force and moment arm

d) All a, b and c are correct

9. The minimum number of unequal forces whose resultant will be zero:

a) 2

b) 3

c) 4

d) 5

10. If the line of action of force passes through axis of rotation or the origin, then its torque is:

a) Maximum

b) Unity

c) Zero

d) None of these

11. If the position $\vec{r}$ and force $\vec{F}$ are in same direction, then torque will be:

a) Maximum

b) Minimum

c) Same

d) Negative

12. The direction of torque can be found by:

a) Head to tail rule

b) Right hand rule

c) Left hand rule

d) Fleming rule

13. If value of moment arm is zero, then torque produced will be

a) 1
b) 0
c) doubled
d) decreased
e)

14. According to system international, unit of torque is

a) newton meter
b) newton per meter
c) newton per meter square
d) per newton meter

15. Distance from a point around which body rotates and point at end is called

a) length of object
b) moment arm
c) momentum arm
d) distance of object

16. Pair of forces that cause steering wheel of a car to rotate is called

   a) couple
   b) friction
   c) normal force
   d) weight

17. To form a couple, force should be

a) equal in magnitude
b) parallel and opposite
c) separated by distance
d) all of above

18. Sum of all torques acting on a body is zero, this condition represents equilibrium's

a) first condition
b) second condition
c) third condition
d) fourth condition

19. If 2nd condition of equilibrium is satisfied, body will be in

a) translational equilibrium
b) rotational equilibrium
c) static equilibrium
d) dynamic equilibrium

20. If counter clockwise is taken as positive, then anti-clockwise will be taken as

a) positive
b) negative
c) doubled
d) halved

Explanation: counter means anti

21. If first condition of equilibrium is satisfied, then body will be in

a) translational equilibrium
b) rotational equilibrium
c) static equilibrium
d) dynamic equilibrium

22. A doorknob is normally placed near the edge of the door opposite the hinges. This is because

A. the door would look funny with the knob in any other position.

B. that is where the door's rotational inertia will be lowest.

C. a force at this position will produce a minimum torque.

D. a force at this position will produce a maximum torque.

E. that is where the door's center of mass is located.

23. To obtain the maximum torque for a given force when using a wrench, the force should be applied at a ____degree angle to the handle of the wrench.

A. 30 **B. 90** C. 180 D. 60 E. 45

Part II: End of Chapter Problems Solutions

Problems

1. (a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

   Solution:

   (a) $\tau = r_{\perp}F = 0.850\,\text{m} \times 55.0\,\text{N} = 46.75\,\text{N}\cdot\text{m} = \underline{46.8\,\text{N}\cdot\text{m}}$

2. (b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges
3. Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the **second child** exert to keep the door from moving? Assume friction is negligible.

Solution:

First child torque is: $\tau_1 = 17.5N \times 0.60m = 10.5\ N.m$

Second child opposite torque is $\tau_2 = F\ N \times 0.45m$

Second child torque must exceeds the first child torque, that is

$$\tau_2 \geq \tau_1 \rightarrow 10.5Nm = F\ N\ \times 0.45\ m$$

Solve for F,

$$F = \frac{10.5Nm}{0.45\ m} = 23.33\ N$$

Thus F should be greater than $23.33\ N$.

4. Two children of mass $20.0\ kg$ and $30.0\ kg$ sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of $3.00\ m$, at what distance from the pivot point is the small child sitting in order to maintain the balance

   Solution:

   Net $\tau = 0, \tau_{cw} = \tau_{ccw}$

$$(20.0kg \times g) \times r_1 m = (30.0kg \times g) \times r_2 m$$

$$r_1 = \frac{30.0}{20.0} \times r_2 \rightarrow r_1 = 1.5\ r_2$$

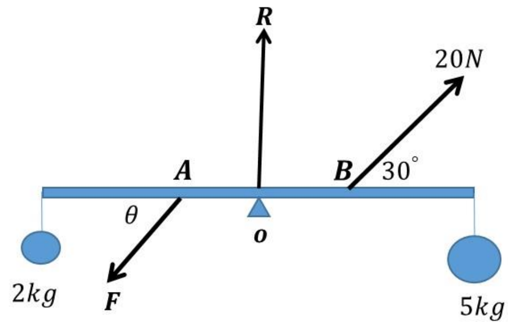
   But

$$r_1 + r_2 = 3.0\ m$$

$$1.5r_2 + r_2 = 3.0\ m \rightarrow 2.5\ r_2 = 3.0 \rightarrow r_2 = \frac{3.0}{2.5} = 1.2\ m$$

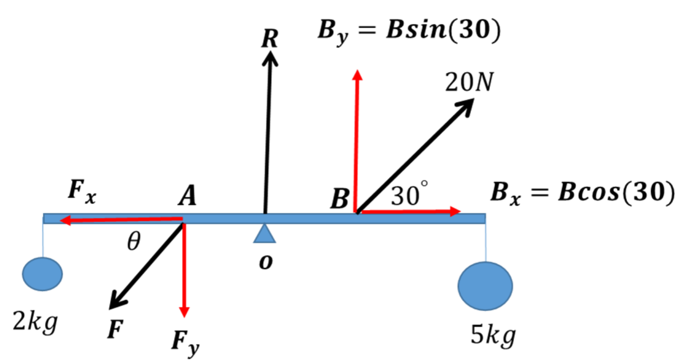
   And $r_1 = 1.8\ m$

5. Find the force F that makes the shown system below in equilibrium, where the bar is weightless and is $60\ cm$ in length. The pivot O is in the middle of the bar, and the points A and B are locate $12\ cm$ to the left and $15\ cm$ to the right of the pivot, respectively. What is the magnitude of the reaction force on the pivot?



Solution:

In this question we need to solve for R and F. You may have noticed that the system is in equilibrium. Thus, you can apply the equilibrium conditions to solve for your unknowns, F and R. First, resolve vectors,



Now, apply the second law of equilibrium, we have (assume $g = 10m/s^2$)

$$2kg \times 10m/s^2 \times \frac{L}{2} + F_y \times L_A + B \times sin(30) \times L_B - 5kg \times \frac{L}{2} \times 10m/s^2 \; =$$

Where, $L_A = 0.12m, L_B = 0.15m, and\ L = 0.6m$

Apply and solve for F_y, we have $F_y = 62.5\ N$

$$\sum F_x = 20\cos 30 - F_x = 0 \rightarrow F_x = 20\cos 30 = 17.32\ N$$

$$F = -17.32\, i - 62.5\, j \quad N$$

$\theta = tan^{-1}\left(\frac{62.5}{17.32}\right) = 74.5^{\circ}$ Below the x-axis

Applying the second condition of equilibrium, (notice the question didn't ask for F)

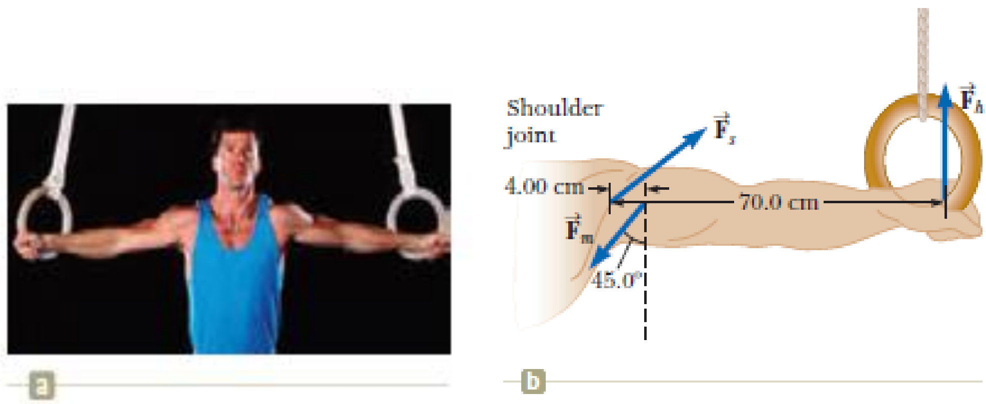
Apply the $\sum all(F_y) = 0$, we have

$$R + 20N \times sin(30) - 2kg \times \frac{10m}{s^2} - 5kg \times \frac{10m}{s^2} - 62.5N = 0$$

Solve for R

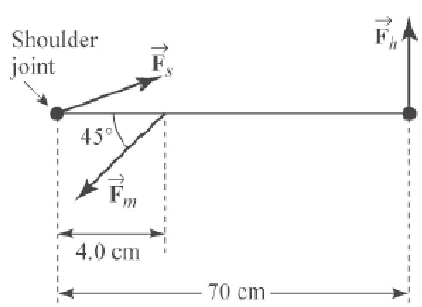
$$R = 122.5N$$

6. When a gymnast performing on the rings executes the iron cross, he maintains the position at rest shown in Figure below (a). In this maneuver, the gymnast's feet (not shown) are off the floor. The primary muscles involved in supporting this position are the latissimus dorsi ("lats") and the pectoralis major ("pecs"). One of the rings exerts an upward force $\boldsymbol{F_h}$ on a hand as shown in Figure below (b). The force $\boldsymbol{F_s}$ is exerted by the shoulder joint on the arm. The latissimus dorsi and pectoralis major muscles exert a total force $\boldsymbol{F_m}$on the arm. (a) Using the information in the figure, find the magnitude of the force $\boldsymbol{F_m}$ for an athlete of weight $750\ N$.



Solution:

We can draw the FBD as follow:



From the symmetry of the situation we can conclude that the magnitude of the upward force on each hand is half the athlete's weight. That is,

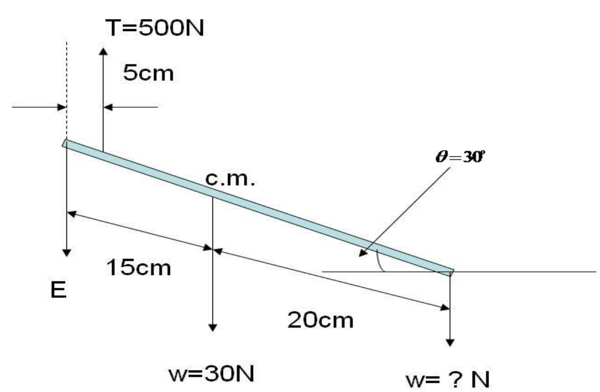
$$F_h = \frac{750N}{2} = 375N$$

Considering the shoulder joint as the pivot, applying the second condition of equilibrium, we have

$$\sum \tau = 0 \rightarrow F_h N \times 0.7m - F_m \sin 45\ N \times 0.04m$$

Solving for F_m, we have $F_m = 9.28kN$

7. An athlete his arm is inclined by 30°, as shown in figure below. If the maximum tension can be tolerated by athlete is $500\ N$. Calculate the maximum Weight can be held in his hand



Solution:

$$T\ cos\left(30\right)\times 0.05 - w\ cos\left(30\right)\times 0.35 - 30 cos\left(30\right)\times 0.15 = 0$$

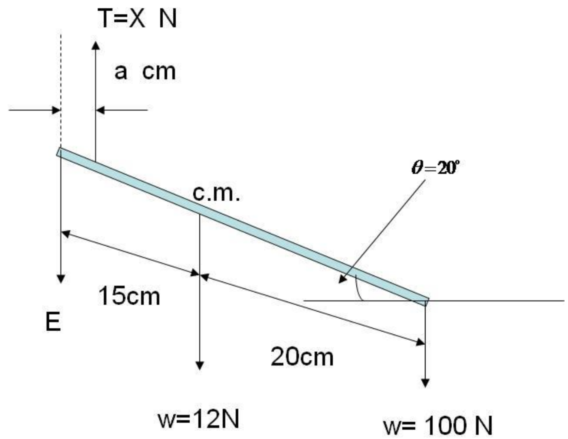
$$cancel\ \ cos\left(30\right), becomes$$

$$500\times 0.05 - w\ \times 0.35 - 30\times 0.15 = 0,$$

$$w\ = \frac{500\times 0.05 - 30\times 0.15}{0.35} = 58.6\ N$$

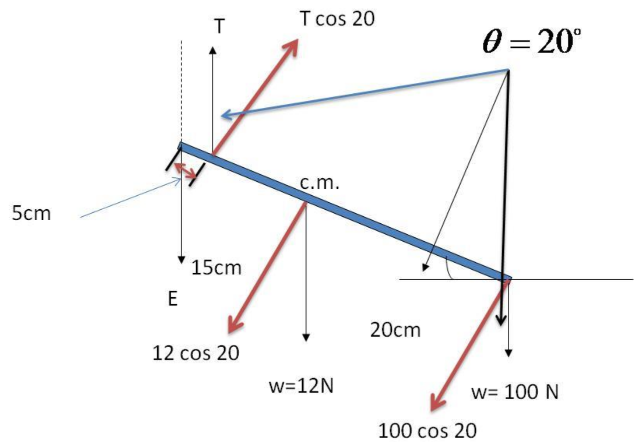
$$m \cong 6\,kg$$

8. An athlete his arm is inclined by 20° , as shown in figure below. Find the maximum tension. Consider $a = 5$ cm



Solution:

The free body diagram is



$$T\ cos\left(20\right)\times 0.05-100cos\left(20\right)\times 0.35-12cos\left(20\right)\times 0.15=0$$

$$cancel\ \ cos\left(20\right),becomes$$

$$T\ \times 0.05-100\times 0.35-12\times 0.15=0,$$

$$T\ =\frac{100\times 0.35+12\times 0.15}{0.05}=736N$$

9. Repeat, Q2 with the athlete arm inclined 20 deg above the horizontal.

Consider $a = 7$ cm

Solution

The analysis applies, but with $a = 7\ cm$

$$T\ cos\,(20) \times 0.07 - 100 cos\,(20) \times 0.35 - 12 cos\,(20) \times 0.15 = 0$$
$$cancel\ \ cos\,(20), becomes$$
$$T\ \times 0.07 - 100 \times 0.35 - 12 \times 0.15 = 0,$$
$$T\ = \frac{100 \times 0.35 + 12 \times 0.15}{0.07} = 525.7N$$

10. Compare the results of the solved example and with Q7 and Q8 result!

Solution

The further the insertion point the less tension in the muscles

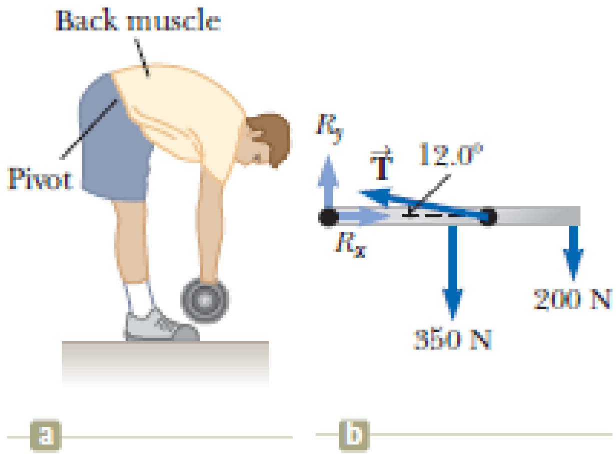
11. Assume a person bends forward to lift a load "with his back" as shown in Figure below. The spine pivots mainly at the fifth lumbar vertebra, with the principal supporting force provided by the erector spinal is muscle in the back. To see the magnitude of the forces involved, consider the model shown in Figure below (b) for a person bending forward to lift a $200\ N$ object. The spine and upper body are represented as a uniform horizontal rod of weight $350\ N$, pivoted at the base of the spine. The erector spinal is muscle, attached at a point two-thirds of the way up the spine, maintains the position of the back. The angle between the spine and this muscle is $\theta = 12^{\circ}$. Find

(a) the tension T in the back muscle and

(b) the compressional force in the spine.

(c) Is this method a good way to lift a load? Explain your answer, using the results of parts (a) and (b).

(d) Can you suggest a better method to lift a load



Solution

a. Take the sum of torques about the hip is zero

$$0 = -350 \times \frac{L}{2} - 200 \times L + Tsin(12) \times \frac{2L}{3}$$

Notice that L drops out the calculations! Solve for T

$$T = 2.71\ kN$$

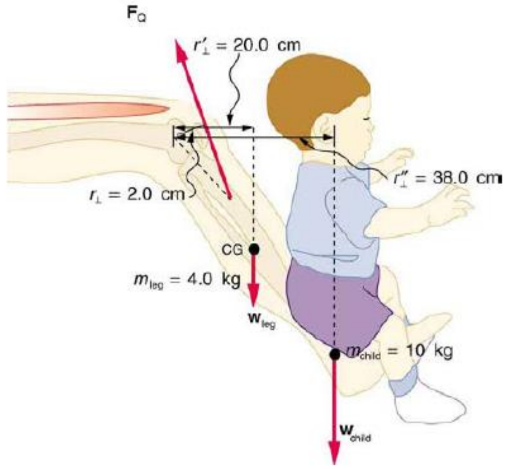
b. R_x is called the compressional force
From the analysis

$$R_x = T_x = Tcos(12) = 2.65\ kN$$

c. You should lift "with your knees" rather than "with your back." In this situation, with a load weighing only $200\ N$, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible.

d. In this situation, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible.

12. A father lifts his child as shown in Figure below. What force should the upper leg muscle exert to lift the child at a constant speed?



Solution

A father lifts his child as shown in figure. What force should the upper leg muscle exert to lift the child at a constant speed?

$$\text{net } \tau = 0 \text{ (pivot at knee joint)}$$
$$F_Q \times 2.00\text{ cm} = (w_{leg})(20.0\text{ cm}) + (w_{child})(38.0\text{ cm})$$
$$w_{leg} = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$$
$$w_{child} = (10.00\text{ kg})(9.80\text{ m/s}^2) = 98.0\text{ N}$$
$$\therefore F_Q = \frac{39.2\text{ N} \times 0.200\text{ m} + 98.0\text{ N} \times 0.380\text{ m}}{0.0200\text{ m}} = 2254\text{ N} = \underline{2.25 \times 10^3\text{ N}}$$

13. When a person stands on tiptoe (a strenuous position), the position of the foot is as shown in Figure q8a. The total weight of the body Fg is supported by the force n exerted by the floor on the toe. A mechanical model for the situation is shown in Figure 2q8b, where T is the force exerted by the Achilles tendon on the foot and R is the force exerted by the tibia on the foot. Find the values of T, R, and when $Fg = 700\ N$

Solution

Since the person stand on his toe is in equilibrium, start applying the laws of equilibrium, that is, apply the first law of equilibrium, But first we have to draw the Free body diagram, and then choose the pivot of the ankle in this position at R, thus the forces.

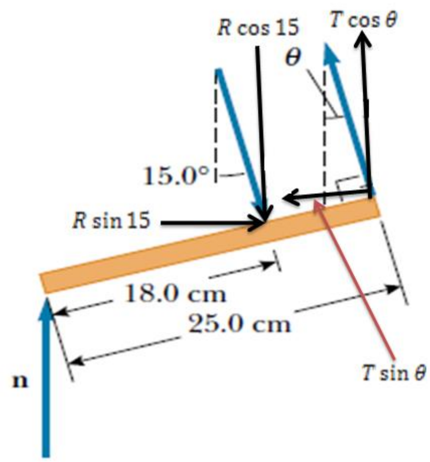
Now apply the first condition of equilibrium, that is

The sum of forces have to equal to zero,

$\sum f_x = R\ sin\ 15 - T\sin\theta = 0$ (1)

$\sum f_y = n - Rcos\ 15 + T\cos\theta = 0$ (2)

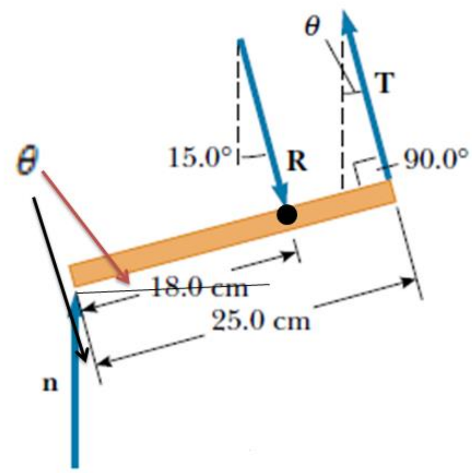
We know that $n = 700N$, equation (2) becomes,



$\sum f_y = 700 - R\ cos\ 15 + T\cos\theta = 0$ (3)

Now, we have two equations with three unknown namely; $R, T, and\ \theta$. Thus we have to invoke the second condition of equilibrium to be able to solve, from figure below, we have

$\sum \tau = -700\cos\theta \times (0.18m)\ c.w. + T \times (0.25 - 0.18)m = 0$ (4)



$$-700\cos\theta \times (0.18m)\ c.w. + T \times (0.25 - 0.18)m = 0$$

$$T = \frac{700N \times 0.18m \times \cos\theta}{0.07m} = 1800 \times \cos\theta\ N$$

From equation (1),

$$R\ sin\ 15 - T\sin\theta = 0 \rightarrow R = 1800\sin\theta\cos\theta / \sin 15$$

Substitute R in equation 3,

$$700 - R\ cos\ 15 + T\cos\theta = 0$$

$$\rightarrow 700 - \frac{1800\sin\theta\cos\theta}{\sin 15} \times \cos 15 + 1800\cos^2\theta = 0,$$

Rearrange for θ, we find

$$700 - 6714\sin\theta\cos\theta + 1800\cos^2\theta = 0,$$

Divide by 1800, we have

$$0.388 - 3.73\ \sin\theta\cos\theta + \cos^2\theta = 0,$$

For small angle we can use the series expansion for the sin and cos functions, that is,

$$\sin\theta = \theta - \frac{\theta^3}{3!} \ and \cos\theta = 1 - \frac{\theta^2}{2!}$$

Now, solve for theta, that is

$$0.388 - 3.77\left(\theta - \frac{\theta^3}{6}\right)\times\left(1 - \frac{\theta^2}{2}\right) + \left(1 - \frac{\theta^2}{2}\right)^2 = 0$$

$$\theta = 21.142^{\circ}$$

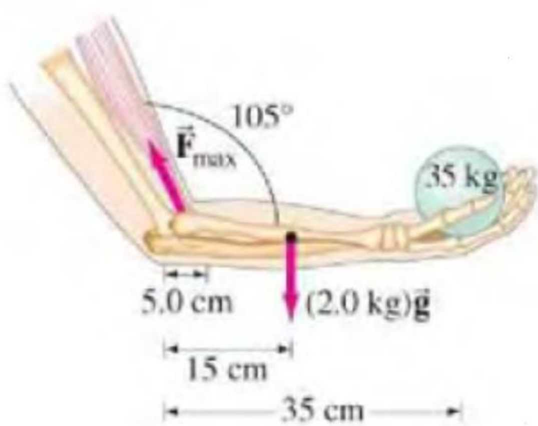
Now,

$$T = 1800 \times \cos 21.142 \ \ N = 1678.84\ N$$

And

$$R = 1800 \sin 21.142 \cos 21.142 \,/\, \sin 15 = 2339.57\ N$$

14. If $35\ kg$ is the maximum mass m that a person can hold in a hand when the arm is positioned with a $105°$ angle at the elbow as shown in Figure, what is the maximum force F_{max} that the biceps muscle exerts on the forearm? Assume the forearm and hand have a total mass of $2.0\ kg$ with a CG that is $15\ cm$ from the elbow, and that the biceps muscle attaches $5.0\ cm$ from the elbow.



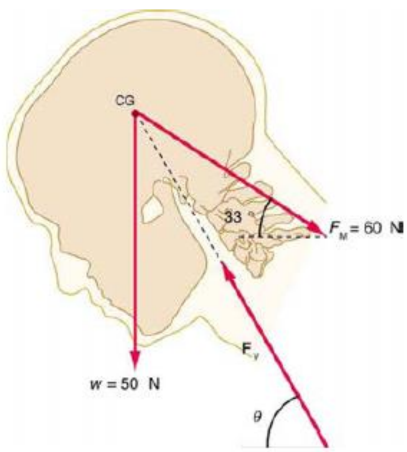
Solution:

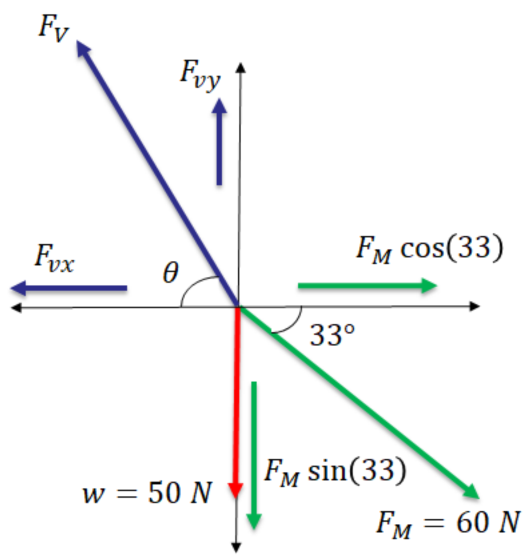
Take the torques about the elbow joint. Let clockwise be positive! Since the arm is in equilibrium, the total torque will be zero, that is

$$\sum\tau = 2kg \times \frac{10m}{s^2} \times 0.15m + 35kg \times \frac{10m}{s^2} \times 0.35m - F_{max} \times 0.05m \times \sin 105 = 0$$

$$F_{max} = \frac{2kg \times \frac{10m}{s^2} \times 0.15m + 35kg \times \frac{10m}{s^2} \times 0.35m}{0.05m \times \sin 105} = 2547N$$

15. A person working at a drafting board may hold her head as shown in Figure, requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae F_V to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.





Solution:

$$\sum F_y = 0 = -50N + F_{vy} - F_M \sin(33)$$

Solving for F_{vy}, we have

$$F_{vy} = 50N + F_M \sin(33) = 50N + 60\sin(33)$$

$$F_{vy} = 82.678\ N$$

$$\sum F_x = 0 = -F_{vx} + F_M \cos(33)$$

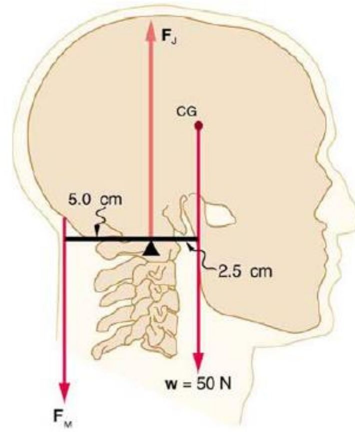
Solve for F_{vx}

$$F_{vx} = F_M \cos(33) = 60\cos(33) = 50.33N$$

$$F = \sqrt{50.33^2 + 82.678^2} = 96.787N$$

$$\theta = 58.674\ degrees$$

16. Even when the head is held erect, as in Figure 9.41, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?



Solution:

a. $\sum \tau = -50N \times 0.025m + F_M \times 0.05m = 0$

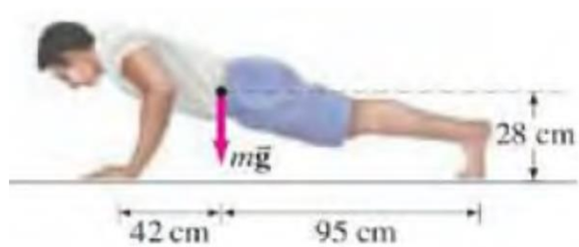
Solve for F_M

$$F_M = \frac{50N \times 0.025m}{0.05m} = 25N$$

b. $\sum F = 0 \rightarrow F_J - w - F_M = 0$

$$F_J = w + F_M = 50N + 25N = 75N$$

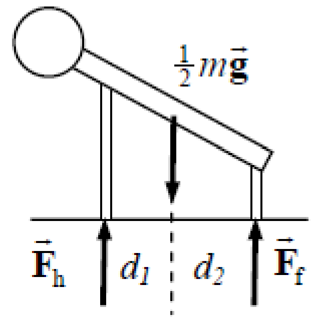
17. A man doing push-ups pauses in the position shown in Fig. 12-86. His mass $m = 68\ kg$. Determine the normal force exerted by the floor (a) on each hand; (b) on each foot.



Solution:

The Free body diagram is

a. The man is in equilibrium, so the net force and the net torque on him must be zero. We use half of his weight (**on each hand**), and then consider the force just on one hand and one foot, **considering him to be symmetric**. Take torques about the point where the foot touches the ground, with counterclockwise as positive.



$$\sum \tau = \frac{1}{2}mg \times d_2 - F_h \times (d_1 + d_2) = 0$$

Solve for F_h

$$F_h = \frac{mg \times d_2}{2(d_1 + d_2)} = \frac{68kg \times 9.8m/s^2 \times 0.95}{2(0.42 + 0.95)} = 231N$$

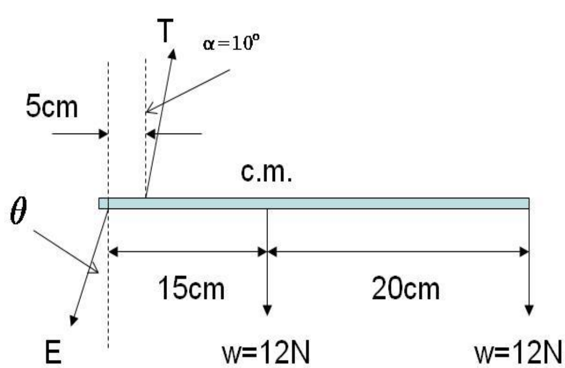
b. Use Newton's second law for vertical forces to find the force on the feet.

$$\sum F_y = 2F_h + 2F_f - mg = 0$$

Rearrange for F_f, we have

$$F_f = \frac{mg - 2F_h}{2} = \frac{68kg \times \frac{9.8m}{s^2} - 2 \times 231N}{2} = 103N$$

18. A realistic free body diagram model for the forearm. In the figure below, $w_1 = 12$ N find the tension T exerted by the muscle and the force exerted by the elbow joint E.



Solution

Use the second law of equilibrium,

$$-12N \times 0.35m - 12N \times 0.15m + (Tcos\ \alpha)N \times 0.05m = 0$$

Solve for T, you will find

$$T = \frac{12N \times 0.5m}{0.05m \times \cos\alpha} = 121.85\ \ N$$

Now, invoke the first law of equilibrium. The Horizontal component is

$$-E_x = T\sin\alpha = 121.85\ N \times \sin\alpha \rightarrow E_x = -21.16\ N$$

The Vertical component is

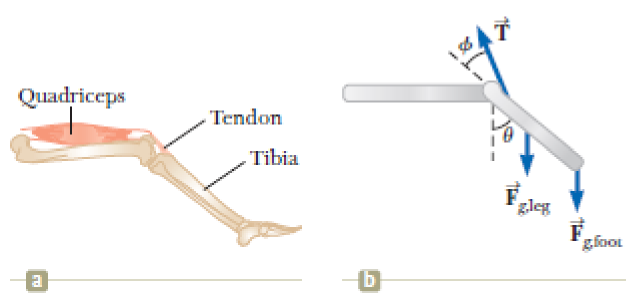
$$T\cos\alpha - 12 - 12 - E_y = 0 \rightarrow E_y = -Tcos\ \alpha + 24$$

$$E_y \cong -96\ \ N$$

$$E = -21.16\ i - 96\ j \qquad N$$

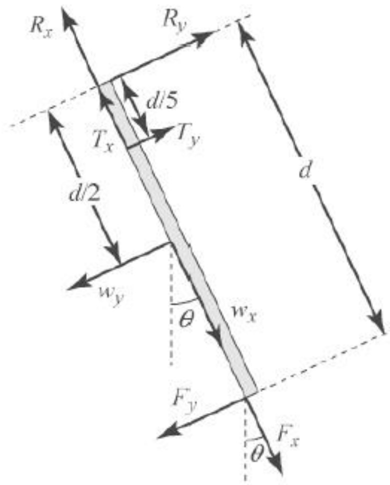
$$\theta = tan^{-1}\left(\frac{E_y}{E_x}\right) = tan^{-1}\left(\frac{-96}{-21.16}\right) = 77.57^{\circ}$$

19. The large quadriceps muscle in the upper leg terminates at its lower end in a tendon attached to the upper end of the tibia (Figure below (a, b)). The forces on the lower leg when the leg is extended are modeled as in Figure b below, where $\boldsymbol{T}$ is the force in the tendon, $\boldsymbol{F_{g,leg}}$ is the gravitational force acting on the lower leg, and $\boldsymbol{F_{g,foot}}$ is the gravitational force acting on the foot. Find $\boldsymbol{T}$ when the tendon is at an angle of $\varphi = 25^{\circ}$ with the tibia, assuming $\boldsymbol{F_{g,leg} = 30.0\ N}$, $\boldsymbol{F_{g,foot} = 12.5\ N}$, and the leg is extended at an angle $\theta = 40^{\circ}$ with respect to the vertical. Also assume the center of gravity of the tibia is at its geometric center and the tendon attaches to the lower leg at a position one-fifth of the way down the leg.



Solution

First, we resolve all forces into components parallel to and perpendicular to the tibia, as shown in free body diagram. Note that $\theta = 40^\circ$ and



$$w_y = 30.0\,N \times \sin 40 = 19.3\,N$$

$$F_y = 12.5\,N \times \sin 40 = 8.03\,N$$

and

$$T_y = T \sin 25$$

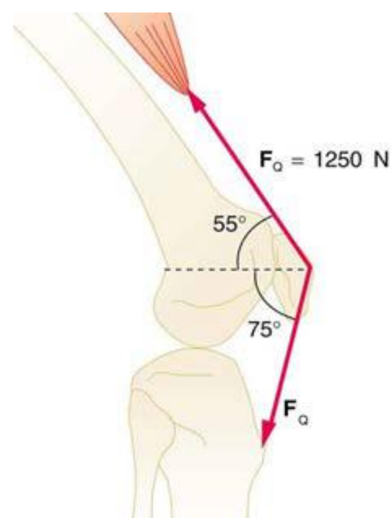
Using the second law of equilibrium, $\sum \tau = 0$, for an axis perpendicular to the page and through the upper end of the tibia gives

$$(T \sin 25) \times \frac{d}{5} - (19.3) \times \frac{d}{2} - (8.03) \times d = 0$$

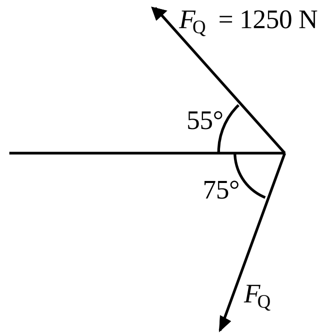
Solve for T

$$T = 209\,N$$

20. The upper leg muscle (quadriceps) exerts a force of $1250\,N$, which is carried by a tendon over the kneecap (the patella) at the angles shown in Figure below. Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).



Solution



$$F_x = (1250\text{ N})\cos 55° + (1250\text{ N})\cos 75° = 1040\text{ N}$$
$$F_y = (1250\text{ N})\sin 55° - (1250\text{ N})\sin 75° = -183\text{ N}$$
$$F = \left[(1040\text{ N})^2 + (-183\text{ N})^2\right]^{1/2} = \underline{1056\text{ N}} = \underline{1.1\times10^3\text{ N}}$$
$$\theta = \tan^{-1}\left(\frac{183}{1040}\right) = 9.98° \text{ or } \underline{190° \text{ ccw from the positive x-axis}}$$

Chapter 3

Solutions

Part I: MCQs

Answered Multiple Choice Questions with some explanations

1. Tensile strain is equal to

   A. Force per unit area
   B. Force per unit volume
   C. Extension per unit length
   D. Force per unit length

2. Hooke's law states that

   A. extension is proportional to load when elastic limit is not exceeded
   B. extension is inversely proportional to load when elastic limit is not exceeded
   C. extension is independent of load when elastic limit is not exceeded
   D. load is dependent on extension

3. In a composite body, consisting of two different materials……….will be same in both materials

   A. Stress
   B. Strain
   C. Both stress and strain
   D. None of these

4. Dimensions of strain are

   A. $[L]$
   B. $[M]\,[L]^{-1}\,[T]^{-2}$
   C. $[L]^{-1}$
   D. It's a dimensionless quantity

5. The modulus of elasticity __________________ temperature increase.

   A. decreases with
   B. increases with
   C. does not depend on
   D. may increase or decrease with

6. When too many people stand on a bridge it collapses, why?
  A. Due to increase in stress
  B. Due to overweight
  C. Due to improper construction
  D. Due to friction

7. If a material is subjected to two incremental true strains namely ε_1 and ε_2, then the total true strain is
  A. $\varepsilon_1 \times \varepsilon_2$
  B. $\varepsilon_1 - \varepsilon_2$
  C. $\boldsymbol{\varepsilon_1 + \varepsilon_2}$
  D. $\varepsilon_1 / \varepsilon_2$

8. Shape of true stress-strain curve for a material depends on
  A. Strain
  B. Strain rate
  C. Temperature
  D. All

9. The slope of Stress-Strain Curve, within elastic limits is called
  A. Modulus of Elasticity
  B. Hooke's Law
  C. Stress
  D. Tensile strain

10. The deformation per unit length is called
  A. Strain
  B. Stress
  C. Elasticity
  D. None of these

11. The ability of the material to deform without breaking is called
  A. Elasticity
  B. Plasticity
  C. Creep
  D. None of these

12. A load of $1\ kN$ acts on a bar having cross-sectional area $0.8\ cm^2$ and length $10\ cm$. The stress developed in the bar is
  A. $\boldsymbol{12.5\ N/mm^2}$
  B. $25\ N/mm^2$
  C. $50\ N/mm^2$
  D. $75\ N/mm^2$

13. If a material has identical properties in all directions, it is called
    A. Elastic
    B. Plastic
    C. Isotropic
    D. Homogeneous

14. Every material obeys the Hooke's law within
    A. Elastic limit
    B. Plastic limit
    C. Limit of proportionality
    D. None of these

15. Dimensional formula for Young's modulus of elasticity is
    A. $\boldsymbol{ML^{-1}T^{-2}}$
    B. MLT^2
    C. $M^{-1}L^{-1}T^{-1}$
    D. $ML^{-2}T^{-2}$

16. A perfectly elastic body
    A. Can move freely
    B. Has perfectly smooth surface
    C. Is not deformed by any external surface
    D. Recovers its original size and shape when the deforming force is removed.

Part II: End of Chapter Problems Solutions

1. A $200 - \text{kg}$ load is hung on a wire with a length of $4\ \text{m}$, a cross-sectional area of $0.2 \times 10^{-4}\ \text{m}^2$, and a Young's modulus of $8 \times 10^{10}\ \text{N/m}^2$. What is its increase in length?

   Solution

$$\Delta l = \frac{F}{A}\frac{l}{Y} = \frac{200 \times 10\ N}{0.2 \times 10^{-4}\ \text{m}^2} \times \frac{4\ m}{8 \times 10^{10}\ \text{N/m}^2} = 5 \times 10^{-3} m = 5\ mm$$

2. During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.

Solution

Use the equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$, where $Y = 1.6\times10^{10}\ \text{N/m}^2$

$L_0 = 0.350\ \text{m}$, $A = \pi\, r^2 = \pi(0.0180\,\text{m})^2 = 1.018\times10^{-3}\ \text{m}^2$,
and
$F_{\text{tot}} = 3w = 3(60.0\,\text{kg})(9.80\,\text{m/s}^2) = 1764\ \text{N}$,
so that the force on each leg is
$F_{\text{leg}} = F_{\text{tot}}/2 = 882\ \text{N}$.
Substituting in the value gives:

$$\Delta L = \frac{1}{1.6\times10^{10}\ \text{N/m}^2}\frac{(882\ \text{N})}{(1.018\times10^{-3}\ \text{m}^2)}(0.350\,\text{m}) = 1.90\times10^{-5}\ \text{m}.$$

So each leg is stretched by $\underline{1.90\times10^{-3}\ \text{cm}}$.

3. During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex his already moribund adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius

   Solution

$$\Delta L = \frac{1}{(9\times10^{9}\ \text{N/m}^2)}\frac{(150\,\text{kg})(9.80\ \text{m/s}^2)}{\pi(0.0210\ \text{m})^2}(0.380\ \text{m}) = \underline{4.5\times10^{-5}\ \text{m}}.$$

4. Assume that Young's modulus for bone is $1.5\times10^{10}\ \text{N/m}^2$ and that a bone will fracture if more than $1.50\times10^{8}\ \text{N/m}^2$ is exerted. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of $2.50\ \text{cm}$? (b) If a force of this magnitude is applied compressively, by how much does the $25.0-\text{cm}-\text{long}$ bone shorten?

   Solution

$$Radius = r = \frac{2.5}{2} = 1.25cm = 0.0125m$$

$$Area = \pi r^2 = \pi(0.125)^2 = 4.91\times10^{-4}m^2$$

$$1.50\times10^{8}\frac{\text{N}}{\text{m}^2} = \frac{\text{F}}{\text{A}} \rightarrow \text{F} = 1.50\times10^{8}\frac{\text{N}}{\text{m}^2}\times4.91\times10^{-4}m^2 = 7.365\times10^{4}N$$

Thus the maximum force is $7.365\times10^{4}N$

$$\Delta l = \frac{l}{Y}\frac{F}{A} = \frac{0.25\text{m}}{1.5\times10^{10}\ \text{N/m}^2}\times1.50\times10^{8}\frac{\text{N}}{\text{m}^2} = \frac{0.25}{100}\text{m} = 2.5\text{mm}$$

5. TV broadcast antennas are the tallest artificial structures on Earth. In 1987, a 72.0-kg physicist placed himself and 400 kg of equipment at the top of one 610-m high antenna to perform gravity experiments. By how much was the antenna compressed, if we consider it to be equivalent to a steel cylinder 0.150 m in radius?

Solution

$$F = w = mg = (72.0\text{ kg} + 400\text{ kg})(9.80\text{ m/s}^2) = 4626\text{ N} = 4630\text{ N}$$
$$L_0 = 610\text{ m};\quad Y = 2.10\times10^{11}\text{ N/m}^2$$
$$A = \pi\,(0.150\text{ m})^2 = 0.07069\text{ m}^2$$
$$\therefore \Delta L = \frac{L_0F}{YA} = \frac{(610\text{ m})(4626\text{ N})}{(2.10\times10^{11}\text{ N/m}^2)(0.07069\text{ m}^2)} = 0000190\text{ m} = \underline{0.190\text{ mm}}$$

6. A steel wire $1\ mm$ in diameter can support a tension of $0.2\ kN$. Suppose you need a cable made of these wires to support a tension of $20\ kN$. The cable's diameter should be of what order of magnitude?

Solution

If we are going to hold this force with $1\ mm$ wires we need $\frac{20kN}{0.2kN} = 100$ wire to support such a force.

We know that the cross sectional area is proportional to the square of the radius/diameter. Thus, the needed diameter is

$$1mm \times \sqrt{100} = 10mm = 1cm$$

7. If the elastic limit of copper is $1.5 \times 10^8\ \text{N/m}^2$, determine the minimum diameter a copper wire can have under a load of $10.0\ kg$ if its elastic limit is not to be exceeded.

Solution

$$\sigma = \frac{F}{A} \rightarrow 1.5\times10^8\,\frac{\text{N}}{\text{m}^2} = \frac{10\times10\text{N}}{\text{A}} \rightarrow \text{A} = \frac{100\text{N}}{1.5\times10^8\,\frac{\text{N}}{\text{m}^2}} = 6.667\times10^{-7}\text{m}^2$$
$$A = \pi r^2 \rightarrow r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{6.667\times10^{-7}\text{m}^2}{\pi}} = 0.46mm$$
$$d = 2r = 2\times0.46mm = 0.92mm$$

8. (a) Find the minimum diameter of a steel wire $18\ m$ long that elongates no more than $9\ mm$ when a load of $380\ kg$ is hung on its lower end. (b) If the elastic limit for this steel is $3 \times 10^8\ \mathrm{N/m^2}$, does permanent deformation occur with this load?

Solution

$$\frac{F}{A} = E\frac{\Delta l}{l}$$

$$\frac{380 \times 10\ \ N}{A\ \ m^2} = 20 \times 10^{10}\ N/m^2 \times \frac{9 \times 10^{-3} m}{18\ m}$$

Solve for A, you will find,

$$A = 0.000038\ m^2 \to \pi r^2 = A \to r = \sqrt{\frac{0.000038}{\pi}} = 0.00348m$$

$$r \cong 3.48mm$$

Thus, $d \cong\ 6.955mm$

Test:

$\frac{F}{A} = \frac{3800N}{0.000038m^2} = 10^5 N/m^2 \ll 3 \times 10^8\ \mathrm{N/m^2}$

Thus, the deformation doesn't occur.

9. For safety in climbing, a mountaineer uses a $50\ m$ nylon rope that is $10\ mm$ in diameter. When supporting the $90\ kg$ climber on one end, the rope elongates by $1.60\ m$. Find Young's modulus for the rope material.

Solution

$$Y = \frac{F/A}{\Delta l/l} = \frac{90 \times 10N\Big/\left(\pi(0.5 \times 10^{-2})\right)m^2}{1.6m/50m} = 3.58 \times 10^8 N/m^2$$

10. A pipe has an inner radius of $0.02\ m$ and an outer radius of $0.023\ m$. If subjected to a tension stress of $5 \times 10^7 Nm^{-2}$, how large is the applied force?

Solution

$$\sigma = \frac{F}{A} \to F = \sigma A = 5 \times 10^7 \mathrm{Nm^{-2}} \times \pi(0.023^2 - 0.02^2)m^2 = 2.026 \times 10^4 N$$

11. A man leg can be thought of as a shaft of bone $1.2\ m$ long. If the strain is 1.3×10^{-4} when the leg supports his weight, by how much is his leg shortened?

Solution

$$\varepsilon=\frac{\Delta l}{l}=1.3\times10^{-4}=\frac{\Delta l}{1.2m}\rightarrow\Delta l=1.3\times10^{-4}\times1.2\text{m}=0.156\text{ mm}$$

12. What is the spring constant of a human femur under compression of average cross-sectional area $10^{-3}m^2$ and length $0.4\ m$?

Solution

$$\frac{F}{A}=Y\frac{\Delta l}{l}\ and\ F=k\Delta l$$

$$\frac{k\Delta l}{A}=Y\frac{\Delta l}{l}\rightarrow k=Y\frac{A}{l}=1.5\times10^{10}\ \text{N/m}^2\frac{10^{-3}m^2}{0.4\text{ m}}=3.75\times10^{7}N/m$$

13. Calculate the force a piano tuner applies to stretch a steel piano wire 8.00 mm, if the wire is originally 0.850 mm in diameter and 1.35 m long.

Solution

$L_0=1.35\text{ m};\ d=0.850\text{ mm}=8.50\times10^{-4}\text{ m}$

$\Delta L=8.00\text{ mm}=8.00\times10^{-3}\text{m};\ Y=2.10\times10^{11}\text{ N/m}^2$

$$\therefore F=\frac{\Delta LYA}{L_0}=\frac{(8.00\times10^{-3}\text{m})(2.10\times10^{11}\text{ N/m}^2)\pi(4.25\times10^{-4}\text{m})^2}{1.35\text{ m}}=\underline{706\text{ N}}$$

14. The average cross-sectional area of a woman femur is $10^{-3}m^2$ and it is $0.4\ m$ long. The woman weights $750\ N$, young's modulus for her bone is $Y_{bone}=0.9\times10^{10}Nm^{-2}$, and the ultimate compression strength is $17\times10^{7}Nm^{-2}$,

a. what is the length change of this bone when it supports half of the weight of the woman?

b. Assuming the stress-strain relationship is linear until fracture, what is the change in length just prior to fracture?

c. Find the spring constant at the femur bone.

Solution

a. when the applied force is twice the women's weight the change is bone length is $Y_{bone}=0.9\times10^{10}\ Nm^{-2}$

$$Y_{bone} = \frac{F/A}{\Delta l/l} \Rightarrow \Delta l = \frac{l}{Y_{bone}}\frac{F}{A} = \frac{0.4m}{0.9\times10^{10}N\cdot m^{-2}}\frac{2\times750N}{10^{-3}m^{2}} =$$

$$\Delta l = \frac{600m}{9\times10^{6}} = 6.667\times10^{-5}m = 66.667\mu m$$

b. $\Delta l = \frac{\sigma_{\max}}{Y_{bone}}l = \frac{17\times10^{7}}{0.9\times10^{10}}\times0.4 = 7.55mm$

You need to apply force

$$\sigma_{\max} = \frac{F}{A} \Rightarrow$$

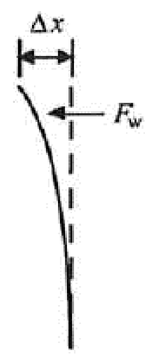
$$F = \sigma_{\max}\times A = 17\times10^{7}\,Nm^{-2}\times10^{-3}m^{2} = 170000N = 226.667\ her\ weight$$

c. The spring constant is

$$k = \frac{YA}{l} = \frac{0.9\times10^{10}Nm^{-2}10^{-3}m^{2}}{0.4m} = 22.5\times10^{6}Nm^{-1}$$

15. A 20.0-m tall hollow aluminum flagpole is equivalent in strength to a solid cylinder 4.00 cm in diameter. A strong wind bends the pole much as a horizontal force of 900 N exerted at the top would. How far to the side does the top of the pole flex?

Solution



$$L_0 = 20.0\,\text{m}$$

$$A = \pi\left(2.00\times10^{-2}\ \text{m}\right)^2 = 1.257\times10^{-3}\ \text{m}^2$$

$$F_\text{w} = 900\ \text{N}$$

$$S = 2.5\times10^{10}\ \text{N/m}^2$$

$$\Delta x = \frac{1}{S}\frac{F_\text{w}}{A}L_0 = \frac{1}{2.5\times10^{10}\ \text{N/m}^2}\times\frac{900\ \text{N}}{1.257\times10^{-3}\ \text{m}^2}\times20.0\,\text{m}$$

$$= 5.73\times10^{-4}\ \text{m} = \underline{0.57\ \text{mm}}$$

16. A vertebra is subjected to a shearing force of $500\ N$. Find the shear deformation, taking the vertebra to be a cylinder $3.00\ cm$ high and $4.00\ cm$ in diameter

Solution

$$\Delta x = \frac{1}{S}\frac{F}{A}L_0 = \frac{1}{\left(8.0\times 10^{10}\ \text{N/m}^2\right)}\frac{500\ \text{N}}{\pi(0.0200\ \text{m})^2}0.0300\ \text{m} = \underline{1.49\times 10^{-7}\ \text{m}}$$

17. A disk between vertebrae in the spine is subjected to a shearing force of 600 N. Find its shear deformation, taking it to have the shear modulus of $1.0 \times 10^9 N/m^2$. The disk is equivalent to a solid cylinder $0.700\ cm$ high and $4.00\ cm$ in diameter.

Solution

$$\Delta x = \frac{1}{S}\frac{F}{A}L_0 = \frac{1}{\left(1\times 10^{9}\ \text{N/m}^2\right)}\frac{600\ \text{N}}{\pi(0.0200\ \text{m})^2}0.00700\ \text{m} = \underline{3.34\times 10^{-6}\ \text{m}}$$

18. A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\frac{\Delta V}{V_0} = 2 \times 10^{-3}$) relative to the space available. Calculate the force exerted by the juice per square centimeter if its bulk modulus is $1.8 \times 10^9 N/m^2$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?

Solution

Using the equation $\Delta V = \frac{1}{B}\frac{F}{A}V_0$ gives:

$$\frac{F}{A} = B\frac{\Delta V}{V_0} = \left(1.8\times 10^{9}\ \text{N/m}^2\right)\left(2\times 10^{-3}\right)$$

$$= 3.6\times 10^{6}\ \text{N/m}^2 = \underline{4\times 10^{6}\ \text{N/m}^2} = \underline{4\times 10^{2}\ \text{N/cm}^2}$$

Since

$1\ \text{atm} = 1.013\times 10^{5}\ \text{N/m}^2$,

The pressure is about 36 atmospheres, far greater than the average jar is designed to withstand.

19. Assume that a $50\ kg$ runner trips and falls on his extended hand. If the bones of one arm absorb all the kinetic energy (neglecting the energy of the fall), what is the minimum speed of the runner that will cause a fracture of the arm bone? Assume that the length of arm is $1\ m$ and that the area of the bone is $4\ cm^2$.

Solution

Energy is mainly a kinetic energy

$$E = \frac{1}{2}mv^2$$

The maximum energy can be absorbed by the bones is

$$\mathcal{E} = \frac{Al{\sigma_B}^2}{2Y}$$

Where

$\sigma_{bone} = 10^8\ N/m^2\ ,\ Y_{bone} = 14 \times 10^9\ N/m^2$

$$\mathcal{E} = \frac{1}{2}\frac{(4 \times 10^{-4}m^2\ )(1.0m)(10^8N/m^2)^2}{14 \times 10^9N/m^2} = 142.86\ J$$

$$142.86\,J = \frac{1}{2}50\ kg\ v^2$$

$$v = \sqrt{\frac{2 \times 142.86J}{50\ kg}} = 2.39\ m/s$$

20. Calculate the density of sea water at a depth of $1000\ m$, where the water pressure is about $1.0 \times 10^7\ \mathrm{N/m^2}$. (The density of sea water is $1.03 \times 10^3\ \mathrm{kg/m^3}$ at the surface.)

Solution

Here, we have say 1 cubic meter of sea water its density is $1.03 \times 10^3\ kg/m^3$ at sea surface. Now, we drag this 1 cubic meter at a depth of $1000m$. This volume suffers a pressure about $1.0 \times 10^7\ N/m^2$. When certain volume under pressure its volume changes due to this pressure.

When the volume changes, then its density change (since density is equal to mass divided by volume!). To find the change in density, we have to find the new volume and then the new density.
Change in volume due to this pressure is

$$\Delta V = -\frac{\Delta P\, V_0}{B} = -\frac{1.0 \times 10^7 \ \mathrm{N/m^2} \times 1\mathrm{m^3}}{2.34 \times 10^9 \ \mathrm{N/m^2}} = \ -0.0043\mathrm{m^3}$$

Thus the new volume is $V_{new} = 1 - 0.0043 = 0.9957 \ \mathrm{m^3}$, but the mass contained in this volume remains unchanged, which is $1030.0 \ kg$
From the definition of density

$$\rho = \frac{m}{V} \rightarrow \rho_{new} = \frac{m}{V_{new}} = \frac{1030kg}{0.9957m^3} = 1.034 \times 10^3 \ \mathrm{kg/m^3}$$

The percentage change in density is not required by the question! I did this step to show my students, water density change is very small under huge pressure.

$$the\ precentage\ change\ in\ density\ is = \frac{1.034 \times 10^3 - 1.03 \times 10^3}{1.03 \times 10^3} \times 100 = 0.4\%$$

You may have noticed that the change is too small.

21. If the shear stress exceeds about $4 \times 10^8 \ \mathrm{N/m^2}$, steel ruptures. Determine the shearing force necessary (a) to shear a steel bolt $1.0 \ cm$ in diameter and (b) to punch a $1.0 \ cm$-diameter hole in a steel plate $0.5 \ cm$ thick.

Solution

a. $Shear\ stress = \frac{F}{A} \rightarrow F = A \times \sigma_S = \pi r^2 \times \sigma_S$

$$F = \pi \times \left(\frac{10^{-2}}{2} m\right)^2 \times 4 \times 10^8 \frac{\mathrm{N}}{\mathrm{m^2}} = \pi \times 10^4 \ \mathrm{N}$$

b. To punch a hole of 1.0 cm in diameter, we need area on which the force will act on; which is the circumference of the hole times the plate thickness,

$$A = 2\pi rt$$

$$F = A \times \sigma_S = 2\pi rt \times \sigma_S$$

$$F = 2\pi \times \frac{10^{-2}}{2} m \times 0.5 \times 10^{-2} m \times 4 \times 10^8 \frac{\mathrm{N}}{\mathrm{m^2}} = 2\pi \times 10^4 \ \mathrm{N}$$

22. When water freezes, it expands by about 9%. What would be the pressure increase inside your automobile's engine block if the water in it froze? (The bulk modulus of ice is $2 \times 10^9\ \mathrm{N/m^2}$.)

Solution

The idea of this question is that: car engine radiator is made of steel or any metal and filled with water. This material bulk modulus is small compared to water, because of that we neglect radiator's change in volume. Now, when the water freezes its volume is going to increase but the radiator volume is not changing! Thus, the freezed water exerts pressure on the walls of the radiator, which prevents it from expanding.

From this picture, you can see when water expands exerts pressure on coke can! Coke can is weak cannot stand the pressure and it explode because of this pressure!

Now, in normal conditions the water when freezes it expands by 9%, but when the water freezes in the container, the container impose pressure on the ice preventing it from expanding this 9%! Thus, our initial (if we are outside the radiator) volume is $1.09\ V_0$ and our final volume is V_0 (if we are inside the radiator, since the radiator stop water from expanding) after being compressed by the container. The bulk modulus of the water is $2.00 \times 10^9 N/m^2$, thus

$$\Delta P = -B\frac{\Delta V}{V_0} = -2.00 \times 10^9 N/m^2 \times \frac{-0.09V_0}{1.09V_0} = 0.165 \times 10^9 N/m^2$$

$$this\ pressure\ equals = \frac{1.65 \times 10^8 N/m^2}{1.013 \times 10^5 N/m^2} \cong 1600\ atm$$

This is a huge amount of pressure! The container has to be very good one to stand this pressure.

Chapter 4

Solutions

Part I: MCQs

Answered Multiple Choice Questions with some explanations

1. The Celsius scale starts from:
a) $32°F$ b) 273 K
c) $0°C$ d) 373 K

2. Unit of thermodynamic scale of temperature is:
a) Kelvin b) Centigrade
c) Fahrenheit d) Celsius

3. Normal human body temperature 98.6°F corresponds to.
a) 37℃ b) 42 ℃
c) 55 ℃ d) 410 ℃

4. Heat is the form of:
a) Power b) Work
c) Energy d) Motion

5. For a gas obeying Boyle's law, if the pressure is doubled, the volume becomes:
a) Double **b) One half**
c) Four times d) One fourth

6. Standard condition STP refer to a gas at
A. 76cm 0℃ B. 760mm 273K
C. 1atm 273K **D. all of the above**

7. Temperature is a property which determines
A. How much heat a body contains
B. Whether a body will feel hot or cold to touch
C. In which direction heat will flow between two systems
D. How much total absolute energy a body has

8. If the volume of a gas is held constant and we increase its temperature then
A. its pressure is constant
B. its pressure rises
C. its pressure falls
D. any of above

9. We prefer mercury as a thermometric substance because
A. Over a wide range of temperature its expansion is uniform
B. It does not stick to thermometer glass
C. It opaque to light
D. All of above

10. The scales of temperature are based on two fixed points which are
A. The temperatures of water at $0℃ - 100℃$
B. The temperature of melting ice and boiling water at atmospheric pressure
C. The temperatures of ice cold and boiling water
D. The temperatures of frozen and boiling mercury

11. The reading on the Fahrenheit scale will be double the reading on the centigrade scale when the temperature on the centigrade scale is
A. $460℃$ B. $280℃$
C. $360℃$ **D. $\mathbf{160}℃$**

12. The size of one degree of Celsius is equal to
A. One degree of Fahrenheit scale
B. 1.8 degrees of Fahrenheit scale
C. 3.2 degrees of Fahrenheit scale
D. 2.12 degrees of Fahrenheit scale

13. At constant temperature the graph between V and 1/P is
A. Hyperbola
B. Parabola
C. A curve of any shape
D. A straight line

14. Which of the following properties of molecules of a gas is same for all gases at particular temperature?
A. momentum
B. mass
C. velocity
D. kinetic energy

15. Boltzman constant K in terms of universal gas constant R and Avagadros number N_a is give as
A. $K = RN_a$ B. $\boldsymbol{K = R/N_a}$
C. $K = N_a/Ra$ D. $K = nRN_a$

16. Which one is true for internal energy?
A. It is sum of all forms of energies associated with molecules of a system
B. It is a state function of a system
C. It is proportional to transnational $K.E.$ of the molecules
D. All are correct

17. Units for thermal conductivity
A. $J/kg.K$
B. $J/mol.K$
C. $J.ohm/sec.K^2$
D. $\boldsymbol{W/m.K}$

18. Thermal expansion of a material has units as
A. $J/kg - K$
B. $J/mol - K$
C. $J.ohm/sec.K^2$
D. 1/°C

19. with increase in temperature, thermal conductivity of a metal _____________.
A. Increases
B. Decreases
C. Either
D. All, depending on metal.

20. The amount by which unit length of a material increases when the temperature is raised one degree is called the coefficient of:

A. cubic expansion
B. superficial expansion
C. linear expansion

21. A material of length L_1 at temperature $\theta_1\ K$ is subjected to a **temperature rise** of $\theta\ K$. The coefficient of linear expansion of the material is $\alpha\ K^{-1}$. The material expands by:

A. $L_2(1 + \alpha\theta)$
B. $L_1\alpha(\theta - \theta_1)$
C. $L_1\,[1 + \alpha(\theta - \theta_1)]$
D. $\boldsymbol{L_1\,\alpha\,\theta}$
Temperature rises means: the change in temperature

22. Some iron has a coefficient of linear expansion of $12 \times 10^{-6}\ K^{-1}$. A $100mm$ length of iron piping is heated through $20\ K$. The pipe extends by:

A. $0.24mm$
$\boldsymbol{B.\ 0.024mm}$
C. $2.4mm$
D. $0.0024mm$

23. If the coefficient of linear expansion is A, the coefficient of superficial expansion is B and the coefficient of cubic expansion is C, which of the following is false?
A. $C = 3A$
B. $A = B/2$
C. $\boldsymbol{B = 3/2C}$
D. $A = C/3$

24. The length of a $100mm$ bar of metal increases by $0.3mm$ when subjected to a temperature rise of $100\ K$. The coefficient of linear expansion of the metal is:
A. $3 \times 10^{-3}\ K^{-1}$
B. $3 \times 10^{-4}\ K^{-1}$
C. $\boldsymbol{3 \times 10^{-5}\ K^{-1}}$
D. $3 \times 10^{-6}\ K^{-1}$

25. A liquid has a volume V_1 at temperature θ_1. The temperature is increased to θ_2. If γ is the coefficient of cubic expansion, the increase in volume is given by:
A. $\boldsymbol{V_1\gamma\,(\theta_2 - \theta_1)}$
B. $V_1\gamma\ \theta_2$
C. $V_1 + V_1\gamma\ \theta_2$
D. $V_1\,[1 + \gamma\,(\theta_2 - \theta_1)]$

26. Which of the following statements is false?
A. Gaps need to be left in lengths of railway lines to prevent buckling in hot weather
B. Bimetallic strips are used in thermostats, a thermostat being a temperature- operated switch
C. As the temperature of water is decreased from $\boldsymbol{4}$℃ to $\boldsymbol{0}$℃ contraction occurs
D. A change of temperature of 15℃ is equivalent to a change of temperature of $15K$

27. When a bimetallic strip is heated, the strip will bend toward the side
A) With the smaller coefficient of linear expansion.
B) With the higher temperature.
C) With the larger coefficient of linear expansion.
D) With the lower temperature

28. The rms speed of a certain sample of carbon dioxide molecules, with a molecular weight of 44 g/mol, is 396 m/s. What is the rms speed of water molecules, with a molecular weight of 18 g/mol, at the same temperature?
A) 387 m/s
B) 506 m/s
C) 253 m/s
D) **619 m/s**

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}} \rightarrow v_{rms}(H_2O) = v_{rms}(CO_2)\sqrt{\frac{M_{CO_2}}{M_{H_2O}}} = 396\frac{m}{s} \times \sqrt{\frac{44}{18}} = 619\ m/s$$

29. Which of the following best explains why sweating is important to humans in maintaining suitable body temperature?
A) Evaporation of moisture from the skin extracts heat from the body.
B) Functioning of the sweat gland absorbs energy that otherwise would go into heating the body.
C) The high specific heat of water on the skin absorbs heat from the body.
D) Moisture on the skin increases thermal conductivity, thereby allowing heat to flow out of the body more effectively.
E) None of the other choices is correct

30. The weather outside is frightful. The temperature is $-22\ °\mathrm{F}$. What is the corresponding temperature on the Celsius scale?
A) −30∘C
B) −12∘C
C) −20∘C
D) −35∘C
E) −22∘C

31. The absolute temperature of an ideal gas is _____ if the volume of the sealed container is halved at constant pressure?

A. $cut\ to\ \frac{1}{4}$ B. $\boldsymbol{cut\ to\ \frac{1}{2}}$

C. The same D. Doubled

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow T_2 = T_1\frac{P_1}{P_1}\frac{\frac{1}{2}V_1}{V_1} = \frac{1}{2}T_1$$

32. A bridge in WV (West Virginia) is a steel arch bridge $518\ m$ in length. How much does its length change between extremes of $-20.0°C$ and $35.0°C$?

A. $5.7\ mm$ B. $\mathbf{34.2\ cm}$

c. $60.5\ cm$ D. $2.59\ m$

$$\Delta L = L_0\alpha\Delta T = 518\ m \times 12 \times 10^{-6}°\mathrm{C}^{-1} \times \left(35°\mathrm{C} - (-20°\mathrm{C})\right) = 0.342\ m$$

33. You stir some juice with a force of $20\ N$ for a total distance of $30\ m$. How much heat have you transferred?
A. $143\ cal$ B. $300\ cal$
C. $600\ cal$ D. $2510\ cal$

$$Work = Fd = 20N \times 30m = 600\ J$$

$$W = \frac{600J}{4.186} = 143\ cal$$

34. A $50.0gal$ steel container is completely filled with $CCl4$ ($\beta = 5.81 \times 10^{-4} {}^\circ C^{-1}$) at $10.0^\circ C$. How much will spill out at $30.0^\circ C$?

A.$2.1\ quarts$

B. $0.581\ gal$

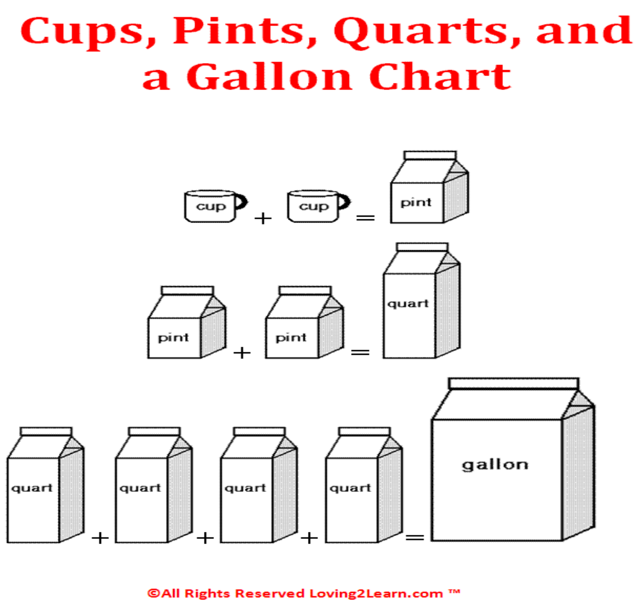
C. $8.74\ cups$

D. $140\ tbsp$

Solution

Here, we assume the volume change of steel container is too small

$$\Delta V = \beta V_0 \Delta T = 5.81 \times 10^{-4} {}^\circ C^{-1} \times 50.0\ gal \times (30 - 10)^\circ C^{-1} = 0.581\ gal$$



Part II: End of Chapter Problems Solutions

1. Liquid nitrogen has a boiling point of $-195.81^\circ C$ at atmospheric pressure. Express this temperature in (a) degrees Fahrenheit and (b) Kelvin's.

Solution

$$Tc = -195.81^\circ C$$

$$T_F = \frac{9}{5} T_C + 32 = \frac{9}{5}(-195.81) + 32 = 320.15\ {}^\circ F$$

$$T_K = T_C + 273.15 = -195.81 + 273.15 = 77.34\ K$$

2. The temperature difference between the inside and the outside of an automobile engine is $450°C$. Express this temperature difference on the (a) Fahrenheit scale and (b) Kelvin scale.

Solution

a. $\Delta T_F = \frac{9}{5} \Delta T_C = \frac{9}{5}(450) = 810\,°F$

b. $\Delta T_K = \Delta T_C = 450\,K$

3. $2.50\ g$ of $XeF4$ gas is placed into an evacuated $3.00\ liter$ container at $80°C$. What is the pressure in the container?

Solution

$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$n = \frac{m}{Mw} = \frac{2.5}{207.28} = 0.012\,mole, R = 8.314\,J/mole.K,$$

$$T_K = T_C + 273.15 = 80 + 273 = 353\,K$$

$$V = 3 \times 10^{-3}\ m^3$$

So,

$$P = \frac{0.012 \times 8.314 \times 353}{3 \times 10^{-5}} = 1.17 \times 10^4\,Pa$$

4. A hydrogen gas thermometer is found to have a volume of $100.0\ cm^3$ when placed in an ice-water bath at $0°C$. When the same thermometer is immersed in boiling liquid chlorine, the volume of hydrogen at the same pressure is found to be $87.2\ cm^3$. What is the temperature of the boiling point of chlorine?

Solution

Since

$$P_1 = P_2 \quad \text{at } T_1 = 273\,K$$

Thus,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \rightarrow T_2 = \frac{V_2}{V_1} T_1$$

$$T_2 = \frac{87.2}{100} \times 273 = 238.056\ K$$

5. (a) Show that the density of an ideal gas occupying a volume V is given by $\rho = \frac{PM}{RT}$ where M is the molar mass. (b) Determine the density of oxygen gas at atmospheric pressure and $20.0°C$.

Solution

a) $PV = nRT$, and $n = \frac{m}{M}$

$$PV = \left(\frac{m}{M}\right) RT \rightarrow \frac{PM}{RT} = \frac{m}{V} = density = \rho$$

b)

$$\rho = \frac{PM}{RT} = \frac{(1.013 \times 10^5) \times 32 \times 10^{-3}}{8.314 \times (273 + 20)} = 1.33\ kg/m^3$$

6. An ideal gas with volume of $3\ L$ is compressed at fixed temperature by increasing its pressure to $250\ kPa$. Find the bulk modulus of this gas.

Solution

At fixed temperature $P_1V_1 = P_2V_2$

Thus,

$$\frac{P_1 - P_2}{P_2} = \frac{V_2 - V_1}{V_1} \rightarrow -\frac{\Delta P}{P_2} = \frac{\Delta V}{V_1}$$

But we know the definition of the bulk modulus is

$$B = -V_1 \frac{\Delta P}{\Delta V}$$

Rearrange $-\frac{\Delta P}{P_2} = \frac{\Delta V}{V_1}$, we have

$$-\frac{\Delta P}{P_2} = \frac{\Delta V}{V_1} \rightarrow \frac{\Delta P}{\Delta V} = -\frac{P_2}{V_1}$$

Substitute in the bulk modulus we have

$$B = -V_1\left(\frac{\Delta P}{\Delta V}\right) = -V_1\left(-\frac{P_2}{V_1}\right) = P_2 = 250\ kPa$$

7. (a) How many atoms of helium gas fill a balloon of diameter $30.0\ cm$ at $20.0°C$ and $1.00\ atm$? (b) What is the average kinetic energy of the helium atoms? (c) What is the root-mean-square speed of each helium atom?

Solution

We know that the ideal gas law can be written as

$$PV = Nk_BT,$$

$$N = \frac{PV}{k_BT} = \frac{1.013\times10^5 N/m^2\times\left(\frac{4}{3}\pi(0.15)^3\right)m^3}{1.38\times10^{-23}JK^{-1}\times293K} = 3.5418\times10^{23}atom$$

a. The average kinetic energy can be obtained using $K.E. = \frac{3}{2}k_BT$

That is,

$$K.E. = \frac{3}{2}1.38\times10^{-23}JK^{-1}\times293K = 6.0651\times10^{-21}J$$

b. $v_{rms} = \sqrt{\frac{3k_BT}{m}}$,

We need to find the mass of a helium atom,

That is

$$m = \frac{M}{N_A} = \frac{4.00\ g/mole}{6.02\times10^{23}molecule/mole} = 6.64\times10^{-24}\ \ g/molecule$$

$$v\_rms = \surd((3k\_B\ T)/m) = \ 1.35\ km/sec$$

8. A cube $10.0\ cm$ on each edge contains air (with equivalent molar mass $28.9\ g/mol$) at atmospheric pressure and temperature $300\ K$. Find (a) the mass of the gas, (b) its weight, and (c) the force it exerts on each face of the cube. (d) Comment on the underlying physical reason why such a small sample can exert such a great force.

Solution

$$PV = nRT \to n = \frac{PV}{RT}$$

$$m = nM = \frac{PV}{RT}M = \frac{1.013 \times \frac{10^5 N}{m^2} \times (0.1)^3 m^3 \times 2.89 \times \frac{10^{-3} kg}{mol}}{8.314 \frac{J}{(mol\,.K)} \times 300K}$$

$$= 1.17 \times 10^{-3} kg$$

The mass of the gas is 1.17 grams

Its weight is $F = mg = 1.17 \times 10^{-3} kg \times 10\ ms^{-2} = 11.7\ mN$

The force exerted on the walls of the cube is

$$F = PA = 1.013 \times \frac{10^5 N}{m^2} \times 0.1^2 m^2 = 1013\ N$$

The molecules to exert such a force on the walls have to have a high speed to hit the walls so hard.

9. A copper telephone wire has essentially no sag between poles $35.0\ m$ apart on a winter day when the temperature is $-\ 20.0°C$. How much longer is the wire on a summer day, when $TC\ =\ 35.0\ °C$?

Solution

The initial information for this question is that,

$$l_0 = 35\ m, at\ T_0 = -20\ ℃$$

We need to find the final length of the wire l when the temperature is $T_f = 35\ ℃$

We know that from our class,

$$\Delta l = \alpha l_0 \Delta T$$

So the new length is

$$\Delta T = (T_f - T_0) = 35 - (-20) = 55\ ℃$$

So, the change in length is

$$\Delta l = \alpha l_0 \Delta T = 35\ m \times 1.7\ \times 10^{-5}\ ℃^{-1} \times 55℃ = 0.0327\ m$$

$$\Delta l = 32.725\ mm$$

10. A steel rod $4.00\ cm$ in **diameter** is heated so that its temperature increases by $70.0C^{\circ}$. It is then fastened between two rigid supports. The rod is allowed to cool to its original temperature. Assuming that Young's modulus for the steel is $20.6 \times 10^{10}\ N/m^2$ and that its average coefficient of linear expansion is $11 \times\ 10^{-6}\ ℃^{-1}$, calculate the tension in the rod.

Solution

$$r = 2\ cm = 2 \times 10^{-2} m, Y_{steel} = 20.6 \times 10^{10}\ \frac{N}{m^2}, and\ \alpha = 11 \times\ 10^{-6}\ ℃^{-1}$$

$$\sigma = Y_{steel}\ \varepsilon$$

Where

$$\sigma = \frac{F}{A}, and\ \varepsilon = \frac{\Delta l}{l_0}$$

Thus,

$$F = Y_{steel}\ \times A \times \frac{\Delta l}{l_0}$$

The change in length due to increase in temperature is

$$\Delta l = \alpha l_0 \Delta T \rightarrow \frac{\Delta l}{l_0} = \alpha \Delta T$$

$$F = Y_{steel}\ \times A \times \left(\frac{\Delta l}{l_0}\right) = Y_{steel}\ \times A \times (\alpha \Delta T)$$

$$F = 20.6 \times 10^{10}\ \frac{N}{m^2} \times (\pi(2 \times 10^{-2} m)^2) \times \left(11 \times\ 10^{-6}\ ℃^{-1} \times 70.0C^{\circ}\right)$$

$$F = 1.99 \times 10^5\ N \cong 2 \times 10^5\ N$$

11. A concrete slab has a length of $12\ m$ at $-5\ ºC$ on a winter's day. What is the change in length from winter to summer, when the temperature is $35\ ºC$? The linear expansion coefficient of concrete is $1\ \times\ 10^{-5}℃^{-1}$.

Solution

$\Delta T = \left(T_f - T_0\right) = \left(35 - (-5)\right) = 40\ ℃$

$\Delta l = \alpha l_0 \Delta T = 1\ \times\ 10^{-5}℃^{-1} \times 12m \times 40\ ℃ = 4.80\ mm$

12. A steel rod is initially at $20\ °C$ and has a length of $2\ m$ and a cross sectional area of $10\ cm^2$.

a) If it is heated to $120\ °C$, by how much does its length increase.

b) How large a force must be applied to its end to restore the original length? (Young's Modulus for steel = $2 \times 10^{11}\ N/m^2$, the coefficient of thermal expansion for steel = $1.72 \times 10^{-5}\ K^{-1}$.

Solution

$l_0 = 2m, A = 10^{-3}m^2, \Delta T = 120 - 20 = 100\ C^{\circ} = 100\ K$

a. length increase due temperature increase

$$\Delta l = \alpha l_0 \Delta T = 1.72 \times 10^{-5}\ K^{-1} \times 2m \times 100\ K = 3.44 \times 10^{-3}m = 3.44\ mm$$

b. force to be exerted, see question 10 solution,

$$F = Y_{steel}\ \times A \times \left(\frac{\Delta l}{l_0}\right) = 2 \times 10^{11}\ N/m^2 \times 10^{-3}m^2 \times \frac{3.44 \times 10^{-3}m}{2m}$$

$$F = 3.44 \times 10^5 N$$

13. A clock with a brass pendulum has a period of $1.000\ s$ at $20.0°C$. If the temperature increases to $30.0°C$, (a) by how much does the period change, and (b) how much time does the clock gain or lose in one month?

Solution

Solution:

$\tau = 1\sec\ at\ \ T = 20^{\circ}C$

the time period of simple harmonic oscillator is given by

$$\tau = 2\pi\sqrt{\frac{l}{g}}\ \ldots\ldots..(1)$$

where l is length of the pendulum, and g is the acceleration due to gravity which is equal to $g = 9.8\ m/s^2$

From equation (1) we need to find l which makes the period one second.

Solve equation (1) for you find

$$\tau = 2\pi \sqrt{\frac{l}{g}} \rightarrow 1\, sec = 2\pi \sqrt{\frac{l}{9.8\, m/s^2}}$$

Solving for l, we find $l = 0.24824\, m$

We know $\alpha_{brass} = 3 \times 10^{-5} \text{°C}^{-1}$, so the new length is

$$l' = l(1 + \alpha \Delta T) = 0.24824m(1 + 3 \times 10^{-5} \text{°C}^{-1}(35 - 20)\text{°C})$$

$$l' = 0.24831\, m$$

The new time period becomes, because of the new length,

$$\tau' = 2\pi \sqrt{\frac{l'}{g}}$$

$$\tau' = 1.000149990\, sec$$

which means the period takes longer time than one sec, the clock will delay the time readings! The pendulum should make 60 period in 60 seconds but now it makes 60 period in 60.0089994sec.

The change in time period is

$$\Delta\tau = \tau' - \tau = 0.000149990\, sec$$

The delay in one month is,

$$\tau_{delay} = 30 \times 24 \times 60 \times 60 \times 0.000149990 = 388.77\, sec$$

$$\tau_{delay} \cong 6.5\, min/month$$

The clock will delay by 6.5 minute in one month

14. A thermometer has a mercury-filled glass bulb with a volume of $2 \times 10^{-7}\, m^3$ attached to a thin glass capillary tube with an inner radius of $5 \times 10^{-5}\, m$. If the temperature increases by $100°C$, how far will the mercury rise in the tube? (volume thermal expansion coefficient of mercury= $1.82 \times 10^{-4}\, K^{-1}$).

Solution

$V_0 = 2 \times 10^{-7}\, m^3$ This is the initial volume of mercury, and it is the initial volume of the glass container that contains the mercury!

When temperature increases both mercury and glass bulb are going to increase in volume. The expansion of the mercury is much bigger than of the glass bulb though.

So the extra volume $\Delta V = (\Delta V_{hg} - \Delta V_{blub})$ due to temperature change is going to rise in the capillary tube.

Thus, ΔV is going to fill same cylinder volume $\pi\, r^2 \times h$

Where r is the radius of the capillary, and h is the height of the capillary.

$\Delta V_{hg} = \gamma V_0 \Delta T$, and $\Delta V_{blub} = 3\alpha V_0 \Delta T$

$\Delta V = \left(\Delta V_{hg} - \Delta V_{blub}\right) = \gamma V_0 \Delta T - 3\alpha V_0 \Delta T = V_0 \Delta T(\gamma - 3\alpha)$

$\Delta V = V_0 \Delta T(\gamma - 3\alpha) = \pi\, r^2 \times h$

$solving\ for\ h$

$$h = \frac{V_0 \Delta T(\gamma - 3\alpha)}{\pi\, r^2}$$

$$h = \frac{2 \times 10^{-7}\ m^3 \times 100\ K}{\pi \times 5 \times (10^{-5}\ m)^2} \times \left(1.82 \times 10^{-4}\ K^{-1} - 3 \times 3.2 \times 10^{-6}\ K^{-1}\right)$$

$$h = 0.439\ m$$

If we ignore the expansion of the glass blub, the height h becomes,

$$h = \frac{V_0 \Delta T \gamma}{\pi\, r^2} = 0.4635\ m$$

15. If $46.6\ kJ$ is required to heat $0.15\ kg$ of helium gas from $20°\mathrm{C}$ to $80°C$ at constant pressure, find the specific heat of helium?

Solution

$$Q = mc\,\Delta T\ , \quad \Delta T = 60\ °C = 60\ K$$

$$C = \frac{Q}{m\Delta T} = \frac{46.6 \times 10^3}{0.15 \times 60} = 5.18 \times 10^3\ \ J/kgK$$

16. Sphere of iron is heated to $900K$ and then cooled by placing it in a container filled of $5\ kg$ water of $300K$, what is the mass of the sphere if the final temperature of the mixture is $340K$, the specific heat of iron is $490\ J/kgK$ and that of water is $4200\ J/kgK$. Find the heat loss by the iron in Calorie.

Solution

Using the principle of conservation of energy

$$Q_{gain}(water) + Q_{loss}(iron) = 0$$

$$m_w c_w (T_f - T_w) = m_i c_i (T_i - T_f)$$

$$5 \times 4200\ (340 - 300) = m_i\ 490\ (900 - 340)$$

$$m_i = \frac{5 \times 4200\ (340-300)}{490\ (900-340)} = 3.06\ kg\ .$$

$$Q_{loss} = m_i c_i \left(T_i - T_f\right) = 3.06 \times 490 \times 560 = 8.4 \times 10^5 J$$

17. A box with a total surface area of $1.20\ m^2$ and a wall thickness of $4.00\ cm$ is made of an insulating material. $A\ 10.0 - W$ electric heater inside the box maintains the inside temperature at $15.0°C$ above the outside temperature. Find the thermal conductivity k of the insulating material.

Solution

We have

$$A = 1.2\ m^2, \qquad d = 4 \times 10^{-2} m, \qquad \Delta T = 15°C = 15\ K$$

$$H = k\,A\frac{\Delta T}{d} \rightarrow k = \frac{Hd}{A\Delta T} = \frac{10\ W \times 4 \times 10^{-2} m}{1.2\ m^2 \times 15\ K} = 0.022\ \frac{W}{K}$$

18. The surface of the Sun has a temperature of about $5800\ K$. The radius of the Sun is $6.96 \times 10^8\ m.$ Calculate the total energy radiated by the Sun each second. (Assume that $e = 0.96$)

Solution

$$T_{sun} = 5800K, \quad R_{sun} = 6.96 \times 10^8\ m, \sigma = 5.76 \times 10^{-8}\ W/(m^2 \cdot K^4)$$

$$P = eA\sigma T^4$$

$$P = 0.96 \times (4\pi(6.96 \times 10^8\ m)^2) \times 5.76 \times 10^{-8}\ W/(m^2 \cdot K^4) \times (5800K)^4$$

$$P = 3.76 \times 10^{26} W$$

Energy per second is

$$Q = 3.76 \times 10^{26} J$$

19. A person walking at a modest speed generates heat at rate of $280\ W$. If the surface area of the body is $1.5\ m^2$ and if the heat is assumed to be generated $0.03\ m$ below the skin, what temperature difference between the skin and interior of body would exist if the heat were conducted to the surface? Assume that the thermal conductivity coefficient is $0.2\ Wm^{-1}K^{-1}$.

Solution

$$H = 280\ W, A = 1.5\ m^2, d = 0.03, k = 0.2$$

$$H = k\ A\frac{\Delta T}{d} \rightarrow \Delta T = \frac{Hd}{kA} = \frac{280 \times 0.03}{0.2 \times 1.5} = 28K$$

20. A $2\ m$ length of copper pipe of $10\ cm$ diameter containing hot water at 80°C. If the surroundings are at 20°C, at what rate does the pipe lose thermal energy due to radiation? (take $e = 1$)

Solution

We have,

$$e = 1, \sigma = 5.76 \times 10^{-8} \frac{W}{(m^2 \cdot K^4)}, T_{in} = 80 + 273 = 353\ K,$$

$$T_{out} = 20 + 273 = 293\ K,$$

The surface area is (the surface area of a cylinder)

$$A = 2\pi rl = 2\pi(5 \times 10^{-2} m) \times 2m = 0.628\ m^2$$

Due to radiation

$$P = eA\sigma(T_{in}^4 - T_{out}^4)$$

$$P = 1 \times 0.628\, m^2 \times 5.76 \times 10^{-8} \frac{W}{(m^2 \cdot K^4)} \times ((353\, K)^4 - (293\, K)^4)$$

$$P = 295.07\, W$$

21. A glass window pane has an area of $3.00\ \mathrm{m^2}$ and a thickness of $0.600\ \mathrm{cm}$. If the temperature difference between its surfaces is $75.0°\mathrm{F}$, what is the rate of energy transfer by conduction through the window? $\kappa = 0.8\ \mathrm{W\ m^{-1}K^{-1}}$

Solution

$$\Delta°\mathrm{C} = \frac{5}{9}\Delta°\mathrm{F} \rightarrow \Delta°\mathrm{C} = \frac{5}{9}75 = 41.67°\mathrm{C} = \Delta K$$

$$H = \kappa A \frac{\Delta T}{L} = 0.8\, W\, m^{-1} K^{-1} \times 3.0 m^2 \frac{41.67\, K}{0.6 \times 10^{-2} m} = 16.67\, kW$$

22. A helium balloon has a volume of $1m^3$. As it rises in the earth's atmosphere, its volume expands. What will be its new volume (in cubic meters) if its original temperature and pressure are $20^{\circ}C$ and $1\ atm$ and its final temperature and pressure are $-40^{\circ}C$ and $0.1\ atm$.

Solution:

We know that the equation of state for initial conditions is given by

$$P_i\, V_i = n\, R\, T_i \ldots \ldots \ldots (1)$$

And the equation of state for the final conditions is given by

$$P_f\, V_f = n\, R\, T_f \ldots \ldots \ldots (2)$$

Rearrange equations (1) and (2), we have

$$\frac{P_i\, V_i}{T_i} = n\, R, \qquad and \qquad \frac{P_f\, V_f}{T_f} = n\, R$$

The right hand side of both equations equals $n\, R$, thus

$$\frac{P_i\, V_i}{T_i} = \frac{P_f\, V_f}{T_f} = n\, R = Constant$$

Now, using

$$\frac{P_i\, V_i}{T_i} = \frac{P_f\, V_f}{T_f} \dots\dots\dots (3)$$

We have $T_i = 20℃ = 293K, and\ T_f = -40℃ = 233K,$

apply this in equation (3), we have

$$\frac{P_i\, V_i}{T_i} = \frac{P_f\, V_f}{T_f} \rightarrow\ V_f = \frac{T_f}{P_f}\frac{P_i\, V_i}{T_i}$$

$$V_f = \frac{233K}{0.1atm}\frac{1\ atm \times 1m^3}{293K} = 8.05\ m^3$$

23. A metal rod 30.0 cm long expands by 0.075 cm when its temperature is raised from $0^{\circ}C$ to $100^{\circ}C$. A rod of different metal and of the same length expands by 0.045 cm for the same temperature. A third rod, also 30.0 cm long, is made up of pieces of each of the above metals placed end to end, and expands by 0.065 cm between $0^{\circ}C$ to $100^{\circ}C$. Find the length of each portion of the composite bar.

Solution:

$$Metal\ I: l_1 = 30\,cm, \Delta T = (T_2 - T_1) = (100-0) = 100^{\circ}C, \Delta l_1 = 0.075cm$$
$$Metal\ II: l_2 = 30\,cm, \Delta T = 100^{\circ}C, \Delta l_2 = 0.045cm$$
$$Composite\,bar: l_3 = 30cm, \Delta T = 100^{\circ}C, \Delta l_3 = (\Delta l_1 + \Delta l_2) = 0.065cm$$

from the above information we can find the linear coefficient of expansion of metal1 and metal2 that is,

$$\frac{\Delta l_1}{l_1} \times \frac{1}{\Delta T} = \alpha_1,\ and\ \frac{\Delta l_2}{l_2} \times \frac{1}{\Delta T} = \alpha_2$$

$$\alpha_1 = 2.5 \times 10^{-5}\left(C^{\circ}\right)^{-1}, \alpha_2 = 1.5 \times 10^{-5}\left(C^{\circ}\right)^{-1}$$

now, assume the length of meta one is x and the length of metal 2 is $(l-x)$

$$\Delta l_1 = \alpha_1 x \Delta T,$$
$$and$$
$$\Delta l_2 = \alpha_2 (l-x)\Delta T$$

add the above equations you find,

$\Delta l_1 + \Delta l_2 = \alpha_1 x \Delta T + \alpha_2 \left(l - x\right) \Delta T$

$0.065cm = 2.5\times10^{-5} x100 + 1.5\times10^{-5}\left(30 - x\right)100$

you will find

$x = 20\ cm$ from metal one and $10\ cm$ from metal 2.

24. A cube 10.0 cm on each edge contains air (with equivalent molar mass 28.9 g/mol) at atmospheric pressure and temperature 300 K. Find (a) the mass of the gas, (b) its weight, and (c) the force it exerts on each face of the cube. (d) Comment on the underlying physical reason why such a small sample can exert such a great force.

Solution:

$$PV = nRT \rightarrow n = \frac{PV}{RT}$$

$$m = nM = \frac{PV}{RT} M = \frac{1.013 \times \frac{10^5 N}{m^2} \times (0.1)^3 m^3 \times 2.89 \times \frac{10^{-3} kg}{mol}}{8.314 \frac{J}{(mol\,.K)} \times 300K}$$
$$= 1.17 \times 10^{-3} kg$$

The mass of the gas is 1.17 grams

Its weight is $F = mg = 1.17 \times 10^{-3} kg \times 10\ ms^{-2} = 11.7\ mN$

The force exerted on the walls of the cube is

$$F = PA = 1.013 \times \frac{10^5 N}{m^2} \times 0.1^2 m^2 = 1013\ N$$

The molecules to exert such a force on the walls have to have a high speed to hit the walls so hard.

Chapter 5

Solutions

Part I: MCQs

Answered Multiple Choice Questions with some explanations

1. The study of properties of fluids **in motion** is called
a) Fluid
b) Fluid statics
c) Fluid dynamics
d) None

2. The dimensions of coefficient of viscosity are.
a) MLT^{-1}
b) M^2LT
c) $\boldsymbol{ML^{-1}T^{-1}}$
d) $M^2L^{-1}T^{-1}$

3. η Is denoted for coefficient of:
a) Friction
b) Viscosity
c) Linear expansion
d) Gravitational customer

4. The SI unit of coefficient of viscosity is:
a) $\boldsymbol{kg\ m^{-1}\ s^{-1}}$
b) $kg\ m^{-2}\ s^{-1}$
c) $kg\ m^{-2}\ s^{-2}$
d) $kg\ m^2\ s$

5. As the water falls from tap, its speed increases and cross sectional area:
a) Zero
b) Increases
c) Decreases
d) Remains constant
Decrease, the velocity of water under gravity increase thus the cross-section has to decrease

6. Turbulent flow is:

a) Unsteady and regular
b) Steady and regular
c) Unsteady and irregular
d) Steady and regular

7. The law of conservation of mass gives
a) Bernoulli's equation
b) Equation of continuity
c) Torricelli theorem
d) None of these

8. Bernoulli's theorem applies to:
a) Liquids
b) Plasma state
c) Solids
d) Fluids

9. Velocity of fluid increases where the pressure is:
a) Low
b) High
c) Constant
d) Changes continuously

10. Irregular flow of fluid is called
a) Streamline
b) Turbulent
c) Uniform
d) Laminar

11. According to equation of continuity, $A_1 v_1 = A_2 v_2 = constant$. **The constant** is equal to:
a) Flow rate
b) Volume of fluid
c) Mass of fluid
d) Density of fluid

12. Venturi meter is a device used to measure:
a) Pressure of the fluid
b) Speed of fluid
c) Density of fluid
d) Viscosity of the fluid

13. Bernoulli's equation is obtained by applying law of conservation of.
a) Mass
b) Energy
c) Momentum
d) Fluid
14. Ideal fluid is.
a) Non-viscous
b) Incompressible
c) Steady flow
d) Possess all of the above properties

15. Instrument used to measure blood pressure is called

a) Venturi meter
b) Blood pressure
c) Sphygmomanometer
d) Sonometer

16. The blood pressure in the vessels is always

a) Less than atmosphere
b) Greater than atmosphere
c) Equal to atmosphere
d) 133.3 $N\ m2$

17. In equation of continuity, the units of Av is given as:

a) Cubic meter
b) Cubic meter per second
c) Square meter per second
d) Square meter

18. If cross-sectional area of pipe decreases, the speed of fluid must increase according to:

a) Venturi relation
b) Bernoulli's equation
c) Equation of continuity
d) Torricelli's theorem

Equation of continuity: $A_1v_1 = A_2v_2$

19. For which position, blood pressure in the body have the smallest value?

a) Standing straight
b) Sitting on chair
c) Sitting on ground
d) Lying horizontally

20. What is the correct formula for absolute pressure?

a. $P_{abs} = P_{atm} - P_{gauge}$
b. $P_{abs} = P_{vacuum} - P_{atm}$
c. $P_{abs} = P_{vacuum} + P_{atm}$
d. $\boldsymbol{P_{abs} = P_{atm} + P_{gauge}}$

21. Which of the following sentences are true for Bernoulli's equation?

1. Bernoulli's principle is applicable to ideal incompressible fluid
2. The gravity force and pressure forces are only considered in Bernoulli's principle
3. The flow of fluid is rotational for Bernoulli's principle
4. The heat transfer into or out of fluid should be zero to apply Bernoulli's principle

a. (1), (2) and (3)
b. (1), (3) and (4)
c. (1), (2) and (4)
d. (1), (2), (3) and (4)

22. Blood circulation through arteries is
a laminar flow
b. a turbulent flow
c. not laminar not turbulent
d. not applicable

23. Newton's law of viscosity states that
a. the shear stress applied to the fluid is directly proportional to the velocity gradient (dv/dy)
b. the shear stress applied to the fluid is inversely proportional to the velocity gradient (dv/dy)
c. the shear stress applied to the fluid is directly proportional to the specific weight of the fluid
d. the shear stress applied to the fluid is inversely proportional to the specific weight of the fluid

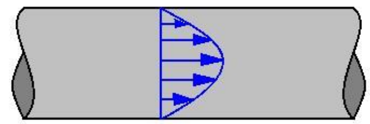
24. How should be the viscosity of the flowing fluid for laminar flow?

a. viscosity of the fluid should be as low as possible, for laminar flow
b. viscosity of the fluid should be as high as possible, for laminar flow
c. change in viscosity of the flowing fluid does not affect its flow
d. unpredictable

25. The flow of fluid will be laminar when,

a. Reynold's number is less than 2000
b. the density of the fluid is low
c. both a. and b.
d. none of the above

26. Below diagram shows the velocity distribution of fluid flow through a pipe. Flow is laminar. What is the ratio of maximum velocity to average velocity?



Velocity Distribution

a. 1
b. 2
c. 4
d. 3.14

27. The imaginary line drawn in the fluid in such a way that the tangent to any point gives the direction of motion at the point, is called as

a. path line
b. streak line
c. filament line
d. stream line

28. Which property of the fluid offers resistance to deformation under the action of shear force?
a. density
b. viscosity
c. permeability
d. specific gravity

29. According to equation of continuity, when water falls its speed increases, while it's cross sectional area

a. increases
b. **decreases**
c. remain same
d. differen

30. If layers of fluid have frictional force between them, then it is known as

a. viscous
b. non-viscous
c. incompressible
d. both a and b

31. Venturi relation is one of applications of

a. equation of continuity
b. Bernoulli's equation
c. light equation
d. speed equation

32. If every particle of fluid has irregular flow, then flow is said to be

a. laminar flow
b. turbulent flow
c. fluid flow
d. both a and b

33. Viscosity of air at 30 °C is

a. 0.019
b. 1.295
c. 0.514
d. 2.564

34. If every particle of fluid follows same path, then flow is said to be

a. laminar flow
b. turbulent flow
c. fluid flow
d. both a and b

35. A racing car has a body with

a. laminated design
b. turbulent design
c. flat design
d. streamlined design

36. Water flowing through hose having diameter $1\ cm$ at speed of $1\ m\ s^{-1}$, if water is to emerge at $21\ m\ s^{-1}$, diameter of nozzle should be

a. 0.2 cm
b. 0.1 cm
c. 0.02 cm
d. 0.01 cm

$$r_1^2 \bar{v}_1 = r_2^2\, \bar{v}_2 \rightarrow r_2 = \sqrt{\frac{r_1^2 \bar{v}_1}{\bar{v}_2}} = \sqrt{\frac{(0.5 \times 10^{-2} m)^2 \times 1\, m\, s^{-1}}{21\, m\, s^{-1}}} = 0.001\, m$$

37. Change in potential energy is measured as difference of

a. mgf
b. mgh
c. mg
d. mgt

38. If fluid has constant density, it is said to be

a. thick
b. in-viscous
c. compressible
d. incompressible

39. Change in potential energy of a body moving from height $10\ m$ to $5\ m$ having mass $3\ kg$ will be

a. 294
b. 295
c. 145
d. 147

$$\boldsymbol{\Delta P.E. = mg\Delta h = 3kg \times 9.8\frac{m}{s^2} \times (10 - 5) = 147J}$$

40. Frictional effect between layers of flowing fluid is known as

a. viscosity
b. friction
c. gravity
d. surface tension

41. If fluid is incompressible and is steady, its mass is

a. increasing
b. decreasing
c. same
d. conserved

42. Force required to slide one layer from another measures the

a. viscosity
b. friction
c. gravity
d. surface tension

43. According to equation of continuity, product of area of pipe and speed of fluid along pipe is

a. 1
b. 0
c. constant
d. different

44. If there is no frictional force between layers of fluid, then it is known as thick

a. non-viscous
b. compressible
c. incompressible
d. viscous

45. Cross-sectional area of water hose having radius of $0.2\ m$ is

a. 1.1
b. 0.23
c. 0.125
d. 1.3
$A = \pi r^2 = 0.1256\ m^2$

46. If density of fluid is not constant, it is said to be

a. viscous
b. incompressible
c. compressible
d. both a and b

47. When net force acting on a droplet becomes zero, its constant speed is known as

a. viscosity
b. friction
c. gravity
d. terminal velocity

48. Torricelli's theorem is one of applications of

a. equation of continuity
b. Bernoulli's equation
c. light equation
d. speed equation

Part II: End of Chapter Problems Solutions

1. As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of $1.5\ cm^2$ and the woman's mass is $55.0\ kg$. Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)

Solution

$$P = \frac{F}{A} = \frac{mg}{A} \qquad \left(1.50\text{ cm}^2 = 1.50\times 0^{-4}\text{ m}^2\right)$$

$$P = \frac{(55.0\text{ kg})(9.80\text{ m/s}^2)}{1.50\times 10^{-4}\text{ m}^2} = \underline{3.59\times 10^6\text{ N/m}^2}$$

$$= 3.59\times 10^6\text{ N/m}^2 \times \frac{1\text{ lb}}{4.448\text{ N}} \times \frac{6.452\times 10^{-4}\text{ m}^2}{1\text{ in.}^2} = \underline{521\text{ lb/in.}^2}$$

2. The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of $1.00\ g$ is supported by a needle, the tip of which is a circle $0.200\ mm$ in radius, what pressure is exerted on the record in N/m^2?

Solution

$$P = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1.00\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{\pi(2.00\times 10^{-4}\text{ m})^2} = \underline{7.80\times 10^4\text{ Pa}}$$

This pressure is approximately 585 mm Hg.

3. What depth of mercury creates a pressure of $1.00\ atm$?

Solution

$$P = h\rho g \Rightarrow h = \frac{P}{\rho g} = \frac{1.013\times 10^5\text{ Pa}}{(13.6\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = \underline{0.760\text{ m}}$$

4. The greatest ocean depths on the Earth are found in the Marianas Trench near the Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is $11.0\ km$ and assuming the density of seawater is constant all the way down

Solution

$$P = h\rho g = (11.0\times 10^3\text{ m})(1.025\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2) = \underline{1.10\times 10^8\text{ Pa}} = \underline{1.09\times 10^3\text{ atm}}$$

5. The aqueous humor in a person's eye is exerting a force of $0.300\ N$ on the $1.10\ cm^2$ area of the cornea. **(a)** What pressure is this in $mm\ Hg$? **(b)** Is this value within the normal range for pressures in the eye?

Solution

$$\text{(a) } P = \frac{F}{A} = \frac{0.300\text{ N}}{1.10\text{ cm}^2} \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)^2 = 2.73\times 10^3\text{ Pa} \times \frac{1\text{ mm Hg}}{133.3\text{ Pa}} = \underline{20.5\text{ mm Hg}}$$

(b) The range of pressures in the eye is $12-24\ mm\ Hg$, so the result in part (a) is within that range.

6. How much force is exerted on one side of an $8.50\ cm$ by $11.0\ cm$ sheet of paper by the atmosphere? How can the paper withstand such a force

Solution

$$F = PA = \left(1.1013\times10^5\ \text{N/m}^2\right)\left(8.50\ \text{in.}\right)\left(11.0\ \text{in.}\right)\left(6.452\times10^{-4}\ \text{m}^2/\text{in.}^2\right) = \underline{6.11\times10^3\ \text{N}}$$

The paper can withstand this force because an equal force is exerted on the other side of the paper in the opposite direction.

7. The left side of the heart creates a pressure of $120\ mm\ Hg$ by exerting a force directly on the blood over an effective area of $15.0\ cm^2$. What force does it exert to accomplish this?

Solution

$$1\ \text{atm} = 1.103\times10^5\ \text{N/m}^2 = 760\ \text{mm Hg}$$

$$F = PA = (120\ \text{mm Hg})\left(\frac{1.103\times10^5\ \text{N/m}^2}{760\ \text{mm Hg}}\right)\left(15.0\ \text{cm}^2\right)\left(\frac{1\ \text{m}^2}{10^4\ \text{cm}^2}\right) = 23.992\ \text{N} = \underline{24.0\ \text{N}}$$

8. **(a)** Convert normal blood pressure readings of 120 over $80\ mm\ Hg$ to newtons per meter squared using the relationship for pressure due to the weight of a fluid $(P = \rho g h)$ rather than a conversion factor. **(b)** Discuss why blood pressures for an infant could be smaller than those for an adult. Specifically, consider the smaller height to which blood must be pumped.

Solution

(a) $P_{120} = h\rho g = \left(0.120\ \text{m}\right)\left(13.6\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right) = \underline{1.60\times10^4\ \text{N/m}^2}$

$P_{80} = \left(0.080\ \text{m}\right)\left(13.6\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right) = \underline{1.07\times10^4\ \text{N/m}^2}$

(b) Since an infant is only approximately $20\ inches$ tall, while an adult is approximately $70\ inches$ tall, the blood pressure for an infant would be expected to be smaller than that of an adult. The blood only feels a pressure of $20\ inches$ rather than $70\ inches$, so the pressure should be smaller.

9. How tall must a water-filled manometer be to measure blood pressures as high as $300\ mm\ Hg$?

Solution

$$\rho_m h_m g = \rho_w h_w g$$

$$h_w = \frac{\rho_m h_m}{\rho_w} = \frac{(13.6\ \text{g/cm}^3)(300\ \text{mm})}{1.00\ \text{g/cm}^3} = 4080\ \text{mm} = \underline{4.08\ \text{m}}$$

10. During forced exhalation, such as when blowing up a balloon, the diaphragm and chest muscles create a pressure of $60.0\ mm\ Hg$ between the lungs and chest wall. What force in newtons does this pressure create on the $600\ cm^2$ surface area of the diaphragm?

Solution

$$F = PA = \left(60.0\ \text{mm Hg} \times \frac{133\ \text{N/m}^2}{1.00\ \text{mm Hg}}\right)\left(600\ \text{cm}^2 \times \frac{1\ \text{m}}{100\ \text{cm}} \times \frac{1\ \text{m}}{100\ \text{cm}}\right) = \underline{479\ \text{N}}$$

11. You can chew through very tough objects with your incisors because they exert a large force on the small area of a pointed tooth. What pressure in pascals can you create by exerting a force of $500N$ with your tooth on an area of $1.00\ mm^2$?

Solution

$$P = \frac{F}{A} = \frac{500\ \text{N}}{1.00\ \text{mm}^2} \times \frac{1000\ \text{mm}}{1\ \text{m}} \times \frac{1000\ \text{mm}}{1\ \text{m}} = 5.00 \times 10^8\ \text{N/m}^2 = \underline{5.00 \times 10^8\ \text{Pa}}$$

12. If the pressure in the esophagus is $-2.00mmHg$ while that in the stomach is$+20.00mmHg$, to what height could stomach fluid rise in the esophagus, assuming a density of $1.10\ g/mL$? (This movement will not occur if the muscle closing the lower end of the esophagus is working properly.)

Solution

$$\Delta P = (22.0\ \text{mm Hg})\left(\frac{133\ \text{N/m}^2}{1.0\ \text{mm Hg}}\right) = \underline{2926\ \text{N/m}^2}$$

$$\Delta P = h\rho g \Rightarrow h = \frac{\Delta P}{\rho g} = \frac{2926\ \text{N/m}^2}{(1.10 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 0.271\ \text{m} = \underline{27.1\ \text{cm}}$$

13. A large storage tank open to the atmosphere at the top and filled with water develops a small hole in the side at a point $16\ m$ below the water level. If the flow rate from the hole is $2.5 \times 10^3\ m^3/min$, determine **a)** the speed at which water leaves the hole and **b)** the diameter of the hole

Solution:

a. Flow Speed

$P_1 = P_2 = 1\,atm,\ v_1 << v_2\ \ so\ consider\ v_1 = 0$,

Bernoulli's Equations

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Apply case conditions in Bernoulli's equation, we have

$$\rho g(h_1 - h_2) = \frac{1}{2}\rho v_2^2$$

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 9.8 \times 16} = 17.71\, m/s$$

b. Diameter of the hole

$$Q = Av = \pi r^2 v$$

Thus r is

$$r = \sqrt{\frac{Q}{\pi v}} = \sqrt{\frac{2.5 \times 10^3\ m^3/\boldsymbol{min}}{\pi \times 17.71\ m/\boldsymbol{s}}}$$

$$Q = 41.67 m^3/\boldsymbol{s}$$

$$r = \sqrt{\frac{41.67 m^3/\boldsymbol{s}}{\pi \times 17.71\ m/\boldsymbol{s}}} = 0.866\, m$$

$$D = 2r = 1.73\, m$$

14. How high can water rise in the pipe of a building if the gauge pressure at the ground floor is 2×10^5 Pa?

Solution

$$\Delta P = \rho g h$$

$$h = \frac{\Delta P}{\rho g} = \frac{2 \times 10^5 N/m^2}{10^3 \times 10} = 20m$$

15. A large artery is cannulated and a saline solution of density $1.3\ gcm^{-3}$ is used as the manometer fluid. What is the gauge pressure of the blood if the high difference in the manometer tube is $0.67\ m$?

Solution

$$\rho = 1.3\ gcm^{-3} = 1.3 \times 10^3 kg/m^3$$

$$\Delta P = \rho gh = 1.3 \times 10^3\ \frac{kg}{m^3} \times 9.8 \frac{m}{s^2} \times 0.67m = 8535.8\ Pa$$

16. A horizontal tube of radius $1\ cm$ is joined to a second horizontal tube of radius $0.5\ cm$. There is a pressure difference of $6660\ Pa$ between the two tubes. **a)** Which tube has the higher water pressure **b)** What volume of water flows through the tubes per second?

Solution

a) The wider tube has the higher water pressure

$P_1 - P_2 = 6660\ Pa.$

Horizontal flow $(P_1 - P_2) = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$

but $r_2^2\, v_2 = r_1^2 v_1,\ r_2 = \frac{1}{2}\, r_1,\ r_2^2 = \frac{1}{4}\, r_1^2$

Thus,

$$r_2^2\, v_2 = 4\, r_2^2 v_1 \rightarrow v_2 = 4v_1 \rightarrow v_2^2 = 16v_1^2$$

Then

$$(P_1 - P_2) = \frac{1}{2}\rho\,(15v_1^2) \rightarrow v_1 = \sqrt{\frac{2 \times (P_1 - P_2)}{15\,\rho}} = 0.942\ m/s$$

$$Q = Av = \pi r_1^2 v_1 = 2.96 \times 10^{-4} m^3/s$$

17. A garden hose has an inner radius of $1\ cm$. The hose has a nozzle with an inside radius of $1.5\ mm$. Water flows through the hose with an average speed of $25\ cm/s$. **a)** what is the speed of water as it emerges from the nozzle? **b)** How long does it take to spray $4\ liter$ of water from the hose?

Solution

$$r_1 = 10^{-2}\ m,\ r_2 = 1.5 \times 10^{-3}\ m, \qquad v_1 = 0.25\ m/s$$

$a)\ v_2 = ??$

We know that, from the continuity equation,

$$v_2 = \frac{r_1^2}{r_2^2} v_1 = \frac{\left(10^{-2}\ m\right)^2}{\left(1.5 \times 10^{-3}\ m\right)^2} \times 0.25 \frac{m}{s} = 11.11\ m/s$$

$b)\ \Delta v = 4L = 4 \times 10^{-3}\ m^3$

$$Q = \frac{\Delta V}{\Delta t} = \pi r_1^2 v_1 \rightarrow \Delta t = \frac{\Delta V}{\pi r_1^2 v_1}$$

$$\Delta t = \frac{\Delta V}{\pi r_1^2 v_1} = \frac{4 \times 10^{-3}\ m^3}{\pi \times (10^{-2}\ m)^2 \times 0.25 \frac{m}{s}} = 50.93\ s$$

18. Water flows through a pipe of radius $30\ cm$ at the rate $0.2\ m^3 / s$. The pressure in the pipe is atmospheric. The pipe slants downhill and feeds into a second pipe of radius $15\ cm$ which is located at $60\ cm$ lower. What is the gauge pressure at the lower part?

Solution

$$r_1 = 0.3\ m\ ,\ P_1 = 1\ atm\ ,\ v_1 = \frac{Q}{\pi r_1^2} = \frac{0.2\ m^3 / s}{\pi \times (0.3m)^2} = 0.707\ m/s$$

$$r_2 = 0.15\ m, P_2 = ??, h_1 - h_2 = 0.6\ m,$$

$$v_2 = \frac{r_1^2}{r_2^2} v_1 \rightarrow v_2 = \frac{(0.3\ m)^2}{(\ 0.15\ m)^2} v_1 = 4v_1 = 2.83\ m/s$$

Apply in Bernoulli's' equations

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2)$$

$$P_2 = 1.013 \times 10^5 \frac{N}{m^2} + \frac{1}{2} \times 1000 \frac{kg}{m^3} \times \left(\left(0.707 \frac{m}{s}\right)^2 - \left(2.83 \frac{m}{s}\right)^2 \right) + 1000 \frac{kg}{m^3} \times 9.8\, m/s^2 \times 0.6m$$

$$P_2 = 1.034 \times 10^5 \frac{N}{m^2}$$

$$Gauge\ pressure\ =\ 0.021\ atm$$

19. Calculate the flow rate in an aorta with a cross sectional area of $2\ cm^2$ if the flow speed is $40\ cm\ /s$.

$$Q\ =\ Av\ \ =\ 2\ \times\ 10^{-4} m^2 \times\ 0.4\ m/s\ =\ 8\ \times 10^5\ m^3/s$$

20. A needle of radius $0.3\ mm$ and length $3\ cm$ is used to give a patient a blood transfusion. Assume the pressure differential across the needle is achieved by elevating the blood $1\ m$ above the patient´s arm. **a)** What is the rate of flow of blood through the needle? **b)** At this rate of flow, how long it take to inject $500\ cm^3$ of blood into patient?

Solution

$$R\ =\ 3 \times 10^{-4}\ m, \qquad L\ =\ 3\ \times 10^{-2}\ m$$

$$\Delta P\ =\ \rho gh\ =\ 1060\ \tfrac{kg}{m^3} \times\ 10 m/s^2 \times\ 1m\ =\ 1.06\ \times\ 10^4\ Pa\,.$$

$$\eta\ =\ 2.08\ \times 10^{-3}\ Pa.s$$

$$a.\quad Q = \frac{\pi \times \Delta P \times R^4}{8 \times \eta \times L} = \frac{\pi \times 1.06\ \times\ 10^4\ Pa\ \times (3 \times 10^{-4} m)^{\,4}}{8 \times 2.08\ \times 10^{-3}\ Pa.s \times 3\ \times 10^{-2}\ m} = 5.403 \times 10^{-7} m^3/s$$

$$b.\qquad Q\ =\ \frac{\Delta V}{\Delta t} \rightarrow \Delta t = \frac{\Delta V}{Q} = \frac{500\ \times 10^{-6}\ m^3}{5.403 \times 10^{-7} m^3/s} = 925.345\ s$$

$$\Delta t = 15.422\ min$$

21. A jet pilot pulls his plane out of dive in such a way that his upward acceleration is $3\ g's$. What you predict as the blood pressure in the brain?

Solution

In our case, we have an upward acceleration, $(a + g)$

With $a\ =\ 3\ g$

Thus, according to equation 5.16 on page 116,

$p_B = p_H + \rho (g + a)(h_H - h_B)$

We have, in the textbook, $P_H = 13.3\, kPa$, $(g + a) = 4g \cong 40\, m/s^2$ and

$\rho = 1060\, kg/m^3$

On the average, the height difference between brain-heart is about $0.3\, m$, that is

$$(h_B - h_H) = 0.3\, m$$

$$p_B = 13.3 \times 10^3 Pa + \left(\frac{1060kg}{m^3}\right)\left(\frac{40m}{s^2}\right)(-0.3m) = 580\, Pa\,.$$

Don't be confuse with minus sign, the brain is higher than the heart

22. If an elevator accelerates upward at $10\, ms^{-2}$, what is the average blood pressure in the brain? What is the average blood pressure in the feet? If the elevator accelerates downward with the same acceleration, what is the average blood pressure in the brain and feet? take $g = 10\, ms^{-2}$.

Solution

$a)\ upward\ acceleration\ \ g + a = 20\, m/s^2$

$$for\ the\ brain\ P_B = P_H + \rho\ (g + a)\ (h_H - H_B)$$

$$P_B = 13.3 \times 10^3 + 1060 \times 20 \times (-0.3) = 6.94\, kPa$$

$$For\ the\ feet \qquad P_F = P_H + \rho\ (g + a)\ (h_H) \qquad take\ h_H = 1.4\, m$$

$$P_F = 13.3 \times 10^3 + 1060 \times 20 \times 1.4 = 43\, kPa$$

$b)\ downward\ acceleration \qquad g - a = 0$

$$P_B = P_H = 13.3\, kPa$$

$$P_F = P_H = 13.3\, kPa$$

23. The pulmonary artery, which connects the heart to the lungs, has an inner radius of $2.6\, mm$ and is $8.4\, cm$ long. If the pressure drop between the heart and lungs is $400\, Pa$, what is the average speed of blood in the pulmonary artery?

Solution

$$R = 2.6 \times 10^{-3}, L = 8.4 \times 10^{-2}, \Delta P = 400\ Pa$$

$$\bar{v} = \frac{\Delta P R^2}{8\eta L} = \frac{400\ Pa \times (2.6 \times 10^{-3} m)^2}{8 \times 2.08 Pa.s \times 10^{-3} \times 8.4 \times 10^{-2} m} = 1.93\ m/s$$

24. The aorta in humans has a diameter of about $2\ cm$ and at certain times, the blood speed through it is about $55\ cm/s$. What is the type of flow? Find also the flow resistance over $30\ cm$ long segment and the power for dissipating the blood through this segment.

Solution

$$N_R = \frac{2\rho\bar{v}R}{\eta} = \frac{2 \times 1060 \frac{kg}{m^3} \times 0.055 \frac{m}{s} \times 10^{-2} m}{2.08 \times 10^{-3} Pa.s} = 560.58$$

$$N_R < 2000, thus\ the\ flow\ is\ laminar$$

$$\mathcal{R}_f = \frac{8\eta L}{\pi R^4} = \frac{8 \times 2.08 \times 10^{-3} Pa.s \times 0.30 m}{\pi (10^{-2} m)^4} = 159\ k\ Pa.s/m^3$$

$$\mathbb{p} = \mathcal{R}_f Q^2 = \mathcal{R}_f (A\overline{v})^2 = 159\ k\ Pa. \frac{s}{m^3} \left(\pi \times (0.01m)^2 \times (0.055 m/s)\right)^2$$

$$p = 47.4\ \mu W$$

25. A standing $1.8\ m$ tall person in an elevator accelerates downward with $1.5\ g$. The height difference between the brain and heart is $40\ cm$. Find the pressure at the brain and foot if the pressure at the heart is $13.3\ kPa$. If this person is suddenly bending so his brain is $30\ cm$ below his heart, calculate the change of the pressure at the brain.

Solution

$$hB - hH = 0.4\ m, \ a = 1.5 \times 10 = 15\ m/s^2$$

$$P_B = P_H + \rho\ (g - a)\ (h_H - H_B)$$

$$P_B = 13300 + 1060 \times (-5) \times (-0.4) = 15.4\ kPa$$

$$P_F = P_H + \rho\ (g - a)\ h_H = 13300 + 1060 \times (-5) \times 1.4 = 5.88\ kPa\ .$$

26. An artery has an inner radius of $2\ mm$. If the flow is laminar and the average flow velocity is $0.03\ ms^{-1}$, What is the (a) maximum velocity (b) the flow rate, (c) and the pressure drop in $0.05\ m$, if the artery is horizontal?

Solution

$$R = 2 \times 10^{-3}\ m, \qquad \bar{v} = 0.03\frac{m}{s}, \qquad L = 0.05\ m$$

$$a)\ v_{max} = 2\,\bar{v} = 0.06\ m/s$$

$$b)\ Q = \pi R^2 \bar{v} = \pi \times (2 \times 10^{-3}\ m)^2 \times 0.03\frac{m}{s} = 3.77 \times 10^{-7} m^3/s$$

$$c)\ \bar{v} = \frac{\Delta P R^2}{8\eta L} \rightarrow \Delta P = \frac{\bar{v} \times 8\eta L}{R^2} = \frac{0.03\frac{m}{s} \times 8 \times 2.1 \times 10^{-3} Pa.s \times 0.05m}{(2 \times 10^{-3}\ m)^2} = 6.3\ Pa$$

27. The pressure drop along a length of a horizontal artery is $100\ Pa$. The radius of the artery is $10\ cm$, and the flow is laminar. (a) what is the net force on blood in this portion of the artery? (b) If the average speed of blood is $0.015\ ms^{-1}$, find the power expended in maintaining the flow?

Solution

$$\varDelta P = 100\ Pa, R = 10^{-1}\ m$$

$$a)\ F = \varDelta PA = \pi R^2\, \varDelta P = \pi \times 10^{-2}\, m^2 \times 100 Pa = \pi\, N\,.$$

$$b)\ \bar{v} = 0.015\ m/s$$

$$\mathbb{P} = Q\,\varDelta P = A\bar{v}\ \frac{F}{A} = F\bar{v}$$

$$\mathbb{P} = \pi\ N \times 0.015\ ms^{-1} = 4.71 \times 10^{-2} W$$

28. The radius of an artery is increased by a factor of 1.5 (a) The pressure drop remains the same, what happens to the flow rate? (b) If the flow rate stays the same, what happens to the pressure drop? (Assume laminar flow).

Solution

a.

$$Q_1 = \frac{\pi}{8}\frac{\Delta P}{L}\frac{R_1^4}{\eta}$$

And

$$Q_2 = \frac{\pi}{8}\frac{\Delta P}{L}\frac{R_2^4}{\eta}$$

Thus,

$$\frac{Q_2}{Q_1} = \frac{\left(\frac{\pi}{8}\frac{\Delta P}{L}\frac{R_2^4}{\eta}\right)}{\left(\frac{\pi}{8}\frac{\Delta P}{L}\frac{R_1^4}{\eta}\right)} = \frac{R_2^4}{R_1^4} = \left(\frac{R_2}{R_1}\right)^4 = \left(\frac{1.5R_1}{R_1}\right)^4 = 5.0625$$

b.

$$Q_1 = \frac{\pi}{8}\frac{\Delta P}{L}\frac{R_1^4}{\eta}$$

Thus,

$$\Delta P_1 = \frac{8\,L\,\eta Q_1}{\pi R_1^4}\ and\ \Delta P_2 = \frac{8\,L\,\eta Q_2}{\pi R_2^4}$$

Divide,

$$\frac{\Delta P_2}{\Delta P_1} = \frac{\frac{8\,L\,\eta Q_2}{\pi R_2^4}}{\frac{8\,L\,\eta Q_1}{\pi R_1^4}} = \left(\frac{R_1}{R_2}\right)^4 = \left(\frac{R_1}{1.5R_1}\right)^4 = 0.1975 \cong 0.2$$

29. The average velocity of water at $20°\text{C}$ in a tube of radius $0.1\ m$ is $0.2\ ms^{-1}$. (a) Is the flow laminar or turbulent? (b) what is the flow rate?

Solution

$$r = 0.1\ m, \qquad \bar{v} = 0.2\ m/s$$

$$\rho_{water} = 1000\ kg/m^3, \eta_{water} = 10^{-3}\ Pa.s$$

$$N_R = \frac{2\rho\bar{v}R}{\eta} = \frac{2 \times 1000\ kg/m^3 \times\ 0.2\ m/s \times 0.1\ m}{10^{-3}\ Pa.s} = 4000$$

a) The flow is turbulent

$$b)\ Q = A\bar{v} = \pi R^2\,\bar{v} = \pi \times 10^{-2}m^2 \times\ 0.2\ m/s = 6.28 \times 10^{-3}m^3/s$$

30. A small artery has a length of $0.11\ cm$ and a radius of $25\ \mu m$. (a) calculate its resistance.(b) If the pressure drop across the artery is $1.3\ kPa$, what is the flow rate?

Solution

$$L = 0.11 \times 10^{-2}\, m, \quad R = 2.5 \times 10^{-5}\, m,$$

$$\rho_{blood} = 1060\, kg/m^3, \eta_{blood} = 2.08 \times 10^{-3}\, Pa.s$$

$$a)\ R_F = \frac{8\eta L}{\pi R^4} = \frac{8 \times 2.08 \times 10^{-3}\, Pa.s \times 0.11 \times 10^{-2}\, m}{\pi(2.5 \times 10^{-5}\, m)^4} = 1.49 \times 10^{13} \quad Pa.s/m^3$$

$$b)\ R_F = \frac{\Delta P}{Q} \rightarrow Q = \frac{\Delta P}{R_F} = \frac{1.3 \times 10^3 Pa}{1.49 \times 10^{13} \quad Pa.s/m^3} = 8.72 \times 10^{-11} m^3/s$$

$$Q = 8.72 \times 10^{-2}\, mm^3/s$$

31. The heart of a resting adult pumps blood at a rate of $5.00\ L/min.$ (a) Convert this to . cm^3/s(b) What is this rate in m^3/s ?

Solution

(a) $\left(\frac{5.00\,\text{L}}{1\,\text{min}}\right)\left(\frac{1000\,\text{cm}^3}{1\,\text{L}}\right)\left(\frac{1\,\text{min}}{3600\,\text{s}}\right) = 1.389\,\text{cm}^3/\text{s} = \underline{1.39\,\text{cm}^3/\text{s}}$

(b) $(1.389\,\text{cm}^3/\text{s}\left(\frac{1\,\text{m}^3}{10^6\,\text{cm}^3}\right) = 1.389 \times 10^{-6}\ \text{m}^3/\text{s} = \underline{1.39 \times 10^{-6}\ \text{m}^3/\text{s}}$

32. Blood is pumped from the heart at a rate of $5.0\ L/min$ into the aorta (of radius $1.0\ cm$). Determine the speed of blood through the aorta.

Solution

$$Q = Av$$

$$v = \frac{Q}{A} = \frac{5000\,\text{cm}^3/60\,\text{s}}{\pi(1.0\,\text{cm})^2} = \underline{27\ \text{cm/s}}$$

33. Blood is flowing through an artery of radius $2\ mm$ at a rate of $40\ cm/s$. Determine the flow rate and the volume that passes through the artery in a period of $30\ s$.

Solution

$Q = Av = \pi r^2 v = \pi(0.20\text{ cm})^2\ (40\text{ cm/s}) = \underline{5.03\text{ cm}^3/\text{s}}$

$V = Qt = (5.03\text{ cm}^3/\text{s})(30\text{s}) = \underline{151\text{ cm}^3}$

34. A major artery with a cross-sectional area of is$1.00cm^2$ branches into 18 smaller arteries, each with an average cross-sectional area of is$0.400cm^2$. By what factor is the average velocity of the blood reduced when it passes into these branches?

Solution

$$Q = A_1\overline{v_1} = 18A_2\overline{v_2}\text{, so that } \overline{v_2} = \frac{A_1}{18A_2}\overline{v_1} = \frac{(1.00\text{ cm}^2)}{18(0.400\text{ cm}^2)}\overline{v_1} = \underline{0.139\overline{v_1}}$$

35. (a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is$10.0cm^2$, what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is $10.0\mu m$?

Solution

(a)

$$Q = A_1\overline{v_1} = A_2\overline{v_2}\text{, so that } A_1 = \frac{\overline{v_2}}{\overline{v_1}}A_2 = 4.00(10.0\text{ cm}^2) = \underline{40.0\text{ cm}^2}$$

(b)

$$A_1 = NA \Rightarrow N = \frac{A_1}{A} = \frac{40.0\text{ cm}^2}{\pi(5.00\times10^{-4}\text{ cm})^2} = \underline{5.09\times10^7}$$

36. The human circulation system has approximately 1×10^9 capillary vessels. Each vessel has a diameter of about $8\mu m$. Assuming cardiac output is $5\ L/min$, determine the average velocity of blood flow through each capillary vessel.

Solution

$Q = Av$

$$v = \frac{Q}{A} = \frac{5000\text{ cm}^3/60\text{s}}{10^9\ \pi r^2} = \frac{83.33\text{ cm}^3/\text{s}}{10^9\ \pi(4.0\times10^{-4}\text{ cm})^2} = \underline{0.166\text{ cm/s}}$$

37. (a) What is the pressure drop due to the Bernoulli effect as water goes into a $3.00 - cm$-diameter nozzle from a $9.00 - cm$-diameter fire hose while carrying a flow of $40.0\ L/s$? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)

Solution

(a)

$$v_1 = \frac{Q}{A_1} = \frac{40.0\times 10^{-3}\ \text{m}^3/\text{s}}{\pi(0.0450\,\text{m})^2} = 6.29\,\text{m/s, and}$$

$$v_2 = \frac{Q}{A_2} = \frac{40.0\times 10^{-3}\ \text{m}^3/\text{s}}{\pi(0.0150\,\text{m})^2} = 56.6\,\text{m/s. So that}$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$= 0.5(1.00\times 10^3\ \text{kg/m}^3)\left[(56.6\,\text{m/s})^2 - (6.29\,\text{m/s})^2\right] = \underline{1.58\times 10^6\ \text{N/m}^2}$$

(b)

$$h = \frac{v_0^2}{2g} = \frac{(56.6\,\text{m/s})^2}{2(9.80\,\text{m/s}^2)} = \underline{163\,\text{m}}$$

38. A glucose solution being administered with an IV has a flow rate of $4.00\ cm^3/min$. What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.

Solution

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l} \propto \frac{1}{\eta}$$

$$Q' \propto \frac{1}{\eta'} \Rightarrow \frac{Q'}{Q} = \frac{\eta}{\eta'}$$

$$Q' = Q\left(\frac{\eta}{\eta'}\right) = (4.00\,\text{cm}^3/\text{min})\left(\frac{1}{2.50}\right) = \underline{1.60\,\text{cm}^3/\text{min}}$$

39. A small artery has a length of $1.1 \times 10^{-3} m$ and a radius of $2.5 \times 10^{-3} m$. If the pressure drop across the artery is $1.3\ kPa$, what is the flow rate through the artery? (Assume that the temperature is $37°C$.)

Solution

$$Q = \frac{(P_2 - P)_1 \pi r^4}{8\eta l}$$

$$= \frac{(1.3\times10^3\ \text{N/m}^2)\pi(2.5\times10^{-5}\ \text{m})^4}{8(2.084\times10^{-3}\ \text{N}\cdot\text{s/m}^2)(1.1\times10^{-3}\ \text{m})} = 8.7\times10^{-11}\,\text{m}^3\text{/s} = \underline{8.7\times10^{-2}\,\text{mm}^3\text{/s}}$$

40. Calculate the Reynolds numbers for the flow of water through (a) a nozzle with a radius of $0.250\ cm$ and (b) a garden hose with a radius of $0.900\ cm$, when the nozzle is attached to the hose. The flow rate through hose and nozzle is $0.500\ L/s$. Can the flow in either possibly be laminar?

Solution

Hose:
$$N_R = \frac{2(1.00\times10^3\ \text{kg/m}^3)(1.96\,\text{m/s})(9.00\times10^{-3}\ \text{m})}{1.005\times10^{-3}\ (\text{N/m}^2)\cdot\text{s}} = \underline{35{,}100} > 2000 \Rightarrow$$

Flow is not laminar.

Nozzle:
$$N_R = \frac{2(1.00\times10^3\ \text{kg/m}^3)(25.5\,\text{m/s})(2.50\times10^{-3}\ \text{m})}{1.005\times10^{-3}\ (\text{N/m}^2)\cdot\text{s}} = \underline{1.27\times10^5} > 2000 \Rightarrow$$

Flow is not laminar.

Chapter 6

Solutions

Part I: MCQs

Answered Multiple Choice Questions with some explanations

1. If 1 A current flows in a circuit, the number of electrons flowing through this circuit is

A. $\mathbf{0.625 \times 10^{19}}$

B. 1.6×10^{19}

C. 1.6×10^{-19}

D. 0.625×10^{-19}

2. The resistivity of the conductor depends on

A. area of the conductor.

B. length of the conductor.

C. **type of material.**

D. none of these.

3. The resistance of a conductor of diameter d and length l is $R\ \Omega$. If the diameter of the conductor is halved and its length is doubled, the resistance will be

A. R Ω

B. 2R Ω

C. 4R Ω

D. **8R Ω**

4. How many coulombs of charge flow through a circuit carrying a current of $10\ A$ in $1\ minute$?

A. 10

B. 60

C. 600

D. 1200

5. A capacitor carries a charge of $0.1\ C$ at $5\ V$. Its capacitance is

A. 0.02 F

B. 0.5 F

C. 0.05 F

D. 0.2 F

6. To obtain a high value of capacitance, the permittivity of dielectric medium should be

A. low

B. zero

C. high

D. unity

7. $1\ F$ is theoretically equal to

A. 1 ohm of resistance

B. ratio of 1 V to 1 C

C. **ratio of 1 C to 1 V**

D. none of these

8. The unit of resistivity is

A. Ω.

B. $\boldsymbol{\Omega - metre}$.

C. $\Omega\ /\ metre$.

D. $\Omega\ /m^2$.

9. Internal resistance of ideal voltage source is

A. zero

B. infinite

C. finite

D. 100 ohms

10. Which quantity should be measured by the voltmeter?

A. Current

B. Voltage

C. Power

D. Speed

11. Which quantity consists of a unit $1kWh$?

A. **Energy**

B. Time

C. Power

D. Charge

12. Which of the following quantities consists of SI unit as $WATT$?

A. Force

B. Charge

C. Current

D. **Power**

13. KCL (Kirchhoff's Current Law) works on the principle of which of the following

A. **law of conservation of charge.**

B. law of conservation of energy.

C. both.

D. None of the above.

14. KVL (Kirchhoff's Voltage Law) works on the principle of

A. law of conservation of charge.

B. law of conservation of energy.

C. both.

D. None of the above.

15. When electric charge moves through electrical conductor, there is a production of

- **A. electric current**
- B. electrical charge
- C. electric cell
- D. electric circuit

16. Measure of rate of flow of electric charge through an electrical conductor is known as

- **A. electric current**
- B. electrical charge
- C. electric cell
- D. electric circuit

17. Do not use appliances, if

- A. damaged
- B. wires exposed
- C. working properly
- **D. both a and b**

18. A component which is used to close or break a circuit, is

- A. bulb
- **B. switch**
- C. wire
- D. electric cell

19. A diagram drawn by using symbols which represents electrical components is called

- **A. circuit diagram**
- B. current diagram
- C. charges diagram
- D. electric diagram

20. A component which provides resistance is called

- A. heat
- B. energy
- C. product
- **D. resistor**

21. A circuit which splits into two or more branches is called

A. series circuit
B. parallel circuit
C. open circuit
D. close circuit

22. In a circuit when an electric current flows in a continuous path, it is called

A. close circuit
B. open circuit
C. hot circuit
D. blocked circuit

23. SI unit for electric current is

A. joule
B. watt
C. volt
D. ampere

24. A device that stores separated electrical charges is called ____ .

A. a resistor
B. a conductor
C. a wire
D. a capacitor
E. an insulator

25. A capacitor is a device used to

A. measure the volume of a glass container.
B. convert electricity into heat.
C. force current through a wire.
D. store separated electrical charges.
E. convert electricity into light.

26. A material will be a ____ if it contains charges that are free to move around within the matter.

A. solid
B. poor conductor
C. good conductor
D. good insulator
E. fluid

27. Good electrical conductors are usually

A. good thermal conductors.
B. good thermal insulators.
C. poor thermal conductors.
D. good electical insulators.
E. transparent to light.

28. If a charge of 12 coulombs flows through a wire every 3 seconds, the current in the wire is

A. 12 amperes.
B. **4 amperes.**
C. 9 amperes.
D. 3 amperes.
E. 36 amperes.

29. If a voltage of 10 volts produces a current of 2 amps in an electrical device, the resistance must be

A. 2 ohms.
B. 5 ohms.
C. 10 ohms.
D. 20 ohms.
E. 8 ohms.

30. The resistance of the dry human body is about

A. 100 ohms.
B. 1 ohm.
C. **100,000 ohms**.
D. 1000 ohms.
E. 10 ohms.

31. Electrical impulses gather and accumulate in which part of a neuron, in order to initiate an action potential?

A. Dendrites
B. Axon hillock
C. Axon terminal branches
D. Node of Ranvier

At the very beginning of the axon as the cell soma starts to narrow is the axon hillock, which is where electrical impulses gather and accumulate from the soma. If these accumulated impulses are large enough in the axon hillock, and enough voltage-gated sodium channels are opened, an action potential will be initiated in the axon

32. What is largely responsible for the negative resting membrane potential (around -70 mV) in a neuron?

A. Axonal insulation by Schwann cells.
B. Voltage-gated sodium channels opening.
C. The action potential.
D. **Potassium leak currents**

At rest (no action potentials) an outward current is generated by a small amount of potassium ions leaking out of the cell through potassium-selective leak channels. This slow leak of positively charged potassium ions means that the internal cell membrane becomes slightly negatively charged compared to the outside; this gives rise to the negative resting membrane potential, which for neurons is around -70 mV.

33. What is the role of Schwann cells in neurotransmission?

A. Thermal insulation of neuronal axons
B. Limit the speed of the action potential
C. Enhance the speed of the action potential
D. Protect the neuronal soma from trauma

Schwann cells are neuroglial cells that wrap around some neuronal axons. They provide electrical insulation with small gaps between Schwann cells called nodes of Ranvier. The action potential 'leaps' from node to node during propagation and may therefore travel more rapidly than in a continuous wave. This method of propagation in myelinated axons in called saltatory conduction.

34. The opening of axon membrane voltage-gated potassium channels is responsible for which part of the action potential?

A. Depolarisation of the membrane
B. Repolarisation of the membrane
C. Contraction of the post synaptic muscle fiber
D. Signalling vesicular release of neurotransmitters

As the membrane potential depolarises towards around $+50\,mV$, voltage-gated sodium channels begin to inactivate. The depolarisation of the membrane causes voltage-gated potassium channels to open, which allows potassium ions to flow down their electrochemical gradient out of the cell to repolarise the membrane towards the resting potential.

Part II: End of Chapter Problems Solutions

1. What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

   Solution:

   Using the equation $I = \frac{\Delta Q}{\Delta t}$, we can calculate the current given in the charge and the time, remembering that $1\,\text{A} = 1\,\text{C/s}$

   $$I = \frac{\Delta Q}{\Delta t} = \frac{4.00\,\text{C}}{4.00\,\text{h}}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 2.778 \times 10^{-4}\ \text{A} = \underline{0.278\,\text{mA}}$$

2. During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is $500\,\Omega$ and a 10.0-mA current is needed. What voltage should be applied?

   Solution:

   $V = IR = (10.0 \times 10^{-3}\ \text{A})(500\ \Omega) = \underline{5.00\ \text{V}}$

3. (a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient?

   Solution:

   (a) $I = \frac{\Delta Q}{\Delta t} \Rightarrow \Delta Q = I\Delta t = (12.0\ \text{A})(0.0100\,\text{s}) = \underline{0.120\,\text{C}}$

   (b) $N_\text{e} = \frac{\Delta Q}{q_\text{e}} = (0.120\,\text{C})\left(\frac{1\,\text{electron}}{1.60 \times 10^{-19}\ \text{C}}\right) = \underline{7.50 \times 10^{17}\ \text{electrons}}$

4. A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

   (a) $I = \frac{\Delta Q}{\Delta t} \Rightarrow \Delta t = \frac{\Delta Q}{I} = \frac{10{,}000\,\text{C}}{5.00 \times 10^{-4}\ \text{A}} = \underline{2.00 \times 10^{7}\ \text{s}}$

   (b) $N_\text{e} = \frac{\Delta Q}{q_\text{e}} = (5.00 \times 10^{-4}\ \text{C/s})\left(\frac{1\,\text{electron}}{1.60 \times 10^{-19}\ \text{C}}\right) = \underline{3.13 \times 10^{15}\ \text{electrons/s}}$

5. What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

Solution:

$$R = \frac{\rho L}{A} = \frac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(20.0\ \text{m})}{\pi(1.0265\times10^{-3}\ \text{m})^2} = \underline{0.104\ \Omega}$$

6. (a) To what temperature must you raise a copper wire, originally at $20.0°\text{C}$, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

Solution:

(a) $R = R_0(1+\alpha\Delta T) = 2R_0 \Rightarrow \alpha\Delta T = 1 \Rightarrow \Delta T = \dfrac{1}{\alpha} = \dfrac{1}{3.9\times10^{-3}\ /°\text{C}} = 256°\ \text{C}$

$T = 20.0°\ \text{C} + 256°\ \text{C} = \underline{276°\ \text{C}}$

(b) No, under ordinary circumstances this temperature is unreachable in household wiring. The resistance is more likely to double because of a decrease in radius of the wire.

7. What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

Solution:

$$P = IV = (250\ \text{A})(24.0\ \text{V}) = \underline{6.00\ \text{kW}}$$

8. (a) What is the resistance of ten $275\text{-}\Omega$ resistors connected in series? (b) In parallel?

Solution:

(a) $R_s = R_1 + R_2 + R_3 + \ldots\ldots + R_{10} = (275\ \Omega)(10) = \underline{2.75\ \text{k}\Omega}$

(b) $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots\ldots + \dfrac{1}{R_{10}} = (10)\left(\dfrac{1}{275\,\Omega}\right) = 3.64\times10^{-2}/\Omega$

So that $R_p = \left(\dfrac{1}{3.64\times10^{-2}}\right)\Omega = \underline{27.5\,\Omega}$

9. (a) Given a 48.0-V battery and $24.0\text{-}\Omega$ and $96.0\text{-}\Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

Solution:

(a) $R_s = R_1 + R_2 = 24.0\,\Omega + 96.0\,\Omega = 120\ \Omega$, so that $I = \dfrac{V}{R_s} = \dfrac{48.0\ \text{V}}{120\,\Omega} = \underline{0.400\ \text{A}}$.

Thus, $P_1 = I^2 R_1 = (0.400\text{ A})^2 (24.0\,\Omega) = \underline{3.84\text{ W}}$

And $P_2 = I^2 R_2 = (0.400\text{ A})^2 (96.0\,\Omega) = \underline{15.4\text{ W}}$

(b) $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{24\,\Omega} + \dfrac{1}{96\,\Omega} = 5.208\times10^{-2} \big/ \Omega,\ R_p = \left(\dfrac{1}{5.208\times10^{-2}}\right)\Omega = 19.2\,\Omega$

so that $I = \dfrac{V}{R_p} = \dfrac{48.0\text{ V}}{19.2\,\Omega} = \underline{2.5\text{ A}}$

10. What is the output voltage of a 3.0000-V lithium cell in a digital wristwatch that draws 0.300 mA, if the cell's internal resistance is $2.00\,\Omega$?
Solution:

$$V = E - Ir = 3.0000\text{ V} - (3.00\times10^{-4}\text{ A})(2.00\,\Omega) = \underline{2.9994\text{ V}}$$

11. What is the internal resistance of an automobile battery that has an emf of 12.0 V and a terminal voltage of 15.0 V while a current of 8.00 A is charging it?

Solution:

$$V = E + Ir \Rightarrow r = \frac{V - E}{I} = \frac{15.0\text{ V} - 12.0\text{ V}}{8.00\text{ A}} = \underline{0.375\,\Omega}$$

12. A platinum coil has a resistance of $3.146\ \Omega$ at $40°C$ and $3.767\ \Omega$ at $100°C$. Find the resistance at $0°C$ and then find R_0

Solution:

Since

$$R_T = R_0\left(1 + \alpha(T - T_0)\right)$$

$$3.146 = R_0\left(1 + \alpha\,(40 - 0)\right)$$

$$3.767 = R_0\left(1 + \alpha\,(100 - 0)\right)$$

Divide, we have

$$\frac{3.146}{3.767} = \frac{R_0(1 + \alpha\,40)}{R_0(1 + \alpha 100)} \Rightarrow \alpha = \frac{1}{240}\,℃^{-1}$$

$$3.767 = R_0\left(1 + \frac{1}{240}100\right) \Rightarrow R_0 = 2.732\ \Omega\ at\ \ 0℃$$

13. A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of $50.0\,\Omega$ at $20.0°C$. When immersed in a vessel containing melting indium, its resistance increases to $76.8\,\Omega$. Calculate the melting point of the indium. Hint $\boldsymbol{\alpha = 3.92 \times 10^{-3}}$

Solution:

$R_T = R_0(1 + \alpha\Delta T)$

$76.8\Omega = 50.0\Omega(1 + 3.92 \times 10^{-3}°\mathrm{C}^{-1}\Delta T°\mathrm{C})$

$\Delta T = 136.7°\mathrm{C}$

$\Delta T = T - T_0 \rightarrow 136.7 = T - 20 \rightarrow T = 136.7 + 20$

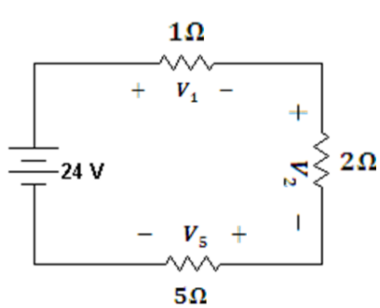
$T = 156.7°\mathrm{C}$

14. A certain resistor has a resistance of 150.4Ωat $20°\mathrm{C}$and resistance of 162.4Ω at $40°\mathrm{C}$. What is its temperature coefficient of resistivity?

Solution:

$$R_{20C} = 150.4\Omega, R_{40C} = 162.4\Omega,$$
$$R_T = R_\circ\left(1 + \alpha\left(T - T_\circ\right)\right),$$
rearrange,
$$\alpha = \frac{1}{R_\circ}\frac{R_T - R_\circ}{T - T_\circ} = \frac{1}{150.4}\frac{(162.4 - 150.4)}{(40 - 20)} = 3.99 \times 10^{-3}\left({}^\circ C\right)^{-1}$$

15. For the circuit shown, calculate i_5; v_1; v_2; v_5; and the power delivered by the source.



Solution:

To find the voltage across each resistor apply the voltage divider law, that is

$$V_1 = 24\frac{1}{1+2+5} = 3\,Volt$$

$$V_2 = 24\frac{2}{1+2+5} = 6\,Volt$$

$$V_5 = 24\frac{5}{1+2+5} = 15\,Volt$$

$$i_5 = \frac{15}{5} = 3\,A$$

This is the circuit current, because the resistances connected in series only.

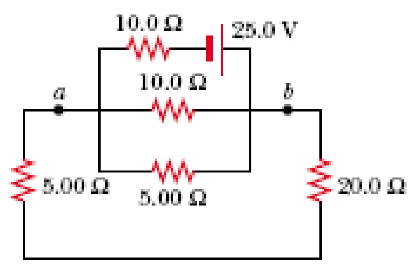
$$\mathcal{P} = IV = 3 \times 24 = 72\,Watt$$

Note that the power delivered to the resistor is

$$\mathcal{P} = i_1V_1 + i_2V_2 + i_5V_5 = 3 \times 3 + 3 \times 6 + 3 \times 15 = 72\,Watt$$

So, the power delivered by the battery equals the power delivered to resistors, which is the conservation of energy.

16. Consider the circuit shown in Figure below. Find (a) the current in the 20.0Ω resistor and (b) the potential difference between points a and b.



Solution:

Before attaching this question! I want you to take a closer look how resistors are connected: I mean which of these connected in parallel and which are connected in series, hum. With me so far, I hope so.

Look at the connecting points $\boldsymbol{a}$ and $\boldsymbol{b}$, what about them, hum. The resistors $20\Omega\ and\ 5\Omega$ are in series, I think you saw that clearly. Redraw the circuit in you mind by replaces these resistors with their equivalent25Ω. Now what do you think, yah. The resistor25Ω, 50Ω and 10Ω are in parallel. SO there equivalent resistance is

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{25} + \frac{1}{5} + \frac{1}{10}\right)^{-1} = 2.94\,\Omega$$

The 25.0 Volt source will distributed on 2.94Ω and the10Ω, via the voltage divider law we have

$$V_{2.94\Omega} = 25\frac{2.94}{2.94 + 10} = 5.68\,Volt$$

So, the potential difference between a and b is 5.68 Volt, and the current in the 20Ω can be found by finding the potential difference across it

$$V_{20\Omega} = 5.68\frac{20}{20 + 5} = 4.54\,Volt$$

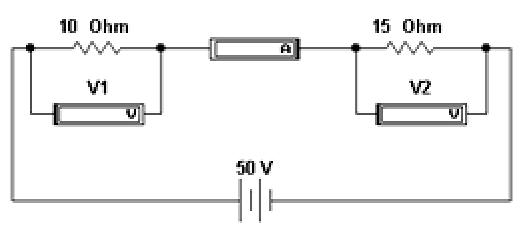
$$I_{20\Omega} = \frac{4.54}{20} = 0.227\,A$$

Or, we can find the branch current, that is

$$I_{25\Omega} = \frac{5.68}{25} = 0.227\,A$$

This current passes through the 20Ω *and* 5Ω resistances

17. In the circuit given below, find the following: the reading of the ammeter (A); voltmeter (V_1); voltmeter (V_2).



Solution:

The reading of the ammeter is the circuit current

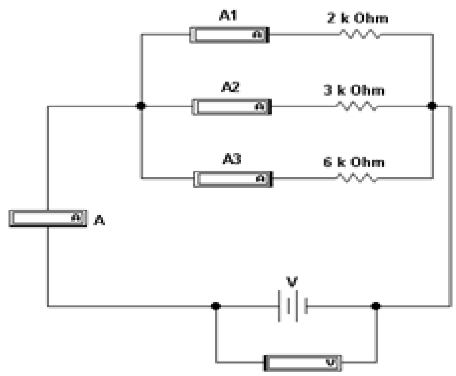
$$\boldsymbol{I = V/R_{eq} = \frac{50}{10 + 15} = 2A}$$

Apply Ohm's law or the voltage divider law to find $V_1$and V_2, that is

$\boldsymbol{V_1 = IR_1 = 2 \times 10 = 20Volt}$
$\boldsymbol{V_2 = IR_2 = 2 \times 15 = 30Volt}$

Notice that $V_1 + V_2 = 50\ Volt$

18. In the circuit given below, if the reading of $A_1 = 5\ mA$, then find the following: the reading of A_2; the reading of A_3; the reading of A; and the reading of V.



Solution:

To find all other currents we need to find the potential difference of the source. Since the resistances are connected in parallel, thus they have the same potential difference. Thus, using Ohm's law we can find

$$V_{R1} = I \times R_1 = 5 \times 10^{-3} A \times 2000\Omega = 10\ Volt$$

So, the battery voltage is 10 Volts, thus the other currents are

$$I_2 = \frac{10}{3000} = 3.33mA;\ I_3 = \frac{10}{6000} = 1.67mA$$

Thus the total current is the sum of these current

$$I = I_1 + I_2 + I_3 = 5mA + 3.33mA + 1.67mA = 10mA$$

This result may obtained by finding the circuit equivalent resistance, that is

$$\boldsymbol{R_{eq}} = \left(\frac{\mathbf{1}}{\boldsymbol{R_1}} + \frac{\mathbf{1}}{\boldsymbol{R_2}} + \frac{\mathbf{1}}{\boldsymbol{R_3}}\right)^{-\mathbf{1}} = \left(\frac{\mathbf{1}}{\mathbf{2000}} + \frac{\mathbf{1}}{\mathbf{3000}} + \frac{\mathbf{1}}{\mathbf{6000}}\right)^{-\mathbf{1}} = \mathbf{1000\Omega}$$

So, again the circuit current is

$$I = \frac{10}{1000} = 10mA$$

19. The timing device in an automobile's intermittent wiper system is based on an RC time constant and utilizes a $0.500\text{-}\mu\text{F}$ capacitor and a variable resistor. Over what range must R be made to vary to achieve time constants from 2.00 to 15.0 s?

Solution:

From the equation $\tau = RC$, we know that:

$$R = \frac{\tau}{C} = \frac{2.00\,\text{s}}{5.00\times10^{-7}\ \text{F}} = 4.00\times10^{6}\ \Omega \text{ and } R = \frac{\tau}{C} = \frac{15.0\,\text{s}}{5.00\times10^{-7}\ \text{F}} = 3.00\times10^{7}\ \Omega$$

Therefore, the range for R is: $\underline{4.00\times10^{6}\ \Omega - 3.00\times10^{7}\ \Omega} = \underline{4.00 \text{ to } 30.0\ \text{M}\Omega}$

20. A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

Solution:

$$r = \frac{60\,\text{s}}{72} = 0.833\,\text{s}\,;\, R = \frac{r}{C} = \frac{0.833\,\text{s}}{2.50\times10^{-8}\ \text{F}} = \underline{3.33\times10^{7}\ \Omega}$$

21. The duration of a photographic flash is related to an RC time constant, which is $0.100\ \mu\text{s}$ for a certain camera. (a) If the resistance of the flash lamp is $0.0400\,\Omega$ during discharge, what is the size of the capacitor supplying its energy? (b) What is the time constant for charging the capacitor, if the charging resistance is $800\ \text{k}\Omega$?

Solution:

(a) $C = \dfrac{r}{R} = \dfrac{1.00\times10^{-7}\ \text{s}}{0.0400\,\Omega} = \underline{2.50\ \mu\text{F}}$

(b) $\tau = RC = \left(8.00\times10^{5}\ \Omega\right)\left(2.50\times10^{-6}\ \text{F}\right) = \underline{2.00\ \text{s}}$

You have to notice that you have two circuit one for charging and the other for discharging. Discharging circuit resistance is $0.0400\,\Omega$ and charging circuit resistance is $800\ \text{k}\Omega$

22. A parallel-plate capacitor has an area $A = 2.0 \times 10^{-4} m^2$ and a plate separation $d = 1.0\ mm$. Find its capacitance.

Solution:

$$C = \frac{\varepsilon_0 A}{d} = 8.85 \times 10^{-12} C^2/Nm^2 \frac{2.0 \times 10^{-4} m^2}{1.0\ \times 10^{-3} m} = \ 1.77 \times 10^{-12} F$$

$$C = 1.77pF$$

23. After **two time** constants, what percentage of the final voltage, emf, is on an initially uncharged capacitor C, charged through a resistance R?

Solution:

$$t = 2RC$$

$$V = E\left(1 - e^{-2}\right) \Rightarrow \frac{V}{E} = 0.865 \Rightarrow \underline{86.5\,\%}$$

24. When two unknown resistors are connected in series with a battery, $225\ W$ is delivered to the combination with a total current of $5.00\ A$. For the same total current, 50.0 W is delivered when the resistors are connected in parallel. Determine the values of the two resistors.

Solution:

Notice that the same current value flows in the circuit for both connections

$$P_S = 225 = i_S^2 R_{eqS} \Rightarrow R_{eqS} = \frac{225}{5^2} = 9\Omega$$

$$P_P = 50 = i_P^2 R_{eqP} \Rightarrow R_{eqP} = \frac{50}{5^2} = 2\Omega$$

The equivalent resistance for resistance in series is

$$R_{eqS} = R_1 + R_2 = 9\Omega \qquad \mathbf{1}$$

The equivalent resistance for resistance in parallel is

$$R_{eqP} = \frac{R_1 R_2}{R_1 + R_2} = 2\Omega \qquad \mathbf{2}$$

Use 2 in 1, we have

$$\frac{R_1 R_2}{9\Omega} = 2\Omega \Rightarrow R_1 R_2 = 18\Omega \Rightarrow R_1 = \frac{18}{R_2}$$

Substitute R_1 in equation 1 and solve for R_2, that is

$$\frac{18}{R_2} + R_2 = 9 \Rightarrow R_2^2 - 9R_2 + 18 = 0$$

Solve,

$$R_2 = 3, 6\Omega$$

This means when

$R_2 = 6\Omega,\ R_1 = 3\Omega\ Or\ R_2 = 3\Omega,\ R_1 = 6\Omega$

25. A $500\text{-}\Omega$ resistor, an **uncharged** $1.50\text{-}\mu F$ capacitor, and a 6.16-V emf are connected in series. (a) What is the initial current? (b) What is the RC time constant? (c) What is the **current** after one time constant? (d) What is the voltage on the capacitor after one time constant?

Solution:

a.

$$I = \frac{V}{R} = \frac{6.16\ V}{500\Omega} = 12.32mA$$

b.

$$\tau = RC = 500\Omega \times 1.50 \times 10^{-6}F = 7.50 \times 10^{-4}s$$

c.

$$I = I_{max}e^{-t/RC}, \qquad I_{max} = 12.32mA$$

$$I = I_{max}e^{-1} = 0.37 \times 12.32mA = 4.56mA$$

d.

$$V_C(t) = V\left(1 - e^{-\frac{t}{RC}}\right) = V(1 - e^{-1}) = 0.63\ \times V$$

$$V_C = 0.63 \times 6.16 = 3.88V$$

26. A $2.00 - nF$ capacitor with an initial charge of $5.10\ \mu C$ is discharged through a $1.30k\Omega$ resistor. (a) Calculate the current through the resistor $9.00\mu s$ after the resistor is connected across the terminals of the capacitor. (b) What charge remains on the capacitor after $8.00\ \mu s$? (c) What is the maximum current in the resistor?

Solution:

a. We know the discharging current is given by

$I = -\frac{Q}{RC}e^{-\frac{t}{RC}} = -I_0e^{-\frac{t}{RC}}$

Thus,

$$I_0 = \frac{Q}{RC} = \frac{5.10\mu C}{2nF \times 1.3k\Omega} = 1.962\ \ A$$

$$I(9\mu s) = -1.962 \times exp\left(-\frac{9 \times 10^{-6}}{2 \times 10^{-9} \times 1.3 \times 10^{3}}\right)$$

$$I(9\mu s) = 62\ mA$$

b. The charge after 8 microseconds

$$q(t) = Q_0 \exp\left(-\frac{t}{RC}\right)$$

$$q(8\mu s) = 5.10\mu C \times exp\left(-\frac{8 \times 10^{-6}}{2 \times 10^{-9} \times 1.3 \times 10^{3}}\right)$$

$$q(8\mu s) = 0.235\ \mu C$$

c. The maximum current is $I_0 = 1.962\ A$

27. Consider a series RC circuit for which $R = 1.0M\Omega\ , C = 5.0\mu F$ and $\mathcal{E} = 30.0Volt$ Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after the switch is closed. (c) If the switch is closed at $t = 0$ find the current in the resistor $10.0\ s$ later.

Solution:

a. The time constant $RC = 5\ second$

b. The maximum charge is given by $Q = C\mathcal{E} = 150\ \mu C$

c. Since it is charging circuit the current in the circuit is given by

$$I(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$$

$$I(10seconds) = \frac{30}{10^6} \exp\left(-\frac{10}{5}\right) = 4.06\ \mu A$$

28. In RC circuit the voltage and current expressions are

$$v = 100 \exp(-1000t)\ V, \qquad t \geq 0;$$

$$i = 5 \exp(-1000t)\quad mA, \qquad t \geq 0^+$$

Find $R, C,$ and the time constant τ

Solution:

In this circuit

$$I_0 = 5\ mA = \frac{\mathcal{E}}{R} = \frac{100}{R} \Longrightarrow R = \frac{100}{5mA} = 20k\Omega$$

The time constant of this circuit is

$$\tau = RC = 10^{-3} \Rightarrow C = \frac{10^{-3}}{20k\Omega} = 50\ nF$$

Also we can find the initial charge on the capacitor's plates

$$Q = C\mathcal{E} = 50nF \times 100 = 5\ \mu C$$

29. A heart defibrillator being used on a patient has an RC time constant of $10.0\ ms$ due to the resistance of the patient and the capacitance of the defibrillator. (a) If the defibrillator has an $8.00\mu F$ capacitance, what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.) (b) If the initial voltage is $12.0\ kV$, how long does it take to decline to $6.00 \times 10^2 V$?

Solution:

(a) $R = \frac{\tau}{C} = \frac{1.00 \times 10^{-2}\ \text{s}}{8.00 \times 10^{-6}\ \text{F}} = 1.25 \times 10^3\ \Omega = \underline{1.25\ \text{k}\Omega}$

(b) $\tau = -RC\ \ln\left(\frac{V}{V_0}\right)$

$$= -\left(1.25 \times 10^3\ \Omega\right)\left(8.00 \times 10^{-6}\ \text{F}\right)\ln\left(\frac{600\ \text{V}}{1.20 \times 10^4\ \text{V}}\right) = 2.996 \times 10^{-2}\ \text{s} = \underline{30.0\ \text{ms}}$$

30. Using the exact exponential treatment, find how much time is required to discharge a $250\text{-}\mu\text{F}$ capacitor through a $500\text{-}\Omega$ resistor down to 1.00% of its original voltage

Solution:

$\tau = RC = (500\ \Omega)\left(2.50 \times 10^{-4}\ \text{F}\right) = 0.125\ \text{s}$; and

$V = V_0 e^{-t/\tau} \Rightarrow t = -\tau \ln\left(\frac{V}{V_0}\right) = (0.125\ \text{s})\ln(0.0100) = \underline{0.576\ \text{s}}$

31. Using the exact exponential treatment, find how much time is required to charge an initially uncharged $100 - pF$ capacitor through a $75.0\text{-}\text{M}\Omega$ resistor to 90.0% of its final voltage.

Solution:

$\tau = RC = \left(7.50 \times 10^7\ \Omega\right)\left(1.00 \times 10^{-10}\ \text{F}\right) = 7.50 \times 10^{-3}\ \text{s}$

$V = E\left(1 - e^{-t/\tau}\right) \Rightarrow \frac{V}{E} = 1 - e^{-t/\tau}$, or

$t = -\tau \ln\left(1 - \frac{V}{E}\right) = -\left(7.50 \times 10^{-3}\ \text{s}\right)\ln(1 - 0.900) = \underline{1.73 \times 10^{-2}\ \text{s}}$