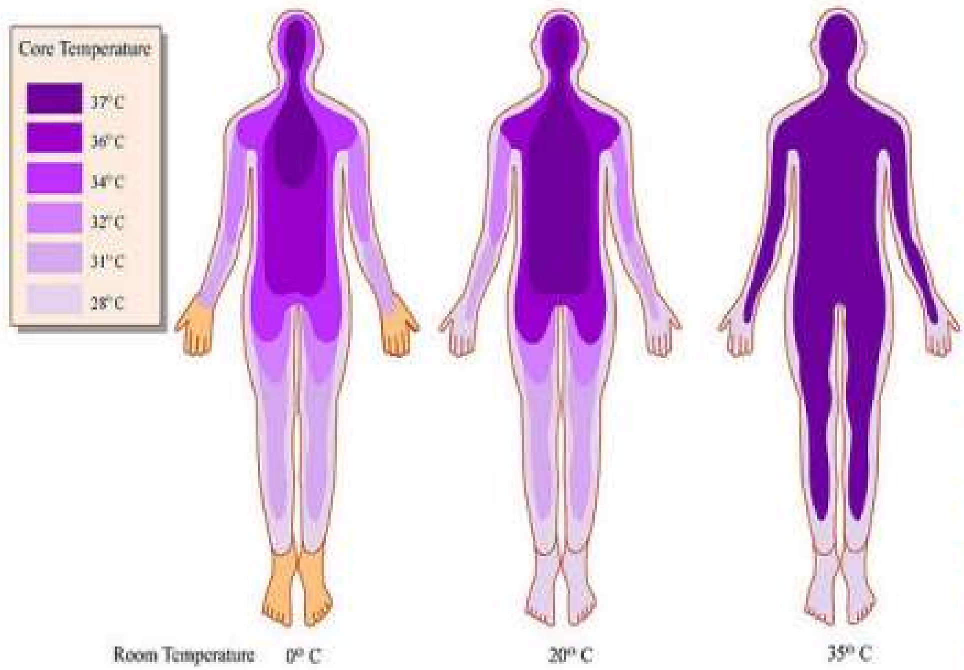
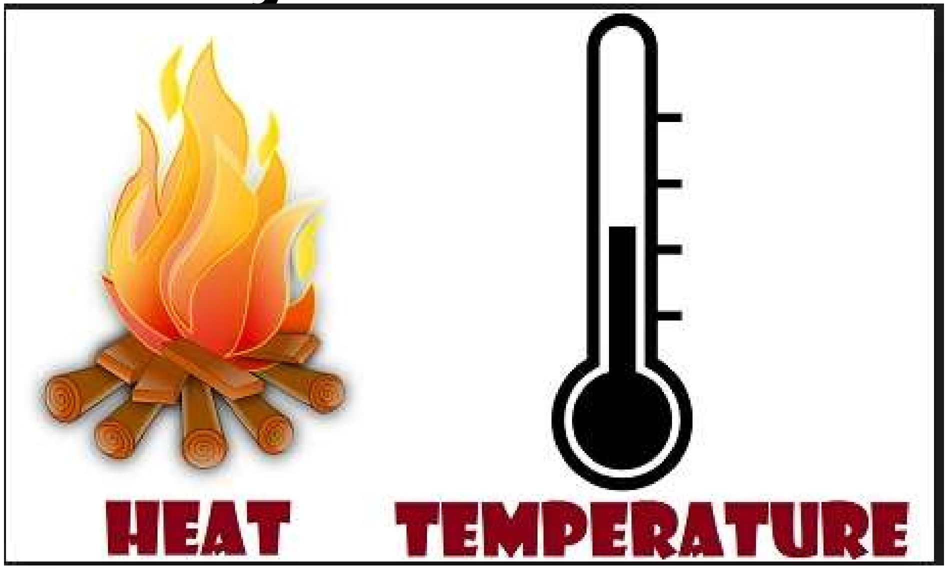
Thermal Properties of Matter



Definitions

- **Heat and Temperature (what is the difference)**
- **Thermal Contact – if energy can be exchanged between two objects, then they are in thermal contact**
- **Thermal Equilibrium – if two objects are in thermal contact and there is no exchange of energy, then they are in thermal equilibrium**



The Law of Equilibrium

- Zeroth Law of Thermodynamics
  - If objects A and B are separately in thermal equilibrium with object C, then A and B are in thermal equilibrium with each other

Definition of Temperature

- **If objects A and B are in thermal equilibrium, then they are at the same temperature.**

How to measure the Temperature

- ***Many physical properties of materials change sufficiently with temperature to be used as the bases for thermometers:***

⇨ The change in volume of a liquid

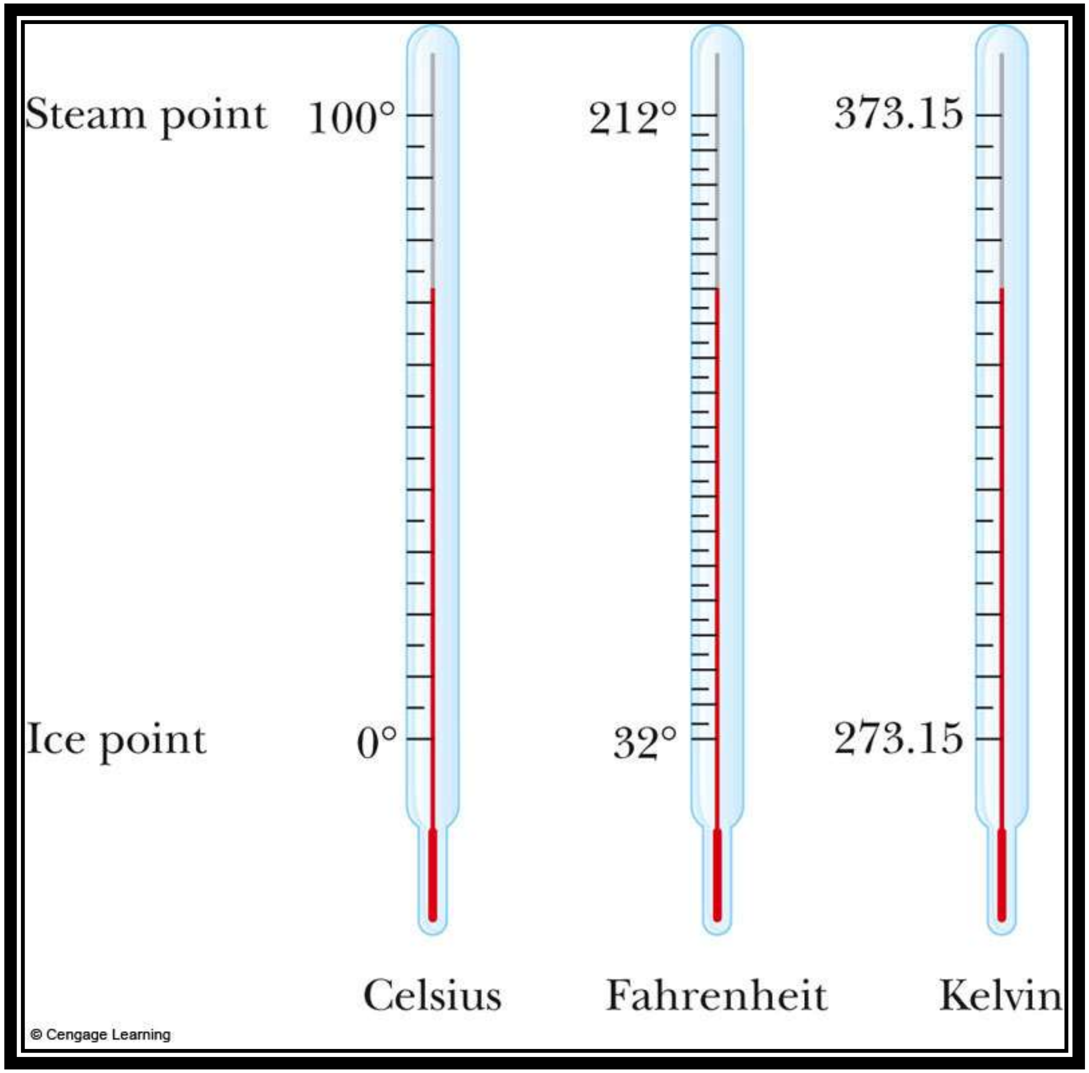
⇨ The change in length of a solid

⇨ The change in pressure of a gas held at constant volume

⇨ The change in electrical resistance of a conductor

⇨ The change in color of a very hot object.

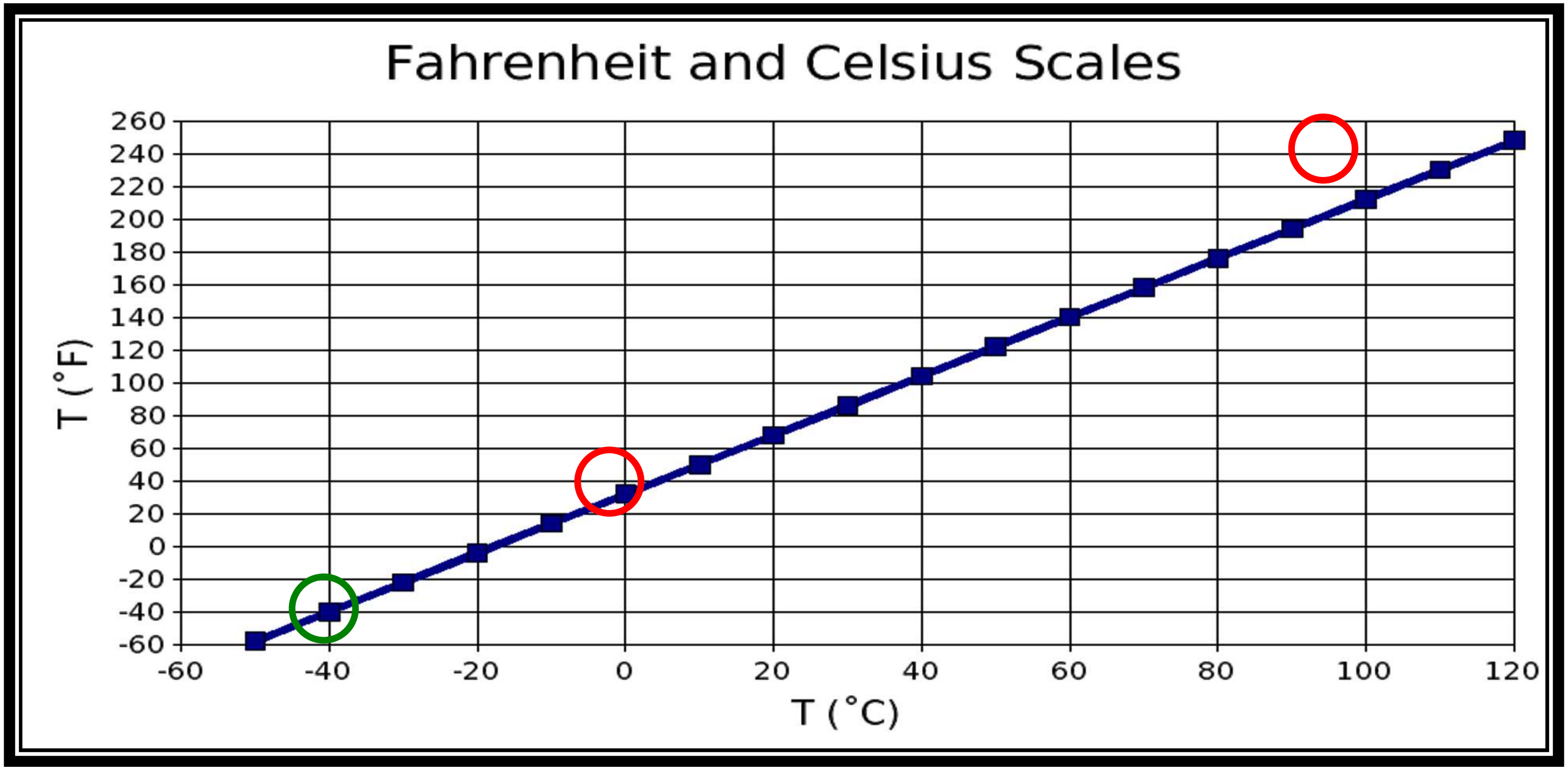
Temperature Scales



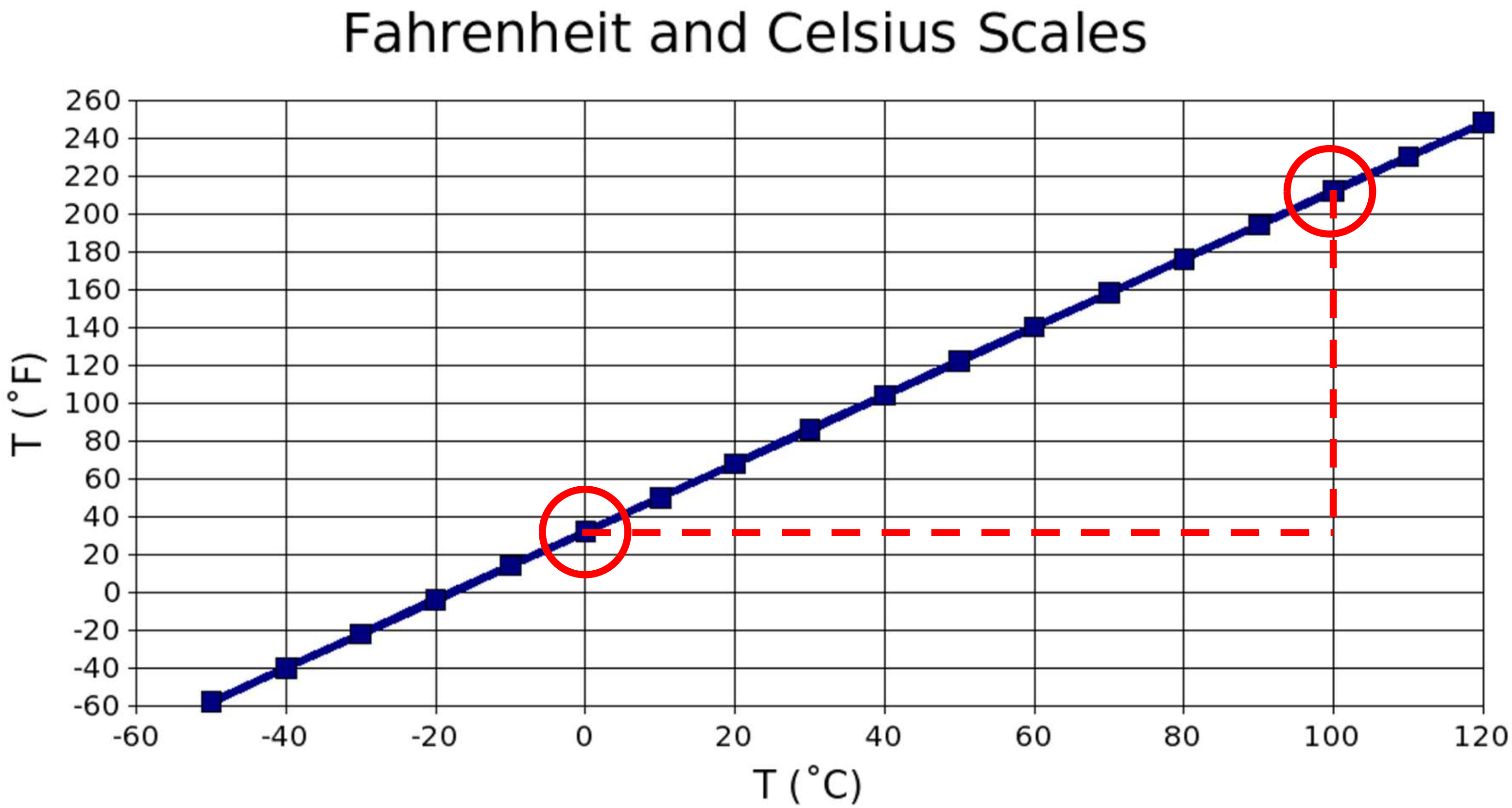
- Column of fluid changes height in response to warmth or coolness of surroundings
- Numbers assigned to the height establishes the *temperature scale*
- Each division in the scale is called a *degree*

Temperature Scales

- **Defined by:**
  - **Height of column when water freezes (0°C = 32°F)**
  - **Height of column when water boils (100°C = 212°F)**
- **Note: −40°C = −40°F**



Conversions



Slope: $$\frac{\Delta T_F}{\Delta T_C} = \frac{212°\mathrm{F} - 32°\mathrm{F}}{100°\mathrm{C} - 0°\mathrm{C}} = \frac{180°\mathrm{F}}{100°\mathrm{C}} = \frac{9}{5}$$

Intercept = $32°\mathrm{F}$

Conversions

	From °C to °F	From °F to °C
Temperature Reading	$T_F = \frac{9}{5} T_C + 32°$	$T_C = \frac{5}{9}\left(T_F - 32°\right)$
Temperature Change	$\Delta T_F = \frac{9}{5} \Delta T_C$	$\Delta T_C = \frac{5}{9} \Delta T_F$

Absolute Zero and Kelvin Scale

- Temperature is in units called kelvins (K)
- $T = 0$ K is called absolute zero
- Represents the temperature at which an ideal gas

$$T = T_C + 273.15\,\mathrm{K}$$

$$\Delta T = \Delta T_C$$

I. Characteristics of a Gas

A) Gases assume the shape and volume of a container.

B) Gases are the most compressible of all the states of matter.

C) Gases will mix evenly and completely when confined to the same container.

D) Gases have lower densities than liquids or solids.

C) The study of gases and its results formed the primary experimental evidence for our belief that matter is made up of particles. MATTER IS NOT CONTINUOUS.

III. Measurable Properties of a Gas

A) Mass - moles - symbol is n

B) Pressure - symbol is P

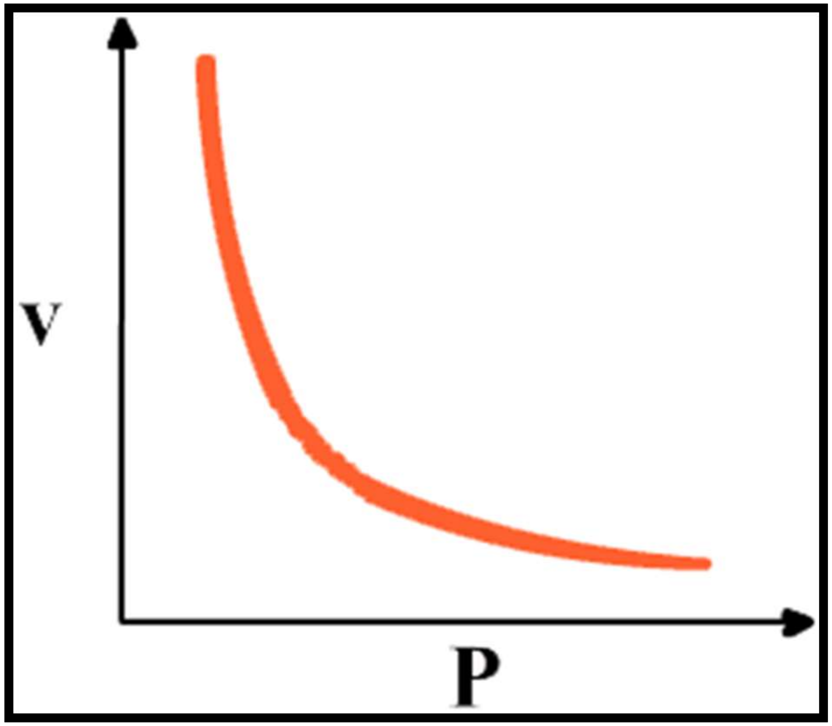
C) Volume - symbol is V

D) Temperature - must be in K - symbol is T

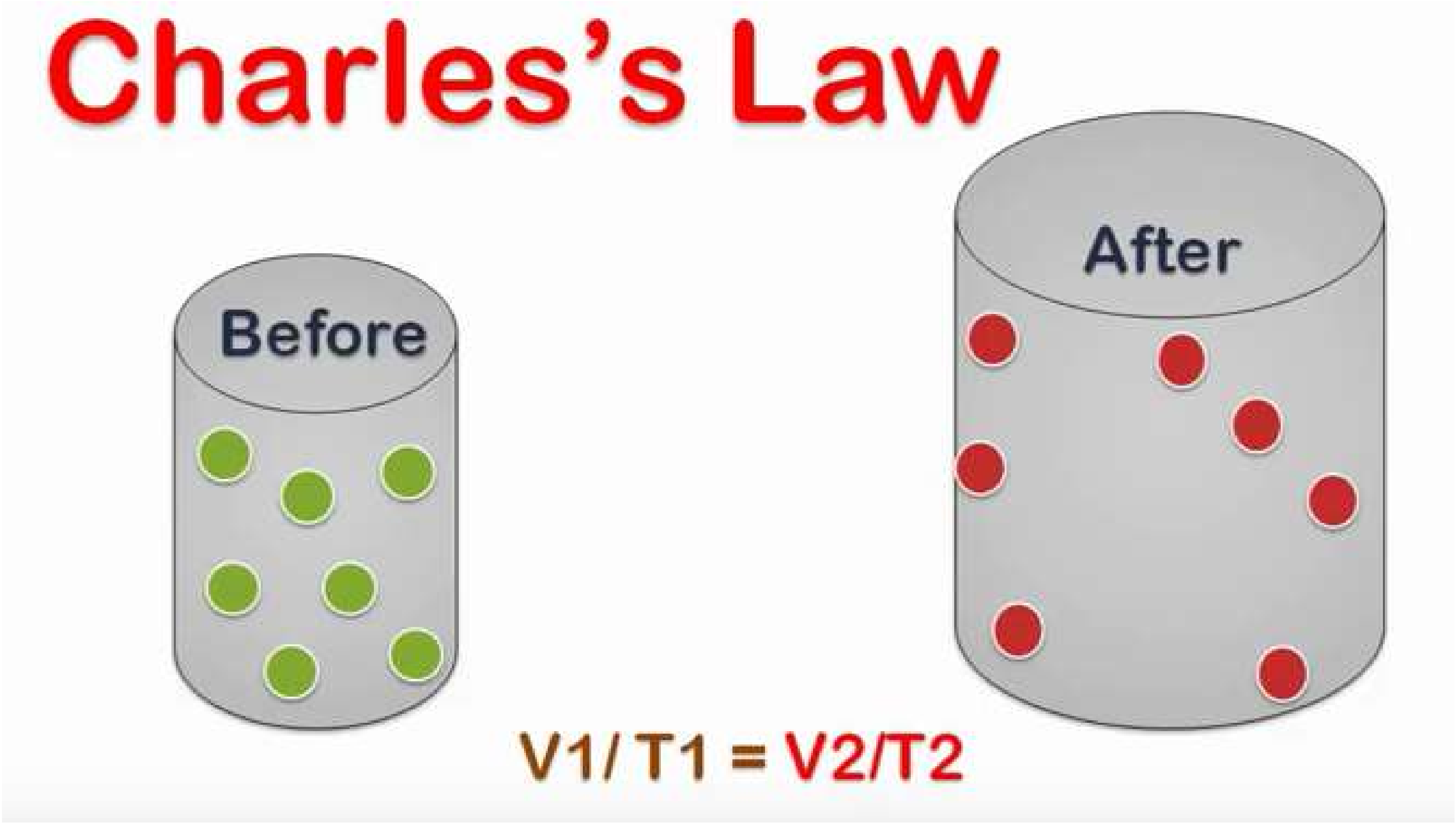
Boyle's Law: For a fixed amount of gas and constant temperature, **PV = constant.**

$$P_1V_1 = P_2V_2$$

T is constant



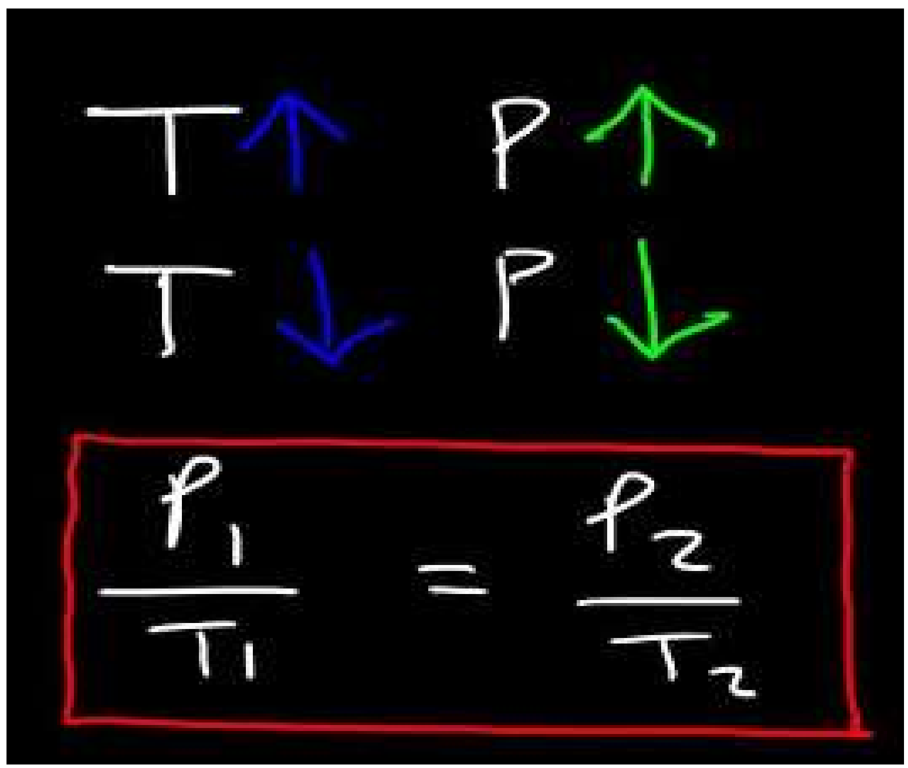
Charles's Law: at constant pressure the volume is linearly proportional to temperature. V/T = constant



Gay-Lussac's Law

Old man Lussac studied the direct relationship between temperature and pressure of a gas at constant volume.

As the temperature increases the pressure of a gas exerts on its container increases.



The Ideal Gas Law

$PV = nRT$

P = Pressure (in kPa)   V = Volume (in L)

T = Temperature (in K) n = moles

$$R = 8.31 \frac{\text{kPa} \cdot \text{L}}{\text{K} \cdot \text{mol}}$$

R is constant. If we are given three of P, V, n, or T, we can solve for the unknown value.

From Boyle's Law:

$P_iV_i = P_fV_f$ or PV = constant

From combined gas law:

$P_iV_i/T_i = P_fV_f/T_f$ or PV/T = constant

Ideal gas law the functional relationship between the pressure, volume, temperature and moles of a gas. PV = nRT; all gases are ideal at low pressure. V =nRT. Each of the individual laws is contained in this equation.

Sample problems

How many moles of H_2 is in a 3.1 L sample of H_2 measured at 300 kPa and 20°C?

PV = nRT   P = 300 kPa, V = 3.1 L, T = 293 K

(300 kPa)(3.1 L) = n (8.31 kPa•L/K•mol)(293 K)

$$\frac{(300\ \text{kPa})(3.1\ \text{L})}{(8.31\ \text{kPa}\cdot\text{L/K}\cdot\text{mol})(293\ \text{K})} = n = 0.38\ \text{mol}$$

Ideal Gas Law

The pressure, P, volume V, and temperature T of an ideal gas are related by a simple formula called the **ideal gas law**.

$$\boxed{PV = nRT}$$

$$R = 8.31\frac{J}{K \cdot mol}$$

$$R = 0.082\frac{L \cdot atm}{K \cdot mol}$$

$$\boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$$

2.50 g of $XeF4$ gas is placed into an evacuated 3.00 $liter$ container at 80°C. What is the pressure in the container?

Solution

$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$n = \frac{m}{Mw} = \frac{2.5}{207.28} = 0.012\, mole, R = 8.314\, J/mole.K,$$

$$T_K = T_C + 273.15 = 80 + 273 = 353\, K$$

$$V = 3 \times 10^{-3}\, m^3$$

So,

$$P = \frac{0.012 \times 8.314 \times 353}{3 \times 10^{-5}} = 1.17 \times 10^4\, Pa$$

A hydrogen gas thermometer is found to have a volume of 100.0 cm^3 when placed in an ice-water bath at $0°C$. When the same thermometer is immersed in boiling liquid chlorine, the volume of hydrogen at the same pressure is found to be 87.2 cm^3. What is the temperature of the boiling point of chlorine?

Since

$$P_1 = P_2 \quad \text{at } T_1 = 273\,K$$

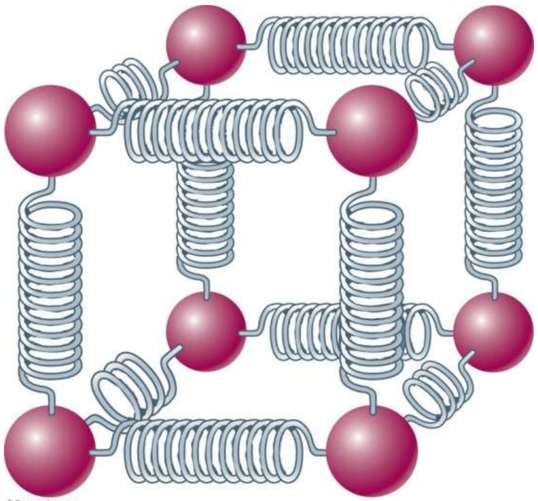
Thus,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \rightarrow T_2 = \frac{V_2}{V_1} T_1$$

$$T_2 = \frac{87.2}{100} \times 273 = 238.056\,K$$

Thermal Expansion

- **For solids and liquids:**
  - **Energy increase via heat input**
  - **Atoms vibrate with greater amplitude**
  - **Average separation increases**
- **Leads to macroscopic expansions**



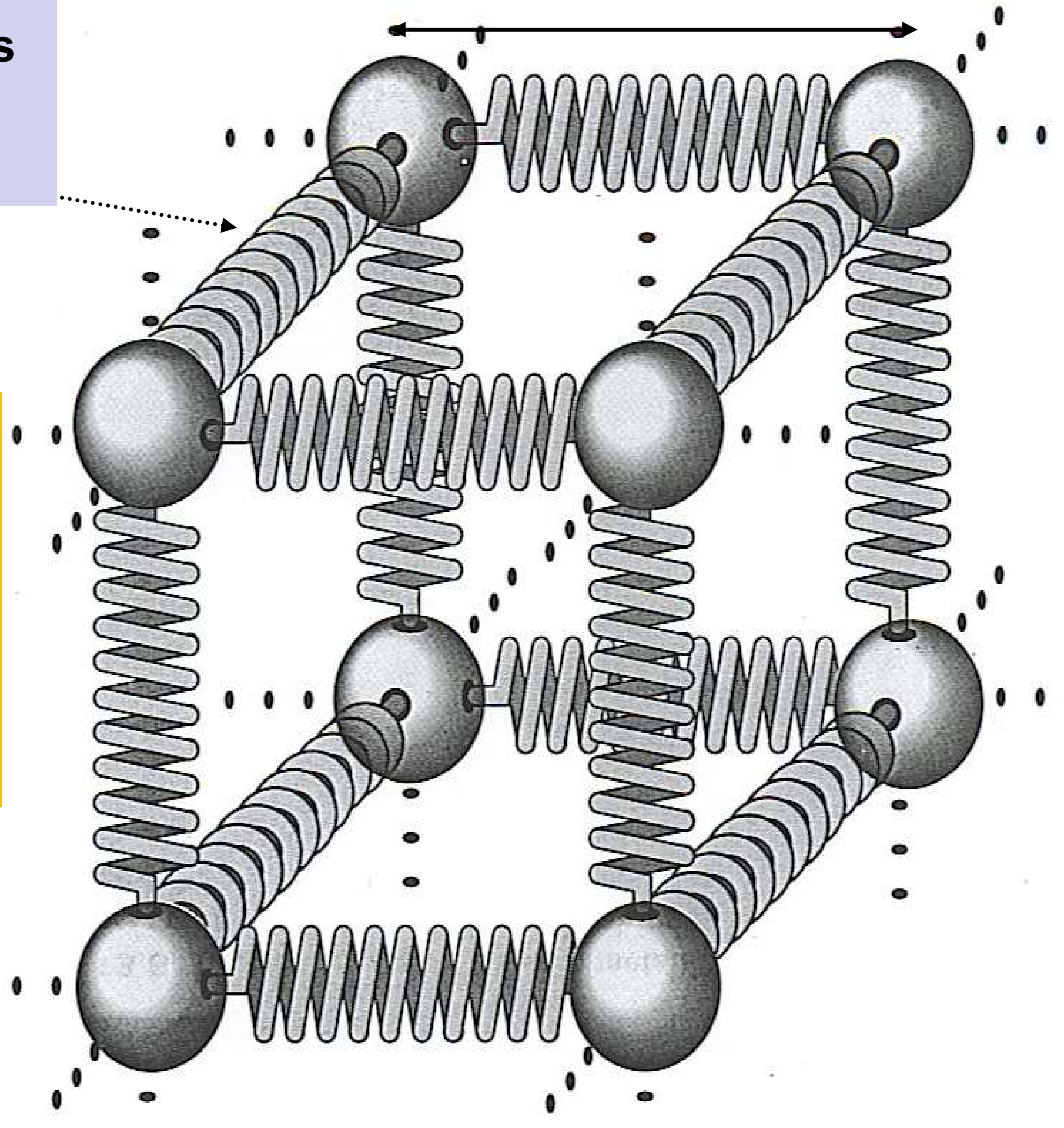
(a)

(b)

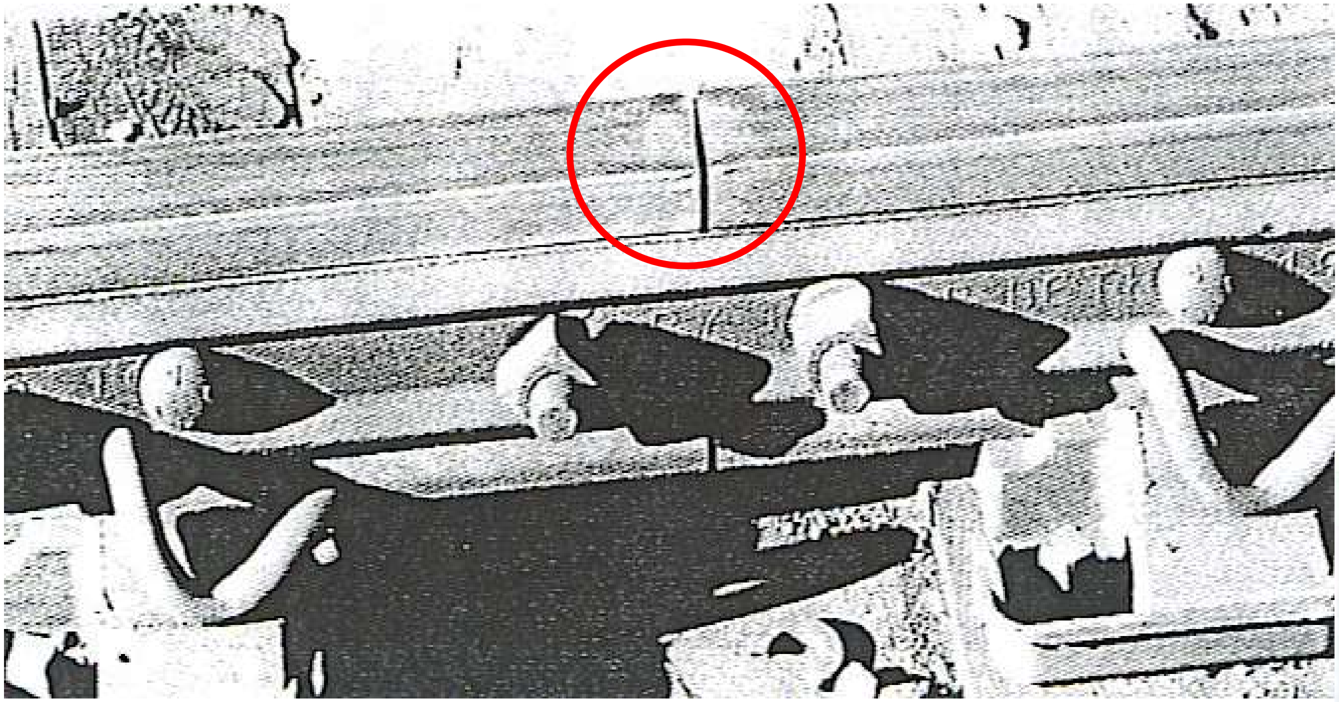
Most solids and liquids expand when heated. Why?

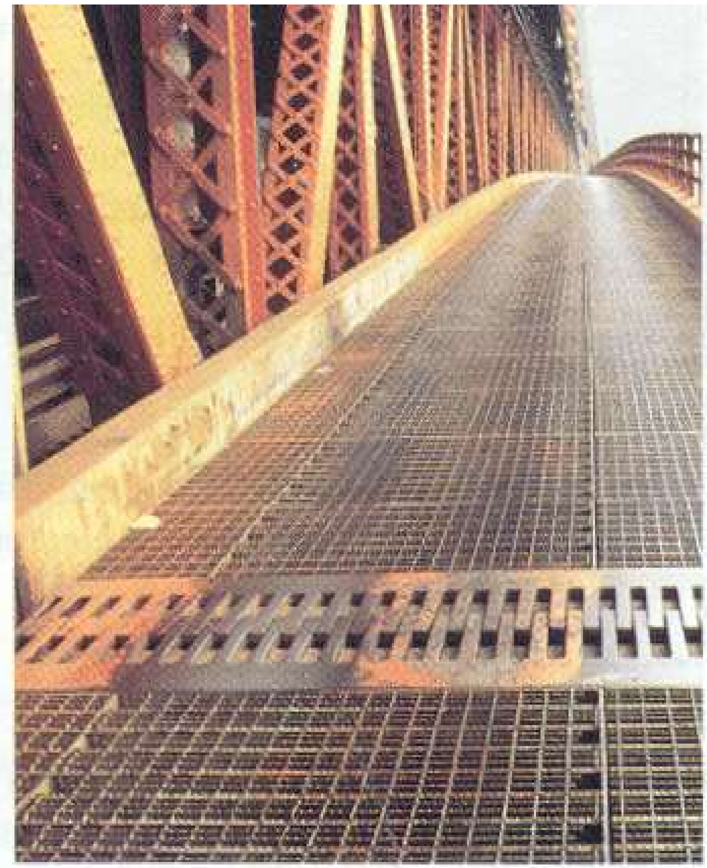
Average distance between atoms

Inter-atomic forces
≡
“springs”



Internal Energy U is associated with the amplitude of the oscillation of the atoms



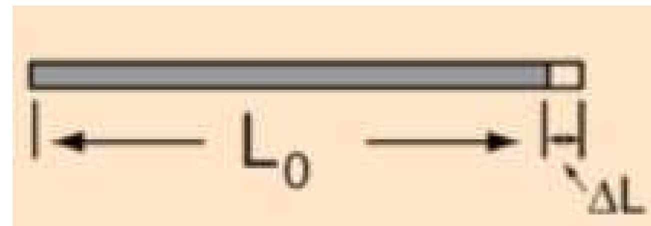


Linear Expansion of Solids

The change in one dimension of a solid (length, width or thickness) is called linear expansion. For small temperature changes, linear expansion is approximately proportional to the change in temperature; $\Delta T = T - T_0$. The fractional change in length is called the thermal strain, which is equal

$$\frac{L - L_0}{L_0} = \frac{\Delta L}{L_0},$$

Thermal strain

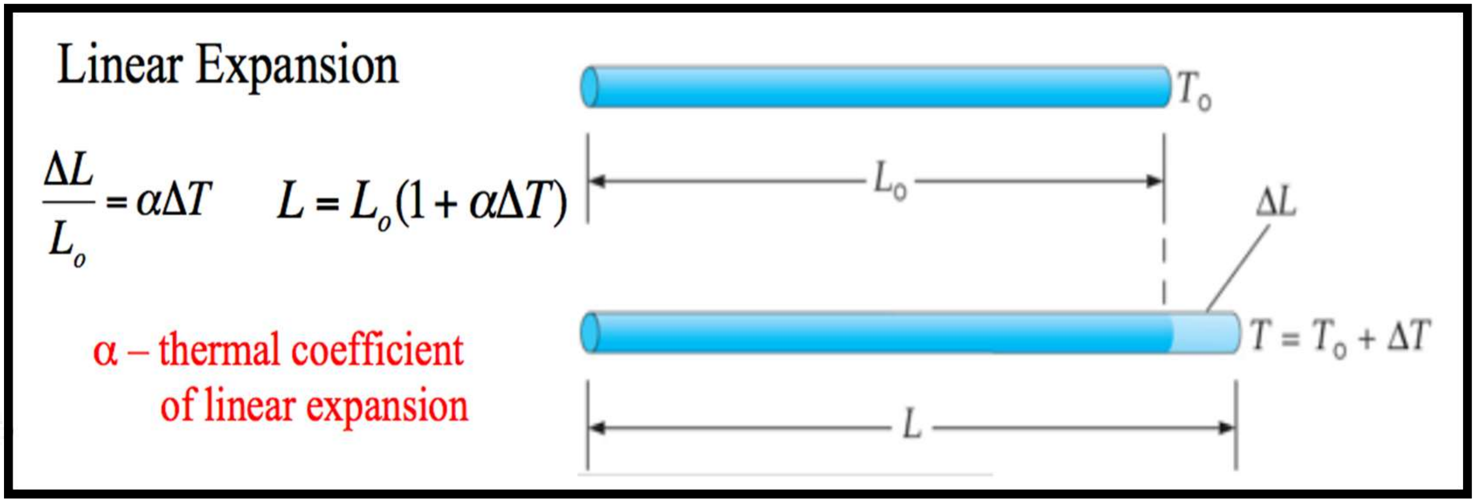


Where L_0 is the original length at the original temperature T_0. This is related to the change in temperature by

$$\frac{\Delta L}{L_0} = \alpha \Delta T \quad \text{or} \quad \Delta L = \alpha L_0 \Delta T, \tag{4.17}$$

Where α is the thermal coefficient of linear expansion, which has units of K^{-1}. We can also write down an expression for the final length L after a temperature change:

$$\begin{aligned}
\Delta L &= \alpha L_0 \Delta T \\
L - L_0 &= \alpha L_0 \Delta T \\
L &= L_0 + \alpha L_0 \Delta T \\
L &= L_0 \left(1 + \alpha \Delta T\right)
\end{aligned}$$



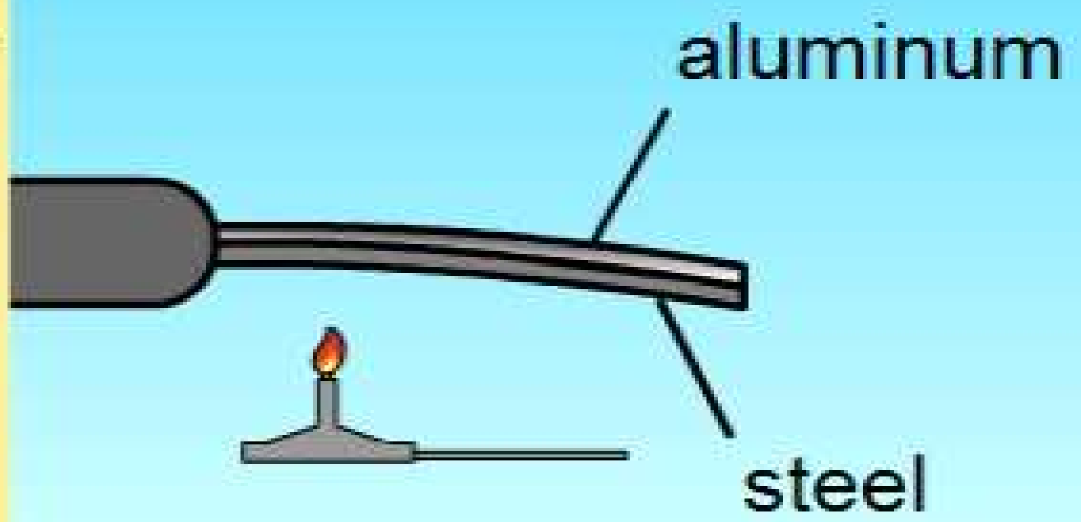
Linear Thermal Expansion Coefficient

The coefficient of expansion may vary slightly for different temperature ranges. Since this variation is negligible for most applications, we (usually) consider α to be a constant and independent of temperature.

Different substances expand at different rates, which because they have different thermal expansion coefficient.

Coefficient of Volume Expansion, at 20 C

Material	Coefficeint of Linear Expansion, α (C)$^{-1}$
Aluminum	25×10^{-6}
Steel	12×10^{-6}
Glass	9×10^{-6}
Concrete	$\approx 12 \times 10^{-6}$



Ceramics
low expansion coefficients
$\alpha \sim 10^{-6}\ K^{-1}$
Polymers **high expansion coefficients**
$\alpha \sim 10^{-4}\ K^{-1}$
Metals
$\alpha \sim 10^{-5}\ K^{-1}$

Linear Thermal Expansion

$$\Delta L = \alpha L_0 \Delta T$$

$$\Delta L = L - L_0$$

$$\Delta T = T - T_0$$

A concrete slab has a length of $12\ m$ at $-5\ ^{o}C$ on a winter's day. What is the change in length from winter to summer, when the temperature is $35\ ^{o}C$? The linear expansion coefficient of concrete is $1 \times 10^{-5}\,^{o}C^{-1}$.

Solution

$$\Delta T = (T_f - T_0) = (35 - (-5)) = 40\ ^{\circ}C$$

$$\Delta l = \alpha l_0 \Delta T = 1 \times 10^{-5}\,^{o}C^{-1} \times 12m \times 40\ ^{\circ}C = 4.80\ mm$$

A steel rod 4.00 cm in **diameter** is heated so that its temperature increases by $70.0C^{\circ}$. It is then fastened between two rigid supports. The rod is allowed to cool to its original temperature. Assuming that Young's modulus for the steel is $20.6 \times 10^{10}\ N/m^2$ and that its average coefficient of linear expansion is $11 \times\ 10^{-6}\ {}^{\circ}\mathrm{C}^{-1}$, calculate the tension in the rod.

$$r = 2\ cm = 2 \times 10^{-2} m, Y_{steel} = 20.6 \times 10^{10}\ \frac{N}{m^2}, and\ \alpha = 11 \times\ 10^{-6}\ {}^{\circ}\mathrm{C}^{-1}$$

$$\sigma = Y_{steel}\ \varepsilon$$

Where

$$\sigma = \frac{F}{A}, and\ \varepsilon = \frac{\Delta l}{l_0}$$

Thus,

$$F = Y_{steel}\ \times A \times \frac{\Delta l}{l_0}$$

$$F = Y_{steel} \times A \times \frac{\Delta l}{l_0}$$

The change in length due to increase in temperature is

$$\Delta l = \alpha l_0 \Delta T \rightarrow \frac{\Delta l}{l_0} = \alpha \Delta T$$

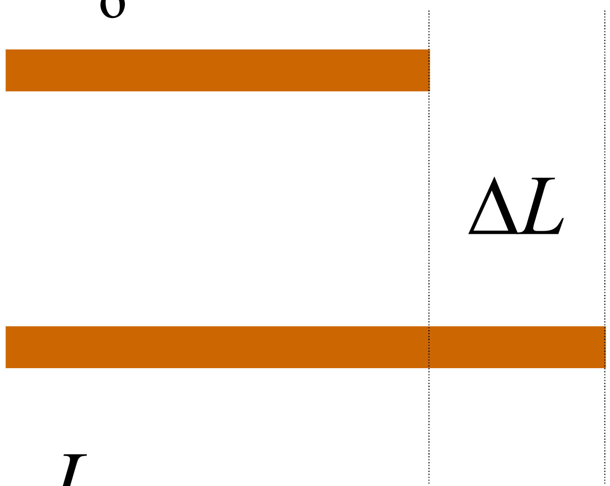
$$F = Y_{steel} \times A \times \left(\frac{\Delta l}{l_0}\right) = Y_{steel} \times A \times (\alpha \Delta T)$$

$$F = 20.6 \times 10^{10} \frac{N}{m^2} \times \pi(2 \times 10^{-2} m)^2 \times \left(11 \times 10^{-6}\,{}^{\circ}\mathrm{C}^{-1} \times 70.0 C^{\circ}\right)$$

$$F = 1.99 \times 10^5\, N \cong 2 \times 10^5\, N$$

Linear

L_o



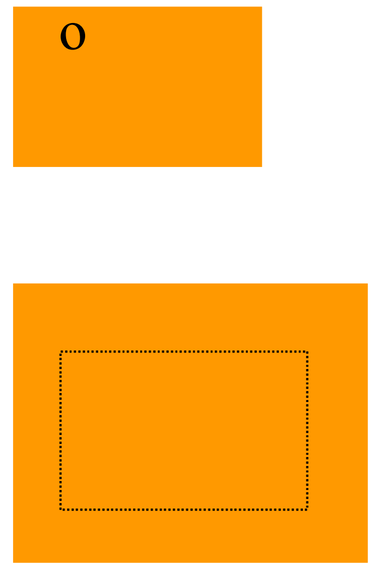
ΔL

L

$$\Delta L = \alpha L_o \Delta T$$

Area

A_o

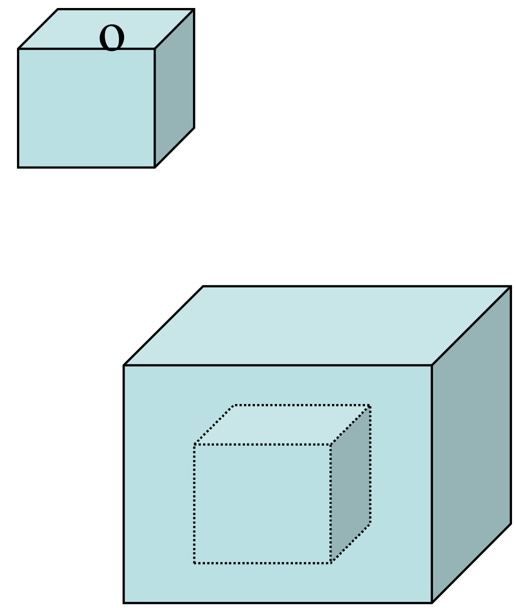


A

$$\Delta A = 2\alpha A_o \Delta T$$

Volume

V_o



V

$$\Delta V = 3\alpha V_o \Delta T = \beta V_o \Delta T$$

Volume expansion – solid cube

$$V_o = L_o^3$$

$$V = L^3 = (L_o + \alpha\, L_o\, \Delta T)^3 = L_o^3(1 + \alpha\, \Delta T)^3$$

$$V = L_o^3\,(1 + 3\,\alpha\,\Delta T + 3\,\alpha^2\,\Delta T^2 + \alpha^3\,\Delta T^3)$$

$$V = L_o^3\,(1 + 3\,\alpha\,\Delta T) \qquad \text{(ignoring higher order terms)}$$

$$V - V_o = \Delta V = 3\,\alpha\, L_o^3 \Delta T = \beta\, V_o\, \Delta T$$

α **coefficient of linear expansion**

β **coefficient of volume expansion**

15. A thermometer has a mercury-filled glass bulb with a volume of $2 \times 10^{-7}\ m^3$ attached to a thin glass capillary tube with an inner radius of $5 \times 10^{-5}\ m$. If the temperature increases by 100°C, how far will the mercury rise in the tube? (volume thermal expansion coefficient of mercury= $1.82 \times 10^{-4}\ K^{-1}$).

$V_0 = \pi r^2 h_0$ = $2 \times 10^{-7}\ m^3$, where $r = 5 \times 10^{-5} m$, *is the inner radius of the capillary,* *and* h_0 *is the initial height of the mercury*, so

$$h_0 = \frac{V_0}{\pi r^2} = \frac{2\times10^{-7}}{3{,}14\times(5\times10^{-5})^2} = 25.4\ m$$

$$\Delta T = 100\ ^0C$$

The new volume of the mercury after the expansion

$V = V_0[1 + \beta\Delta T]$, **where** $\beta = 1.82 \times 10^{-4} K^{-1}$ *and* $V = \pi r^2 h$, *and h is the* **New height of the mercury, so**

$\pi r^2 h = \pi r^2 h_0\ [1 + \beta\Delta T]$ ⋙⋙ $h = h_0\ [1 + \beta\Delta T]$

$$h = 25.4\ [1 + 1.82 \times 10^{-4}(100)] = 25.4 \times 1.0182 = 25.86\ m$$

Expansion Coefficients

$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

Linear Expansion – 1D

$$\boxed{\Delta A = \gamma A_0 \Delta T}$$

Area Expansion – 2D

$$\boxed{\Delta V = \beta V_0 \Delta T}$$

Volume Expansion – 3D

If α is the same in all directions then

$$\gamma = 2\alpha \qquad \beta = 3\alpha$$

Heat as a form of Energy

What is HEAT?

- **Form of energy and measured in JOULES, Ergs, Calorie or BTU (Thermal Units)**
- **Particles move about more and take up more room if heated – this is why things expand if heated**
- **It is also why substances change from: solids liquids gases when heated**

Heating and Cooling

- If an object has become hotter, it means that it has gained heat energy.

  $\Delta T > 0$ (positive)

- If an object cools down, it means it has lost energy

  $\Delta T < 0$ (Negative)

- Heat energy always moves from:

HOT object → COOLER object

Cup of water at 20 °C in a room at 30°C - gains heat energy and heats up – its temperature rises

Cup of water at 20 °C in a room at 10°C loses heat energy and cools down – its temperature will fall.

Heat Capacity

Heat capacity is the amount of energy required to raise the temperature of a substance by 1K or 1ºC.

Different substances will have different thermal capacities, and different masses of the same kind of substance will also have different thermal capacities.

Heat Supplied (Q) = Heat Capacity (C) x Change in temperature (ΔT)

$$\text{Heat Capacity (C)} = \frac{Q\ (Joul)}{\Delta T\ (K)}$$

Specific Heat Capacity

- Specific Heat Capacity can be thought of as a measure of how much heat energy is needed to warm the substance up.
- You will possibly have noticed that it is easier to warm up a saucepan full of oil than it is to warm up one full of water.

- Specific Heat Capacity (C) of a substance is the amount of heat required to raise the temperature of 1g of the substance by 1°C (or by 1 K).
- The units of specific heat capacity are J $^{o}C^{-1}$ g^{-1} or J K^{-1} g^{-1}. Sometimes the mass is expressed in kg so the units could also be J $^{o}C^{-1}$ g^{-1} or J K^{-1} kg^{-1}

The amount of heat energy (q) gained or lost by a substance = mass of substance (m) X specific heat capacity (C) X change in temperature (ΔT)

$$Q = m \times c \times \Delta T$$

- **Approximate values in J / kg ° K of the Specific Heat Capacities of some substances are:**

Air	1000	Lead	125
Aluminium	900	Mercury	14
Asbestos	840	Nylon	1700
Brass	400	Paraffin	2100
Brick	750	Platinum	135
Concrete	3300	Polythene	2200
Cork	2000	Polystyrene	1300
Glass	600	Rubber	1600
Gold	130	Silver	235
Ice	2100	Steel	450
Iron	500	Water	4200

How much energy would be needed to heat 450 grams of copper metal from a temperature of 25.0ºC to a temperature of 75ºC? The specific heat of copper at 25ºC is 0.385 J/g ºC.)

The change in temperature (*ΔT*) is:

75ºC - 25ºC = 50ºC

Given mass, two temperatures, and a specific heat capacity, you have enough values to plug into the specific heat equation

$$q = m \times C \times \Delta T \ .$$

and plugging in your values you get

$$Q = (450\ g) \times (0.385\ J/g\ ºC) \times (50ºC)$$

$$= 8700\ J$$

$$Q = m\,c\,\Delta T$$

Q = energy transfered (joules)
m = mass of water (grams)
c = specific heat capacity
ΔT = temperature change (K or °C)

The sum of heat gain and heat lost equal zero

assume that m_x is the mass of unknown substance, whose specific heat c_x is to be determined, and then it is heated to a certain temperature T_x. Likewise take an amount of water m_w which its specific heat is known c_w and its temperature T_w is known. If both substances are mixed and left until their temperature remain constant (final temperature T_f), so we can apply the conservation of energy as,

$$Q_{gain}(w) + Q_{lost}(x) = 0 \Longrightarrow m_w c_w(T_f - T_w) + m_x c_x(T_f - T_x) = 0$$

Solving this equation for c_x, we find

$$c_x = \frac{m_w c_w (T_f - T_w)}{m_x (T_x - T_f)} \tag{4.25}$$

A quantity of hot water at 91℃ and another cold one at 12℃. How much kilogram of each one is needed to make an 800 liter of water bath at temperature of 35℃.

Th(hot)= 91C, Tc(cold)= 12; Tm (mixture)= 35C

M(hot)+ m(cold) = 800 kg...........................(1)

Q(gain by cold water)+ Q (lost by hot water) = 0
m(cold)x c(water) x (Tm- Tc)+ m(h)x c(water)x (Tm-Th)= 0
23 m(cold) – 79 m(hot) = 0.........................(2)

Heat Transfer

- Heat is transferred through a material by being passed from one particle to the next
- Particles at the warm end move faster and this then causes the next particles to move faster and so on.
- In this way heat in an object travels from:

the HOT end the cold end

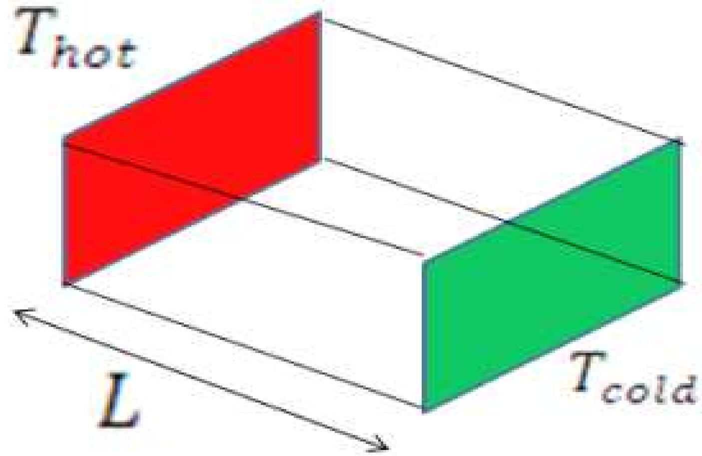
Conduction

- Occurs by the particles hitting each other and so energy is transferred.
- Can happen in solids, liquids and gases,
- Happens **best in solids**-particles very close together
- Conduction does not occur very quickly in liquids or gases

Conduction

- Materials that conduct heat quickly are called conductors
- All metals are good conductors of heat
- Copper is a very good conductor of heat
- Pans for cooking are usually made with a copper or aluminium bottom and plastic handles

Heat Rate $H = \Delta q / \Delta t$



For heat transfer between two plane surfaces, such as heat loss through the wall of a house, the rate of conduction heat transfer (H) is the energy transferred per unit of time:

$$H = \frac{Q}{t} = \kappa A \, \frac{(T_{hot} - T_{cold})}{L}, \quad measured\ in\ Watt \tag{4.26}$$

Where L is the separation distance (thickness) between the hot and the cold sides, A is the contact area, $(T_{hot} - T_{cold})$ is the temperature gradient, and κ is a constant called the thermal conductivity coefficient of the material measured in W/mK. This constant is a characteristic of the material. Table 4.5 shows some values of κ for different materials. Those with high κ like metals are good heat conductor and with small κ like gases and nonmetals are good insulator.

Table 4.5: The thermal conductivity coefficient for different materials.

Substance	($W\,m^{-1}K^{-1}$)	Substance	($W\,m^{-1}K^{-1}$)
Silver	427	Ice	2
Copper	397	Water	0.6
Aluminum	238	Wood	0.08
Gold	314	Air	0.023
Concrete	0.8	Hydrogen	0.1
Glass	0.8	Helium	0.138

Heat Rate $H = \Delta q/\Delta t$

$$H = \kappa \frac{A\Delta T}{L}$$

An aluminum pot contains water that is kept steadily boiling (100 ºC). The bottom surface of the pot, which is 12 mm thick and $1.5x10^4$ mm^2 in area, is maintained at a temperature of 102 ℃ by an electric heating unit. Find the rate at which heat is transferred through the bottom surface. Compare this with a copper based pot. The thermal conductivities for aluminum and copper are shown in the table.

The thickness of ghe pot is L = 12mm= 0.012 m and the temperature differece ΔT= 102-100= 2°C The surface area A= $1.5x10^4$ mm^2 = $1.5x10^{-2}$ m^2

$$H_{al} = \kappa_{Al}\frac{A\Delta T}{L} = 238\frac{1.5\times10^{-2}(2)}{0.012} = 595 \text{ Watt}$$

$$H_{Cu} = \kappa_{Cu}\frac{A\Delta T}{L} = 397\frac{1.5\times10^{-2}(2)}{0.012} = 992 \text{ Watt}$$

A box with a total surface area of 1.20 m2 and a wall thickness of 4.00 cm is made of an insulating material. A 10.0-W electric heater inside the box maintains the inside temperature at 15.0°C above the outside temperature. Find the thermal conductivity k of the insulating material.

Surface area A= 1.2 m^2 and the wall thickness L = 4cm = 0.04 m

The heat rate is the power of the electric heater H= 10 Watt

The temperature difference between inside and outside is ΔT= 15°C

$$H = \kappa \frac{A \Delta T}{L} \to \to \kappa = \frac{H\,L}{A \Delta T} = \frac{10\,(0.04)}{1.2\,(15)} = 0.022\ {}^{W}/_{mK}$$

Heat Transfer by Radiation

- Transfer of heat directly form the source to the object by a wave, travelling as rays.
- Heat radiation is also known as

  INFRA-RED Radiation

- All objects that are hotter than their surroundings give out heat as infra-red radiation
- Heat transfer by radiation does not need particles to occur and is the only way energy can be transferred across empty space

The color and texture of different surfaces determines how well they absorb the radiation.

(1) Black objects absorb more radiation than white objects.

(2) Matt and rough surfaces absorb more than shiny and smooth surfaces.

Radiation

The relationship governing radiation from hot objects is called the Stefan-Boltzmann Law:

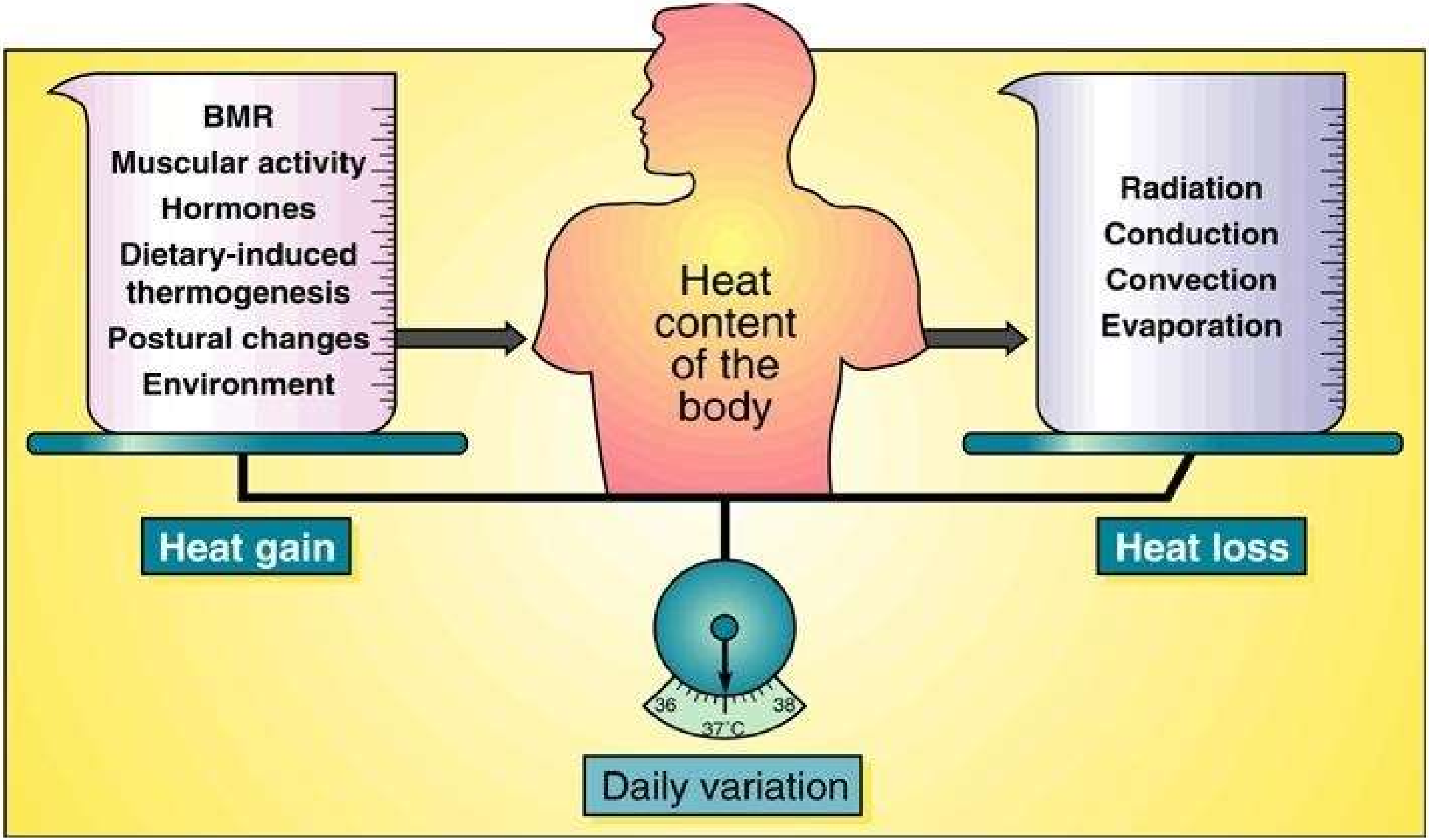
$$P = e\,\sigma\,A\,(T^4 - T_S^4) \tag{4.28}$$

Where P is the net radiated power measured in Watt, e is the emissivity (=1 for ideal radiator), A is the radiation area in m^2, T is the temperature of the radiator in Kelvin, Ts is the temperature of the surroundings in Kelvin, and σ = 5.67×10^{-8} Watt/m^2 K^4 is a constant called Stefan-Boltzmann constant.

Example 4.12

A student tries to decide what to wear is staying in a room that is at 20°C. If the skin temperature is 37°C, how much heat is lost from the body in 10 minutes? Assume that the emissivity of the body is 0.9 and the surface area of the student is 1.5 m^2.

Thermoregulation in Heat



- **Heat Loss by Conduction**
  - Direct transfer of heat through a liquid, solid, or gas from one molecule to another.
  - A small amount of body heat moves by conduction directly through deep tissues to cooler surface. Heat loss involves the warming of air molecules and cooler surfaces in contact with the skin.
  - The rate of conductive heat loss depends on thermal gradient.

- **Heat Loss by Evaporation (~ 55%)**
  - Heat transferred as water is vaporized from respiratory passages and skin surfaces.
  - For each liter of water vaporized, 580 kcal transferred to the environment.
  - When sweat comes in contact with the skin, a cooling effect occurs as sweat evaporates.
  - The cooled skin serves to cool the blood.

A 2 m length of copper pipe of 10 cm diameter containing hot water at 80C. If the surroundings are at 20C, at what rate does the pipe lose thermal energy due to radiation? (take e = 1)

A cylinder of radius r= 0.05 m and length l= 2m

The hot side has temperature = 273+ 80= 353 K

The cold side has temperature= 273+20= 293K

$$P = eA\sigma\left(T_h^4 - T_c^4\right) = 1\,(2\pi r l)5.67 \times 10^{-8}\left[353^4 - 293^4\right]$$

$$= 2(3.14)\,0.05(2)\,5.67 \times 10^{-8}\left[353^4 - 293^4\right]$$

$$= 290.46\ \text{W}$$

A girl having a surface area of 1.2 m^2 wears a down jackets 3 cm thick with thermal conduction coefficient of 0.02 $Wm^{-1}K^{-1}$. If her skin temperature is 34 °C and she can safely lose 85 W by conduction, what is the lowest temperature for which the clothing is adequate?

$$H = \kappa \frac{A\Delta T}{L}$$