FLUID MECHANICS
Biofluids

- **Fluid**

A continuous, amorphous substance whose molecules move freely past one another and that has the ability to flow and assume the shape of its container; a liquid or gas.

- **Fluids share the properties of not resisting deformation and the ability to flow.**

  These properties are typically a function of their inability to support a shear stress in static equilibrium.

While in a solid, stress is a function of strain, in a fluid stress is a function of rate of strain.

Objectives

1. comprehend the concepts necessary to analyse fluids in motion.
2. identify differences between steady/unsteady, uniform/non-uniform and compressible/incompressible flow.
3. construct streamlines and stream tubes.
4. appreciate the Continuity principle through Conservation of Mass and Control Volumes.
5. derive the Bernoulli (energy) equation.
6. familiarise with the momentum equation for a fluid flow.

Characteristics of Fluids

Force & Pressure

Pressure is a normal force exerted by a fluid per unit area

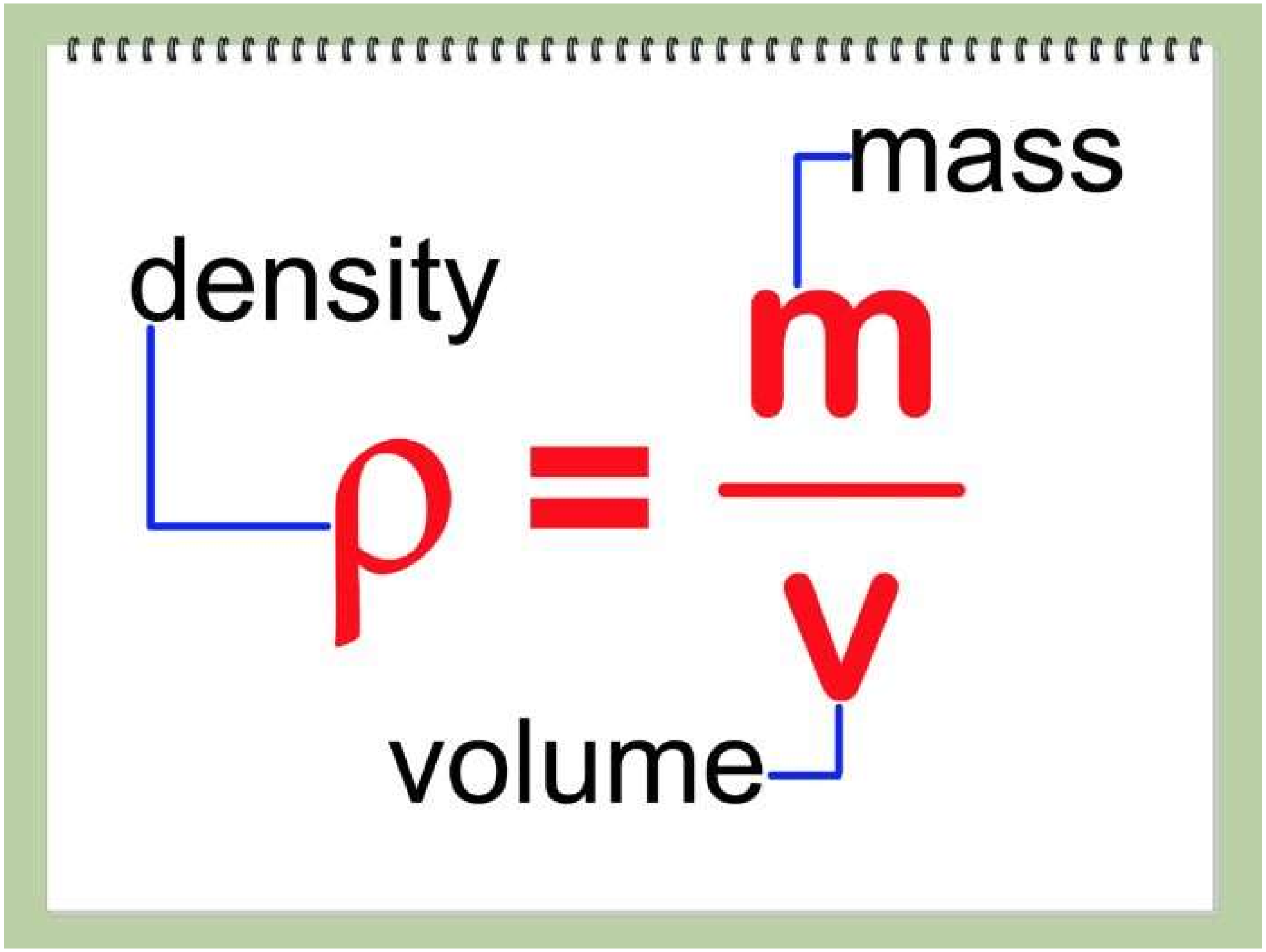
$$= (p_1 - p_2) A = \Delta p A \qquad (5.1)$$

The standard unit for pressure is the Pascal, which is a Newton per square meter (N/m^2). Pressure in a fluid can be seen to be a measure of energy per unit volume by means of the definition of work, that is, this energy is related to other forms of fluid energy by the Bernoulli equation.

$$p = \frac{Force}{Area} = \frac{F}{A} = \frac{F.\Delta x}{A.\Delta x} = \frac{W}{\Delta V} = \frac{Energy}{volume}$$

Where ΔV is the unit volume, which related to the unit mass Δm, of the fluid by the density ρ, as $\Delta m = \rho \Delta V$.

Mass and Density



Kinetic energy density: The kinetic energy of a moving fluid has a great importance when dealing with fluids at motion: more useful in applications like the Bernoulli equation when it is expressed as kinetic energy per unit volume. For a unit mass Δm of a fluid moving with average speed $\bar{v}$, its kinetic energy is given by

$$K.E = \frac{1}{2}\Delta m\overline{v^2} = \frac{1}{2}\rho\Delta V\overline{v^2}$$

So that the kinetic energy density is written as

$$K.E.density = \frac{kinetic\ energy}{volume} = \frac{\frac{1}{2}\rho\Delta V\overline{v^2}}{\Delta V} = \frac{1}{2}\rho\overline{v^2} \qquad (5.2)$$

Potential energy density: Similarly, like the kinetic energy density, it is the potential energy per unit volume, for a certain mass of fluid at distance h above the ground; then the potential energy density is

$$P.E.density = \frac{potential\ energy}{volume} = \frac{\Delta m\,g\,h}{\Delta V} = \frac{\rho\Delta V gh}{\Delta V} = \rho gh \quad (5.3)$$

Uniform Flow, Steady Flow

uniform flow: flow velocity is the same magnitude and direction at every point in the fluid.

non-uniform: If at a given instant, the velocity is not the same at every point the flow. (In practice, by this definition, every fluid that flows near a solid boundary will be non-uniform - as the fluid at the boundary must take the speed of the boundary, usually zero. However if the size and shape of the of the cross-section of the stream of fluid is constant the flow is considered *uniform*.)

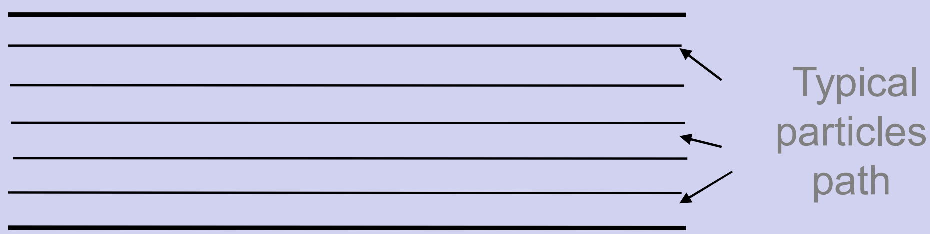
steady: A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but DO NOT change with time.

unsteady: If at any point in the fluid, the conditions change with time, the flow is described as *unsteady*. (In practice there is always slight variations in velocity and pressure, but if the average values are constant, the flow is considered *steady*.)

Laminar and Turbulent Flow

Laminar flow

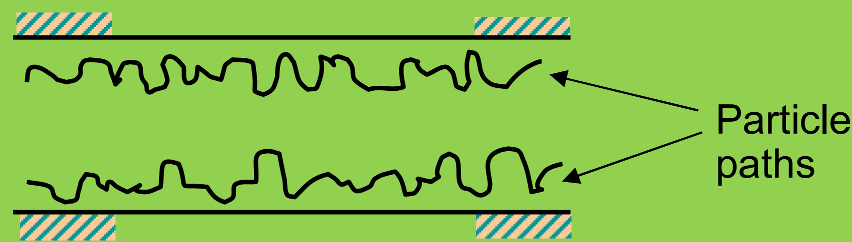
- all the particles proceed along smooth parallel paths (Stream lines) and all particles on any path will follow it without deviation.
- Hence all particles have a low velocity only in the direction of flow.



Laminar flow

Turbulent Flow

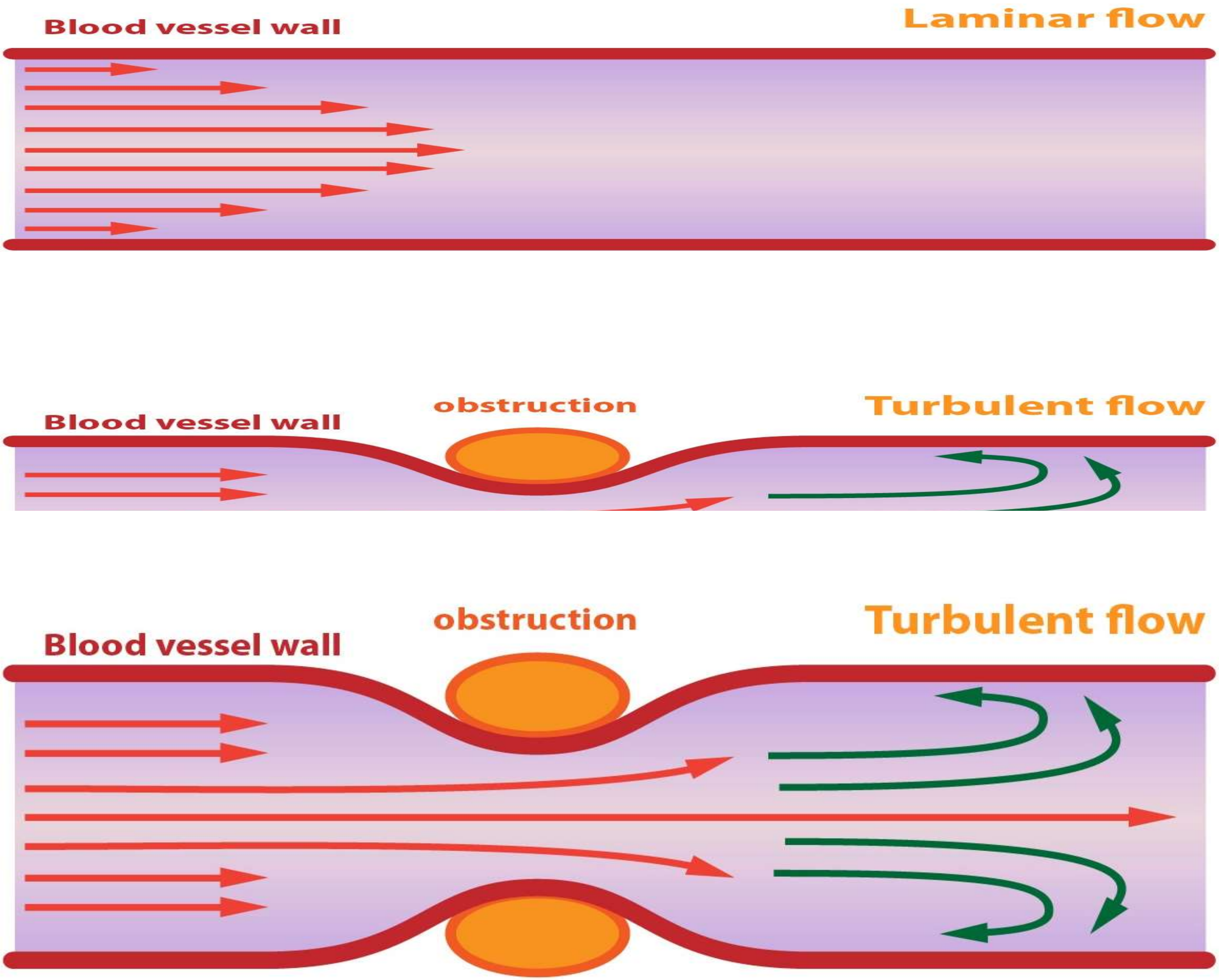
- the particles move very fast (high speed) in an irregular manner through the flow field. Intersection of stream lines
- Each particle has superimposed on its mean velocity fluctuating velocity components both transverse to and in the direction of the net flow.



Turbulent flow

Transition Flow

- exists between laminar and turbulent flow.
- In this region, the flow is very unpredictable and often changeable back and forth between laminar and turbulent states.
- Modern experimentation has demonstrated that this type of flow may comprise short ‘burst’ of turbulence embedded in a laminar flow.


Blood vessel wall
Laminar flow
obstruction
Blood vessel wall
Turbulent flow
obstruction
Blood vessel wall
Turbulent flow

Compressible or Incompressible

- All fluids are compressible - even water - their density will change as pressure changes.
- Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density.
- As you will appreciate, *liquids are quite difficult to compress* - so under most steady conditions they are treated as incompressible.

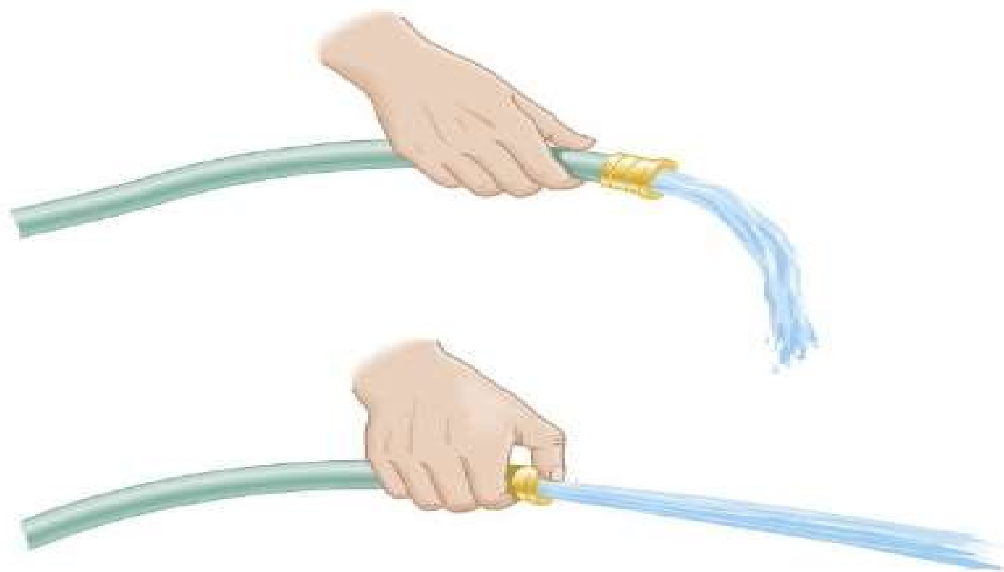
Viscous and Non viscous

Mass flow rate or Volume flow rate

$$mass\ flow\ rate = m = \frac{mass\ of\ fluid}{time\ taken\ to\ collect\ the\ fluid}$$

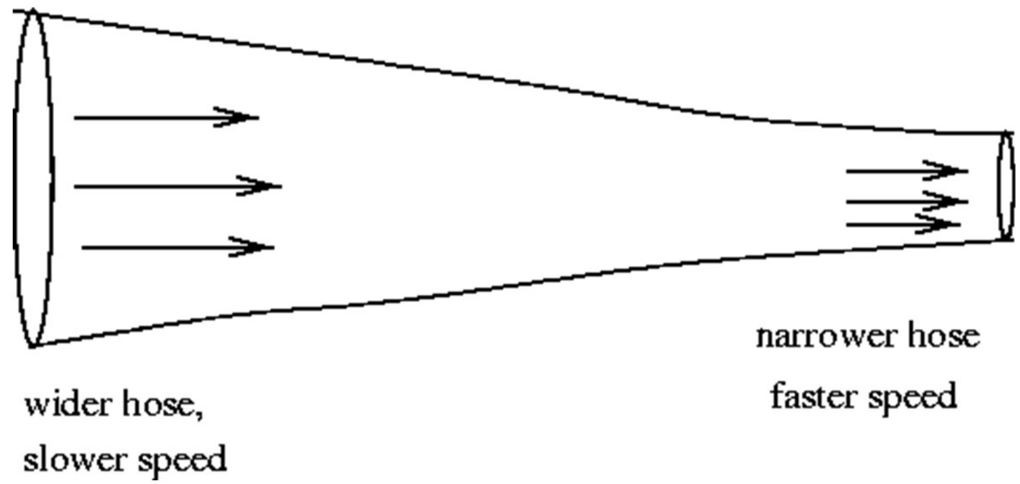
$$time = \frac{mass}{mass\ flow\ rate}$$

The mass of fluid per second that flows through a tube is called the ***mass flow rate***.



Volume flow rate - Discharge

Flow Rate (Discharge) = Q = $\dfrac{\text{volume of fluid} \quad \Delta V}{\text{Time} \quad \Delta t}$



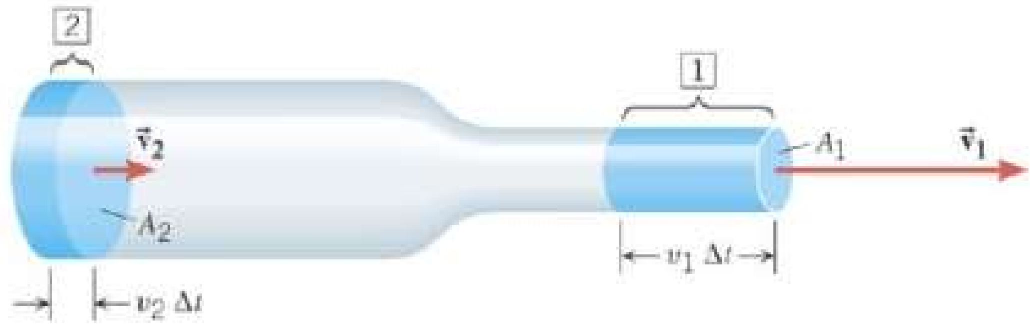
The flow rate is measured in the units of volume per unit time, m^3/s.

"The water all has to go somewhere"

The rate a fluid enters a pipe must equal the rate the fluid leaves the pipe.

$$Q_{in} = Q_{out} \qquad Q = \frac{\Delta V}{\Delta t} = constant$$

Contiuity Equation



$$Q = \frac{\Delta V}{\Delta t} = \frac{A\,\Delta x}{\Delta t} = A\bar{v} = constant \qquad (5.6)$$

Where $\bar{v} = \frac{\Delta x}{\Delta t}$, is the average speed of flow at any point. Take $A = \pi r^2$, so the continuity equation can be rewritten as

$$A_1\,\overline{v_1} = A_2\overline{v_2}$$

Or,

$$r_1^2\overline{v_1} = r_2^2\overline{v_2} \qquad (5.7)$$

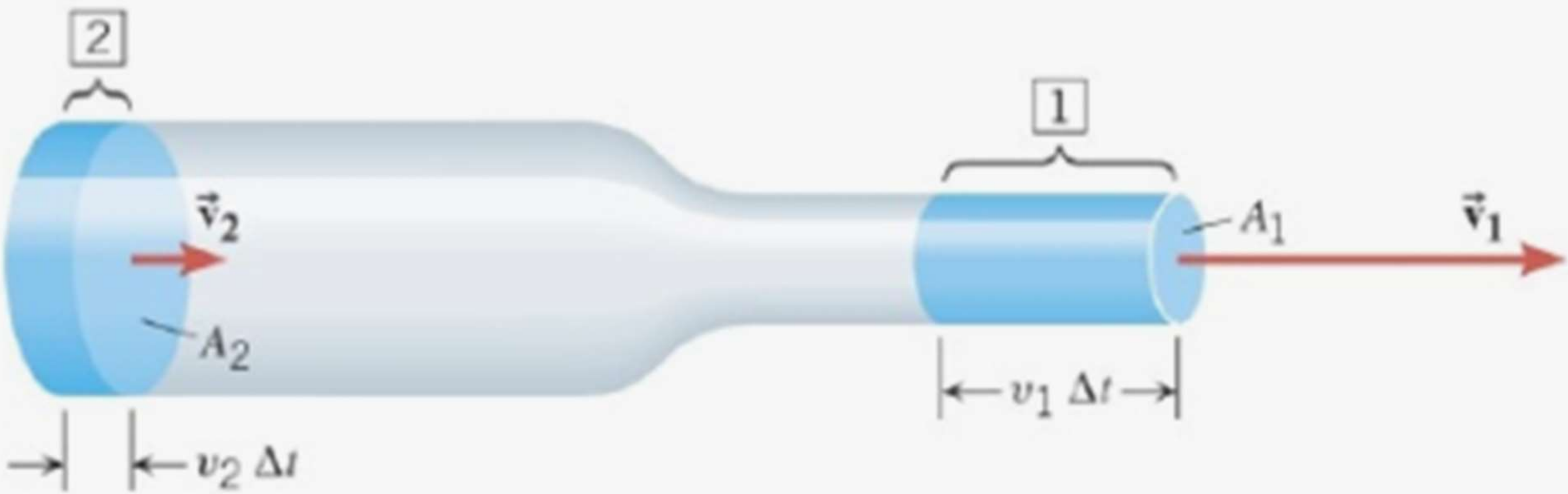
This means that the speed of flow increases obviously by a small decrease of the diameter of the tube.

EQUATION OF CONTINUITY

The mass flow rate has the same value at every position along a tube that has a single entry and a single exit for fluid flow.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Incompressible fluid: $A_1 v_1 = A_2 v_2$



Example 3.2

- If we know the mass flow is 1.7 kg/s, how long will it take to fill a container with 8 kg of fluid?

$$\textit{time} = \frac{\textit{mass}}{\textit{mass flow rate}}$$

$$= \frac{8}{1.7} = 4.7\text{s}$$

Example 3.3

- If the density of the fluid in the above example is 850 kg/m^3 what is the volume per unit time (the discharge)?

$$Q = \frac{\textit{mass fluid rate}}{\textit{density}} = \frac{m/\Delta t}{\rho}$$

$$= \frac{0.857}{850}$$

$$= 0.00108\ m^3/s\ (m^3s^{-1})$$

$$= 1.008 \times 10^{-3}\ m^3/s$$

but *1 litre* $= 1.0 \times 10^{-3} m^3$,

so $Q = 1.008\ l/s$

Example

A garden hose has an unobstructed opening with a cross sectional area of $2.85 \times 10^{-4} m^2$. It fills a bucket with a volume of $8.00 \times 10^{-3} m^3$ in 30 seconds.

Find the speed of the water that leaves the hose through (a) the unobstructed opening and (b) an obstructed opening with half as much area.

(a)

$$Q = Av$$

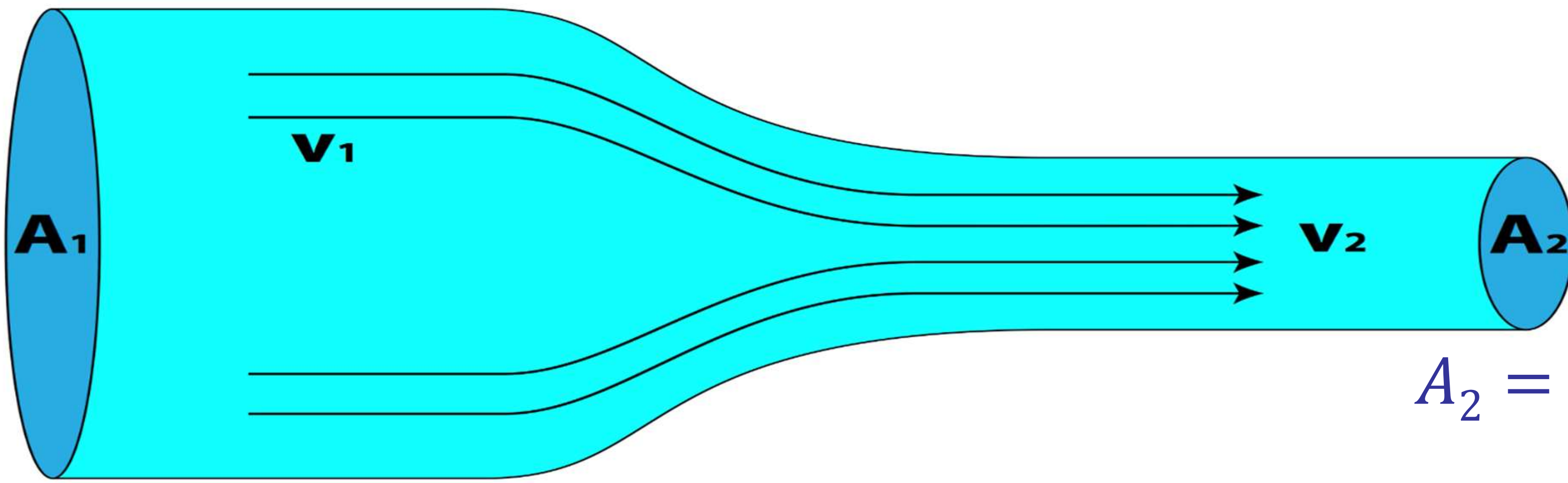
$$v = \frac{Q}{A} = \frac{(8.00 \times 10^{-3}\,\mathrm{m}^3)/(30.0\,\mathrm{s})}{2.85 \times 10^{-4}\,\mathrm{m}^2} = 0.936\,\mathrm{m/s}$$

(b)

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = (2)(0.936\,\mathrm{m/s}) = 1.87\,\mathrm{m/s}$$

A river is 40m wide, 2.2m deep and flows at 4.5 m/s. It passes through a 3.7-m wide gorge, where the flow speed increases to 6.0 m/s. How deep is the gorge?

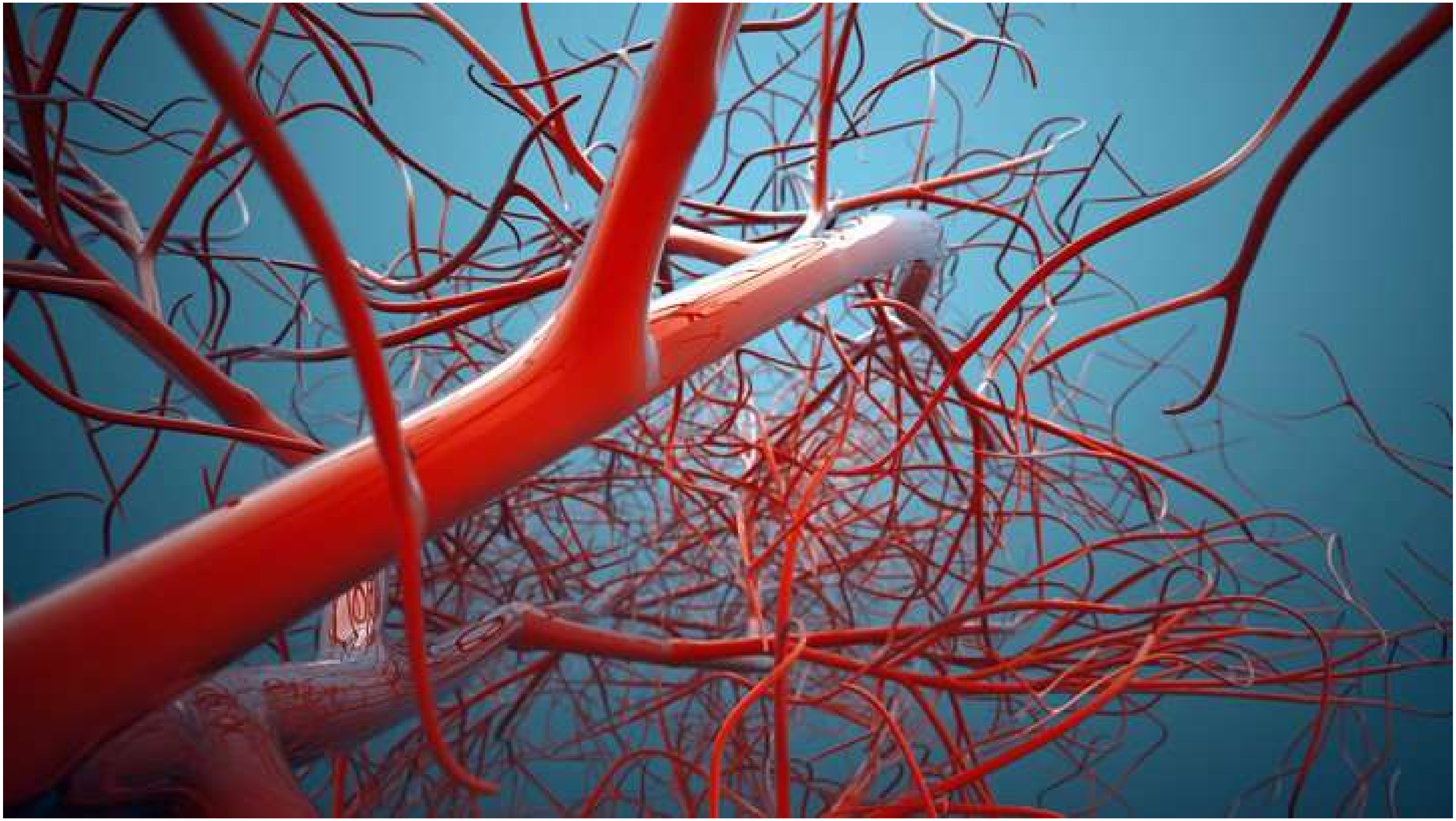


$A_2 = w_2 d_2$

$A_1 = w_1 d_1$

$$Continuity\ equation: A_1 v_1 = A_2 v_2 \rightarrow w_1 d_1 v_1 = w_2 d_2 v_2$$

$$d_2 = \frac{w_1 d_1 v_1}{w_2 v_2} = \frac{40 \times 2.2 \times 4.5}{3.7 \times 6.0} = 18\ m$$



The Blood flows in capillaries with lower speed than in the main blood vessel

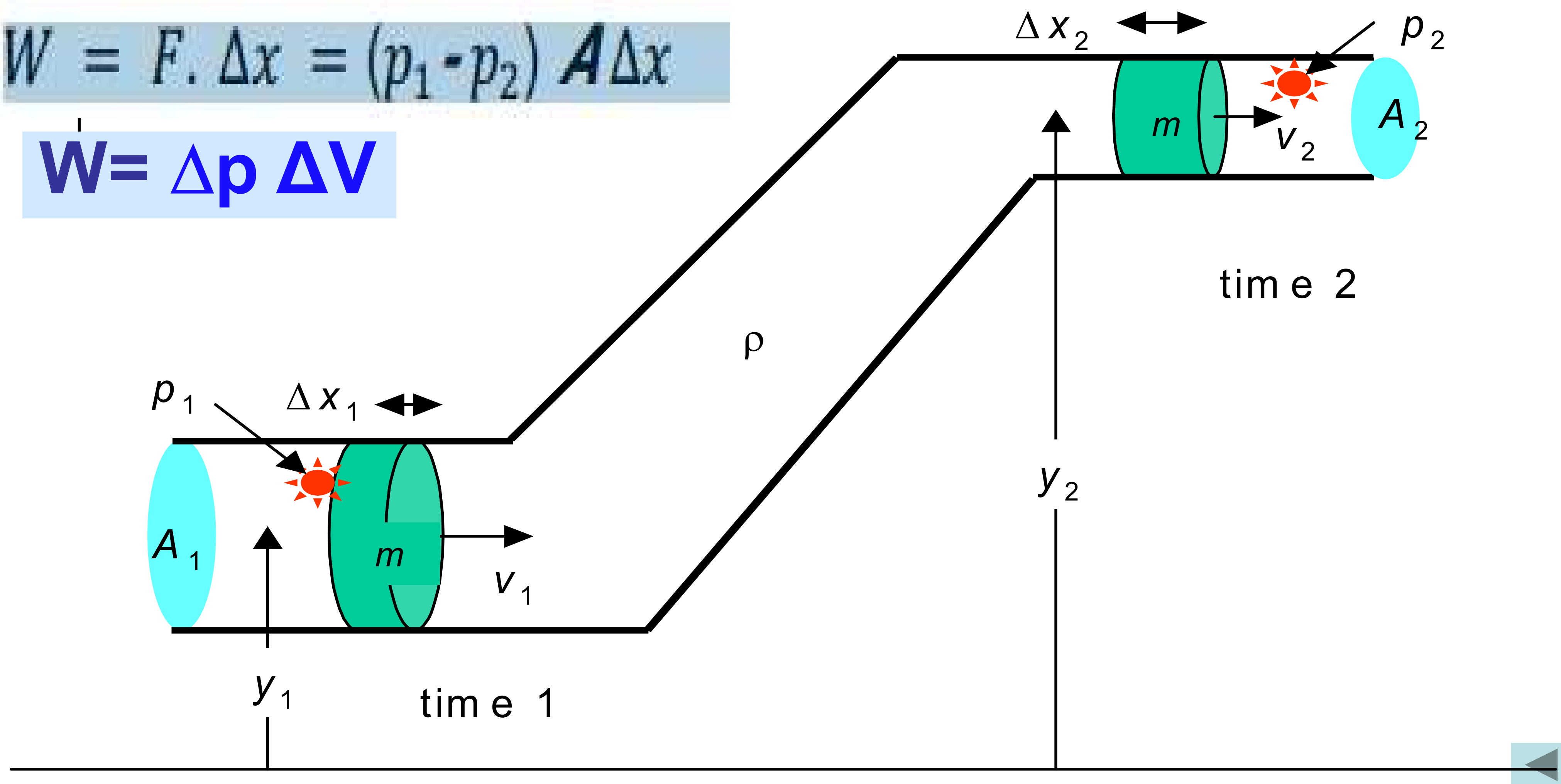
$$A_v < nA_c$$

Conservation of Energy: Bernoulli's Eqn.

The work done on a fluid when it flows from place to another one is equal to the change of its mechanichal energy

$$W = F.\Delta x = (p_1 - p_2)\, A \Delta x$$

$$W = \Delta p\, \Delta V$$



Change in Mechanical Energy

$$\Delta K = \tfrac{1}{2}\,\Delta m\, v_2^2 - \tfrac{1}{2}\,\Delta m\, v_1^2 = \tfrac{1}{2}\,\rho\,\Delta V\, v_2^2 - \tfrac{1}{2}\,\rho\,\Delta V\, v_1^2$$

$$\Delta U = \Delta m\, g\, y_2 - \Delta m\, g\, y_1 = \rho\,\Delta V\, g\, y_2 - \rho\,\Delta V\, g\, y_1$$

Net Work

$$p_1\,\Delta V - p_2\,\Delta V = \tfrac{1}{2}\,\rho\,\Delta V\, v_2^2 - \tfrac{1}{2}\,\rho\,\Delta V\, v_1^2 + \rho\,\Delta V\, g\, y_2 - \rho\,\Delta V\, g\, y_1$$

Rearranging

$$p_1 + \tfrac{1}{2}\,\rho\, v_1^2 + \rho\, g\, y_1 = p_2 + \tfrac{1}{2}\,\rho\, v_2^2 + \rho\, g\, y_2$$

Applies only to an ideal fluid (zero viscosity)

$$p + \tfrac{1}{2}\,\rho\, v^2 + \rho\, g\, y = \text{constant}$$

for any point along a flow tube or streamline

Bernoulli's Equation

for any point along a flow tube or streamline

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Dimensions

p [Pa] = [N.m^{-2}] = [N.m.m^{-3}] = [J.m^{-3}]

$\tfrac{1}{2}\rho v^2$ [kg.m^{-3}.m^{2}.s^{-2}] = [kg.m^{-1}.s^{-2}] = [N.m.m^{-3}] = [J.m^{-3}]

ρ g h [kg.m^{-3} m.s^{-2}. m] = [kg.m.s^{-2}.m.m^{-3}] = [N.m.m^{-3}] = [J.m^{-3}]

Each term has the dimensions of energy / volume or energy density.

$\tfrac{1}{2}\rho v$ 2 KE of bulk motion of fluid

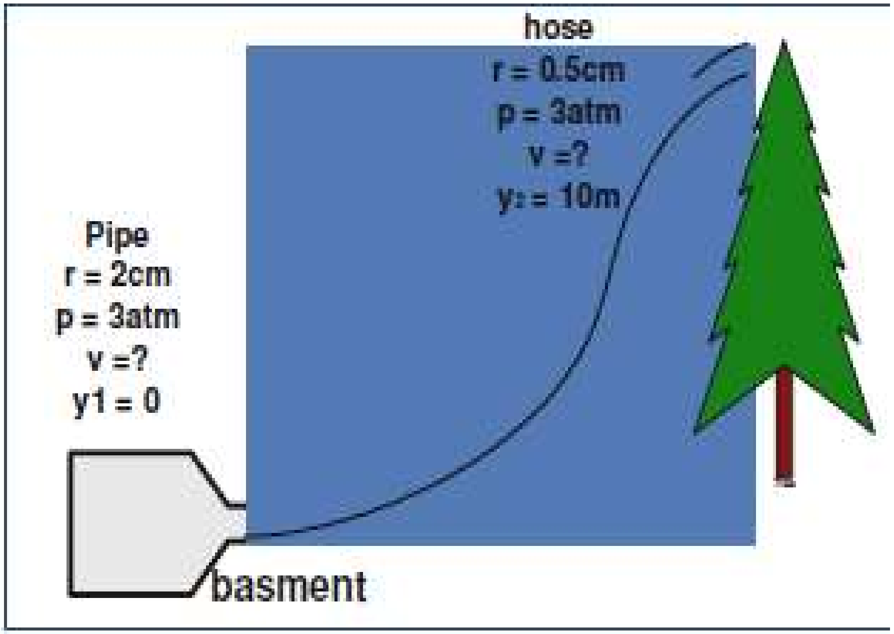
$\rho g h$ GPE for location of fluid

p pressure energy density arising from internal forces within moving fluid (similar to energy stored in a spring)

Example 5.2

Water enters the basement through a pipe 2 cm in radius at an absolute pressure of 3 atm. A hose with a 0.5 cm radius is used to water plants 10 m above the basement. Find the speed of water as it leaves the hose?

We have two points of interest. Point 1 in the pipe at the basement, where $p_1 = 3atm, y_1 = 0m, v_1 = ?$ Point 2 is in the hose just at the moment the water leaving the hose for planting the tree, where $p_2 = 1atm, y_2 = 10m, v_2 = ?$ By applying Bernoulli's equation, we have



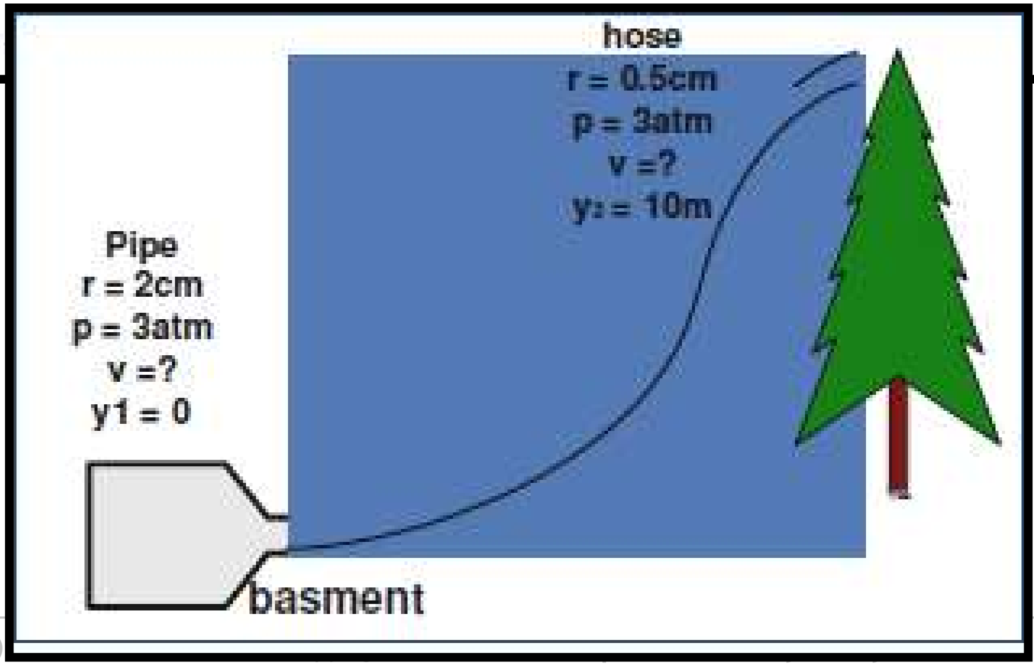
$$p_1 - p_2 = \frac{1}{2}\rho\left(\overline{v_2^2} - \overline{v_1^2}\right) + \rho g (y_2 - y_1)$$

We have two unknowns v_1 and v_2 , so we can reduce them to only one unknown by using the continuity equation, where $v_1 = \frac{r_2^2}{r_1^2} v_2$

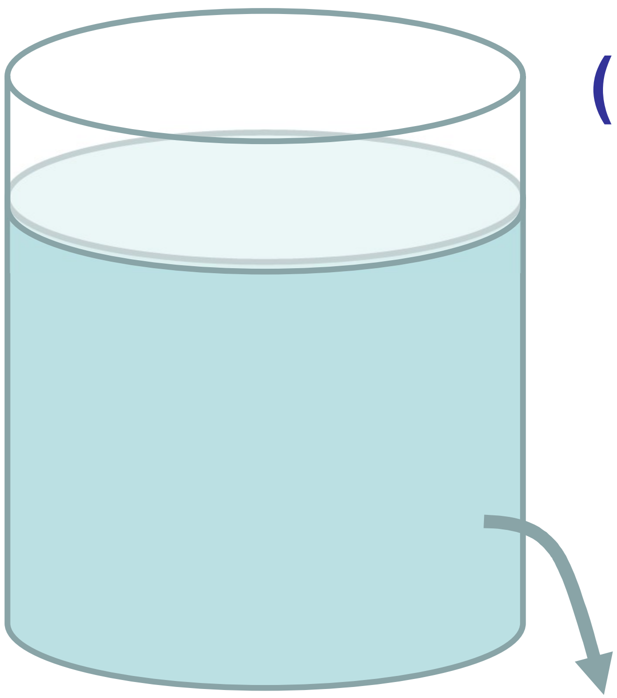
So $v_1 = \frac{1}{16} v_2$, and $\overline{v_2^2} - \overline{v_1^2} = \frac{255}{256} v_2^2$, then we substitute in the last equation to get:

$$(3-1) \times 1.013 \times 10^5 = \frac{1}{2}(1000) \frac{255}{256} v_2^2 + 1000(10)(10-0)$$

Solve for v_2 , we get $v_2 = 14.35\, ms^{-1}$.



Water is flowing from a hole of 1 cm radius at the bottom of a closed cylindrical container of 2m diameter. If the height of the water in the container is 2 m and the pressure over the surface of water is 3 atm., calculate how much time it took until the container became empty?



(1) Point on surface of liquid

(2) Point just outside hole

Applications of Bernoulli Equation

$$p_1 + ½ \rho v_1^2 + \rho g y_1 = p_2 + ½ \rho v_2^2 + \rho g y_2$$

❑ Static Consequences

$v = 0$

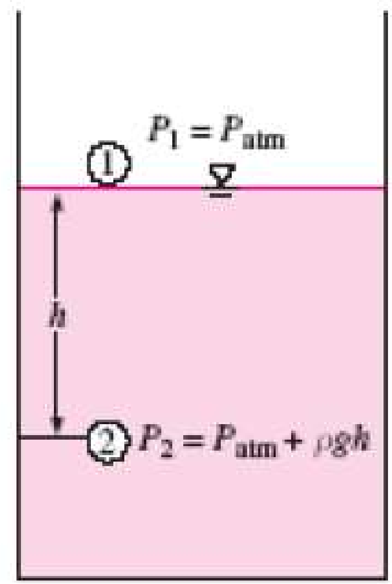
Static Consequence: In this consequence we have zero speed of flow, where the fluid is settle in its container. In this situation the kinetic energy density term leads to zero, and Bernoulli's equation becomes

$$p + \rho g y = constant$$

$$p_1 + \rho g y_1 = p_2 + \rho g y_2$$

$$p_2 = p_a + \rho g(y_1 - y_2) = p_a + \rho g h \qquad (5.13)$$

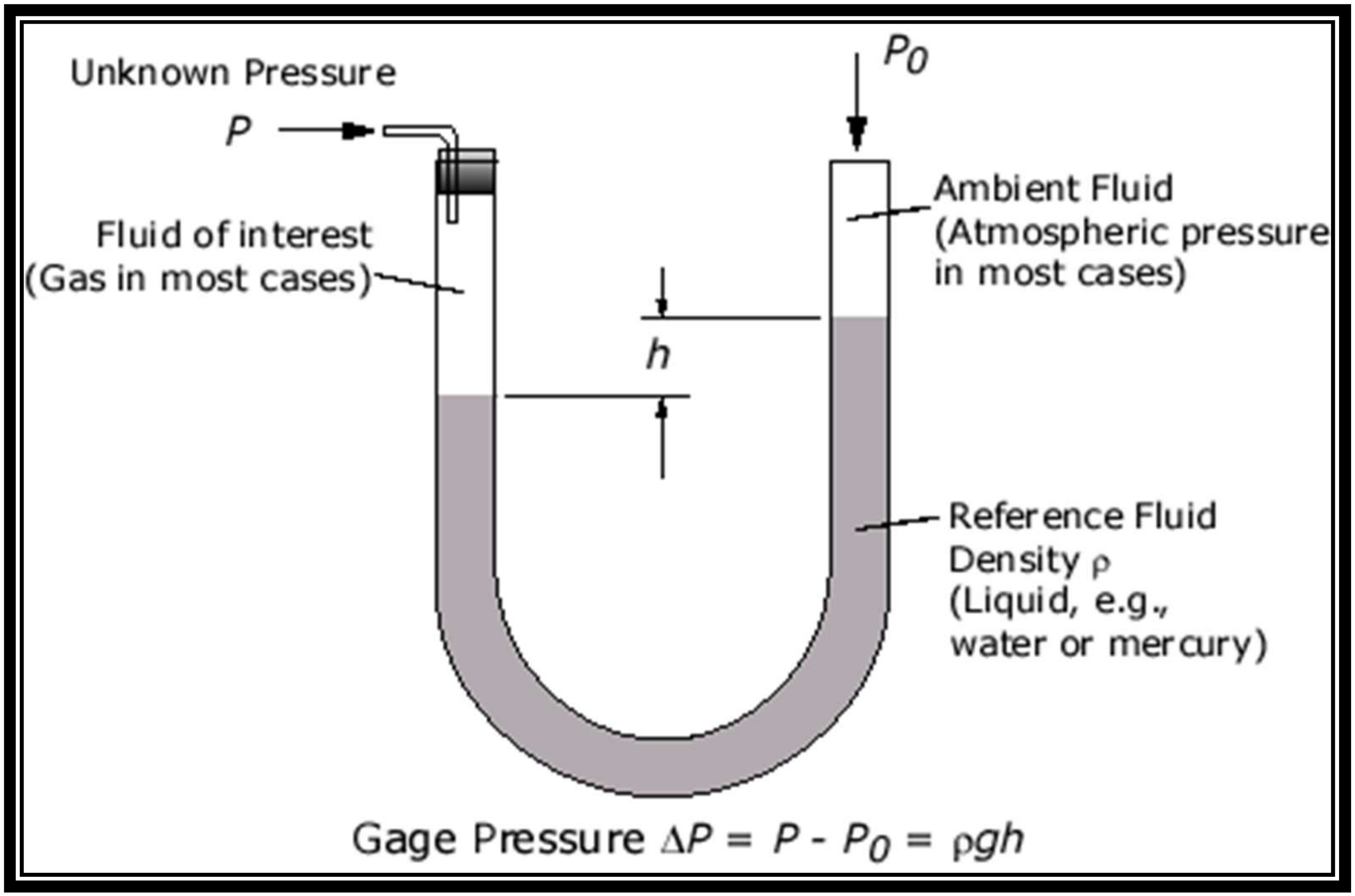
So the pressure inside any container is equal the pressure at the surface plus the potential energy density at that point.



Note: The difference between the absolute pressure at any point and the atmospheric pressure $(p - p_a)$ is called the gauge pressure.

Manometer

A manometer is a device for measuring fluid pressure consisting of a U tube containing one or more liquids of different densities. A known pressure (which may be atmospheric) is applied to one end of the manometer tube and the unknown pressure (to be determined) is applied to the other end. Differential pressure manometer measures only the difference between the two pressures.



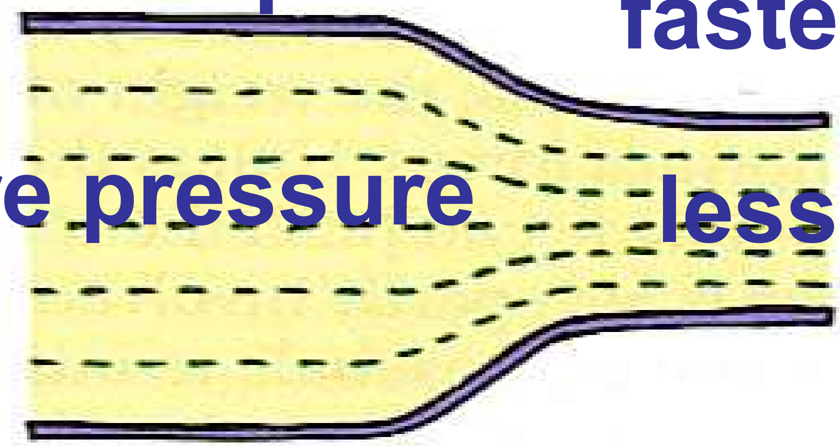
Horizontal flow of fluids

$P + \rho g y + \frac{1}{2} \rho v^2 = \text{constant}$

If no change in height: $P + \frac{1}{2}\rho v^2 = \text{constant}$

slower speed

faster speed



more pressure

less pressure

$$p_1 + \frac{1}{2} \rho \overline{v_1^2} = p_2 + \frac{1}{2} \rho \overline{v_2^2}$$

As the <u>speed</u> of a fluid <u>increases</u>, the <u>pressure</u> in the fluid <u>decreases</u>.

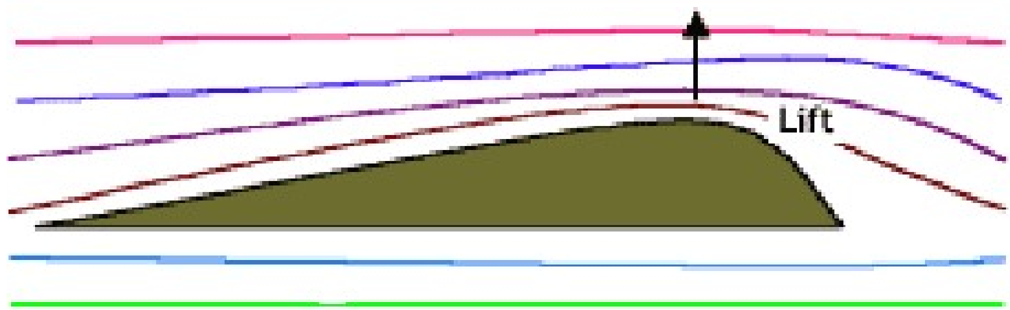
Bernoulli & Flight

Bernoulli's Principle is what allows birds and planes to fly.

The secret behind flight is 'under the wings

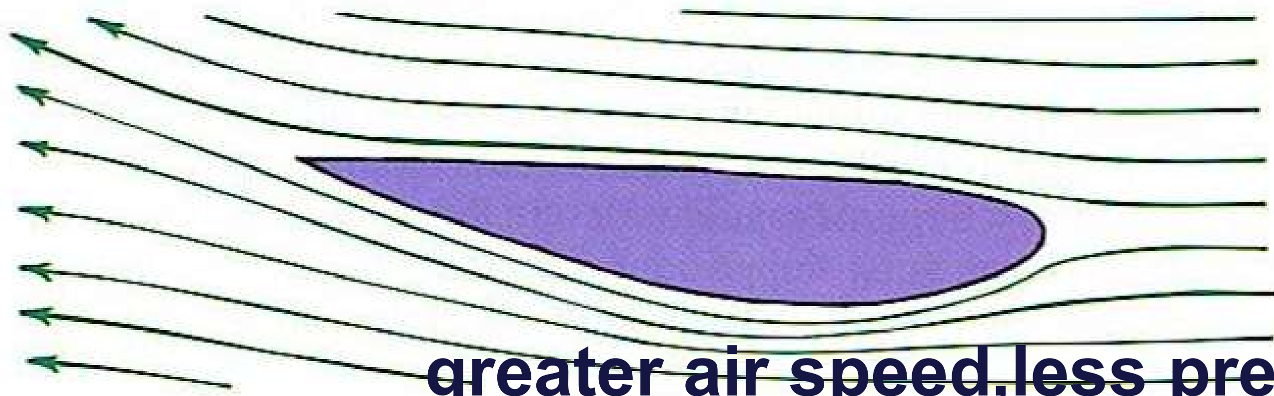
AIRFOIL

On top: greater air speed and less air pressure



On bottom: less air speed and more air pressure

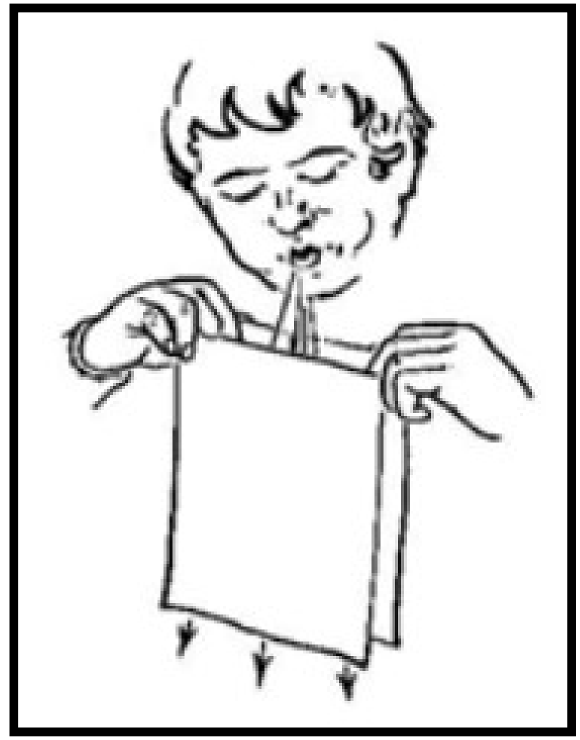
less air speed, more pressure



greater air speed,less pressure

net force: downward

Blowing air between two half sheets of paper



$$p_{out} + \frac{1}{2}\rho\,\overline{v_{out}^2} = p_{in} + \frac{1}{2}\rho\,\overline{v_{in}^2}$$

Rearrange the equation leads to the following

$$p_{out} - p_{in} = \frac{1}{2}\rho\left(\overline{v_{in}^2} - \overline{v_{out}^2}\right)$$

When a person blows between the two sheets, so that the speed of air flowing inside will be larger than that outside ($v_{in} > v_{out}$): this means that the left hand side is positive and therefore $p_{out} > p_{in}$. This pressure difference results in the sheets moving closer toward one another. Thus, the pressure drops when the velocity of the flow increases for a fluid moving at a constant height.

Example 5.5

The diameter of a horizontal blood vessel is reduced from 12mm to 4 mm. What is the flow rate of blood in the vessel, if the pressure at the wide part is 8 kPa and 4 kPa at the narrow one. (Take the density of blood to be 1060 kgm^{-3}.)

$the\ radius\ of\ the\ wider\ part\ r_w = 6mm,$

$the\ radius\ of\ the\ narrow\ part\ r_n = 2mm,$

$the\ pressure\ at\ the\ widew\ part\ P_w = 8\ kPa,$

$the\ pressure\ at\ the\ narrow\ part\ P_n = 4\ kPa$

From the continuity equation, $\frac{v_w}{v_n} = \frac{r_n^2}{r_w^2} = \frac{1}{9},\quad \ggg v_n = 9v_w$

Apply BE

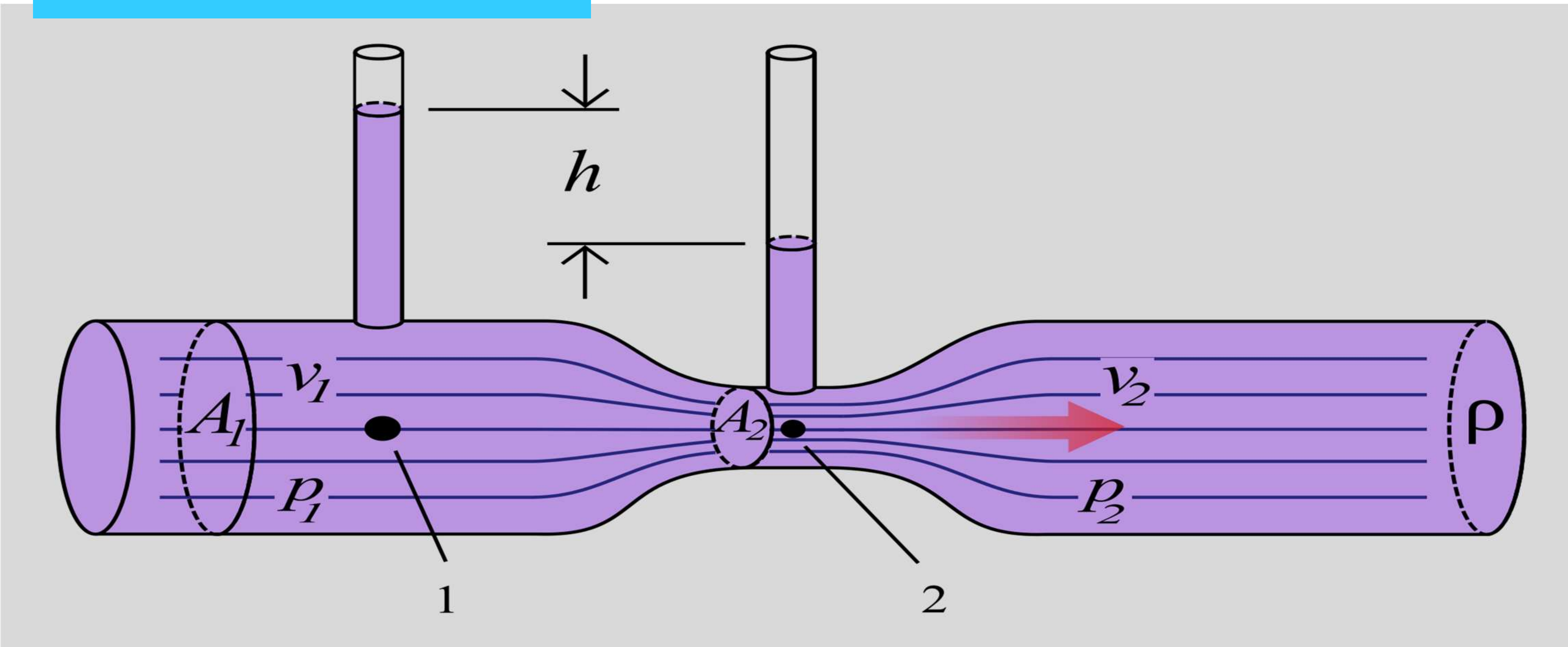
$$P_w - p_n = \frac{1}{2}\rho\left(v_n^2 - v_w^2\right)$$

$$4\times 10^3 = \frac{1}{2}\times 1060 \times \left(80v_w^2\right)$$

$$\rightarrow \quad v_w = 0.307\ m/s$$

Flow rate $Q = A_w v_w = \pi r_w^2 v_w = 3.14 \times 36 \times 10^{-6} \times 0.307$
$= 3.5 \times 10^{-5} m^3/s$

Venturi Tube



$$p_1 - p_2 = \frac{1}{2}\rho\left(\overline{v_2^2} - \overline{v_1^2}\right)$$

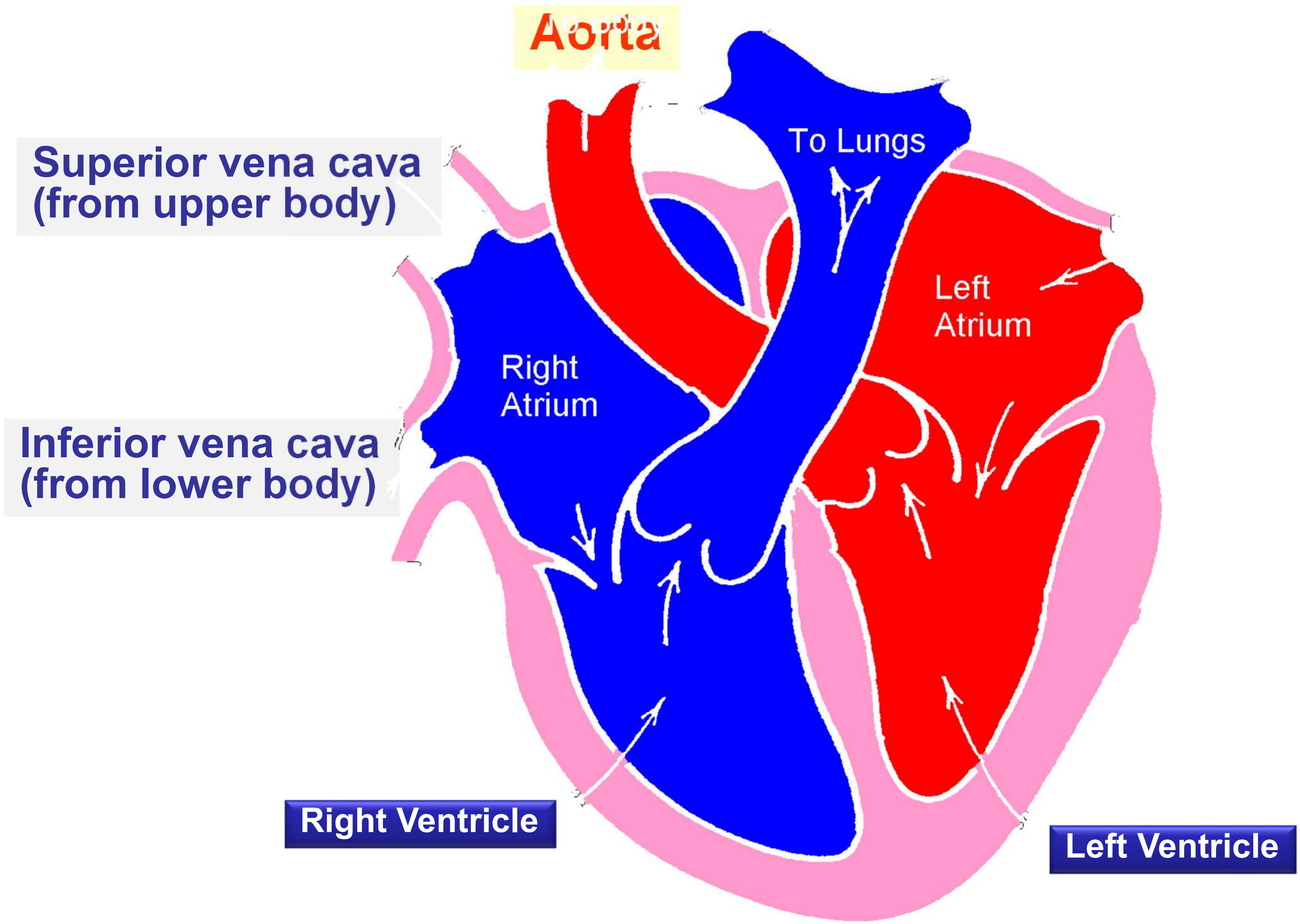
And from the continuity equation we have $v_2 = \frac{A_1}{A_2} v_1$, which can be substituted to get

$$v_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[\left({}^{A_1}/_{A_2}\right)^2 - 1\right]}}$$

Example 5.6

The flow of blood through a large artery in a dog is diverted through a Venturi flow meter. The wider part of the flow meter has an area of $0.08\ cm^2$, which equals the cross sectional area of the artery. The narrower part of the flow meter has an area of $0.04\ cm^2$. If the speed of the blood in the artery was $12.5\ \mathrm{cm\ s^{-1}}$, find the drop in pressure through the flow meter?

Blood Transport System



The role of gravity on blood circulation

We learned from Bernoulli's principle that the pressure of the fluid change according to its kinetic energy density and as well as it potential energy density. Because of that, the blood pressure in human organs is affected by its location from earth.

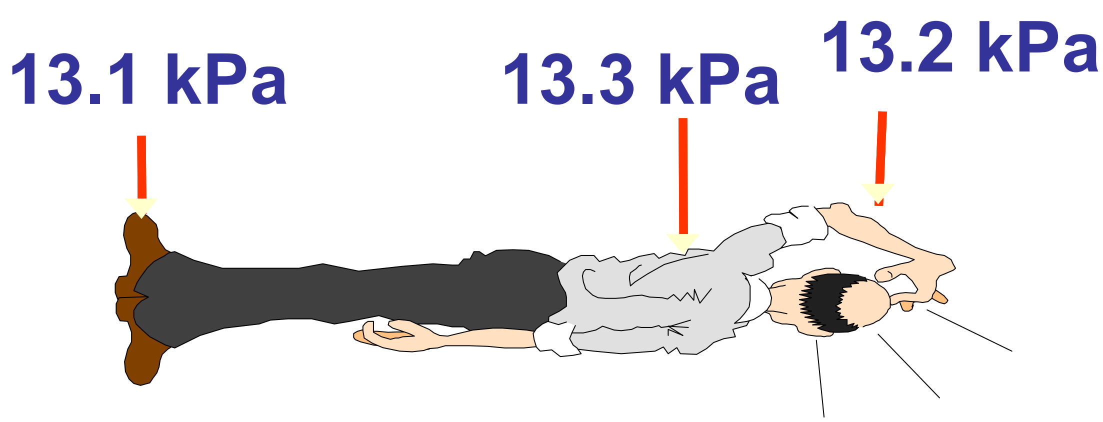
$$p_B + \tfrac{1}{2}\rho v_B^2 + \rho g y_B = p_H + \tfrac{1}{2}\rho v_H^2 + \rho g y_H = p_F + \tfrac{1}{2}\rho v_F^2 + \rho g y_F$$

Note that B, H, and F refer to the brain, heart and feet, respectively

Reclining State

The velocities in the three main arteries (Brain, heart, and feet) are small and roughly equal, so that the kinetic energy density term can be ignored

$$p_B \sim p_H \sim p_F$$



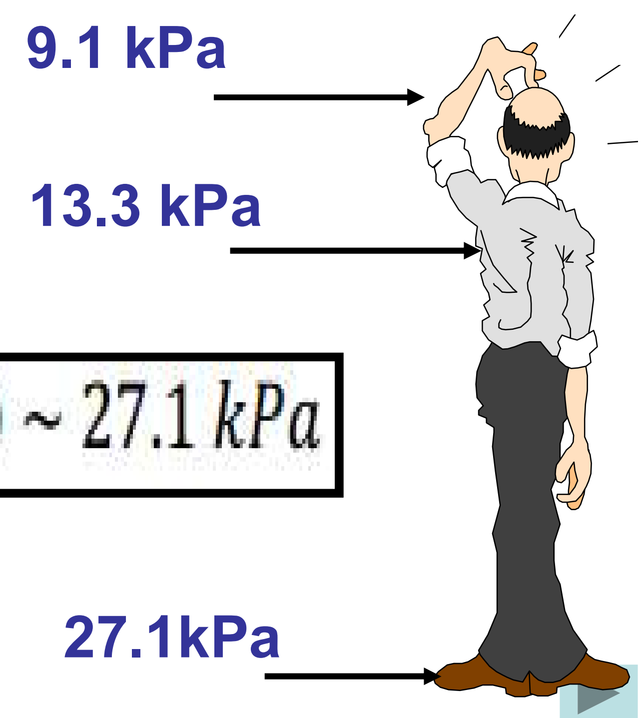
During the blood circulation, the venous system is used to return the blood from the lower extremities to the heart. It is expected to have a problem of lifting blood long distances to the heart against the force of gravity.

Standing Position

The kinetic energy density can be only ignored

$$p_B + \rho g y_B = p_H + \rho g y_H = p_F$$

Typical values for adults standing upward h_H = 1.3 m, h_B = 1.7 m. Typical value of the blood pressure at the heart is gä = 13.3 KPa , and take the blood density to be 1060 kg/ m^3, we find:



$$p_F = p_H + \rho g h_H = 13.3\times 10^3 + (1060)(10)(1.3) \sim 27.1\, kPa$$

The blood pressure at brain

In a similar way, we find that:

$$p_B = p_H + \rho g(h_H - h_B) = 13.3 \times 10^3 + (1060)(10)(-0.4) = 9.06\,kPa$$

- This explains why the pressures in the lower and upper parts of the body are very different when the person is standing, although they are about equal in the reclining.
- The high blood pressure at the foot explain the possibility of lifting blood uphill to the heart, and in addition the muscles surrounding the veins contract and cause constriction.

Example 5.7

When a $1.7\,m$ tall man stands, his brain is $0.5\,m$ above his heart. If he bends so that his brain is 0.4 m below his heart, by how much does the blood pressure in his brain changes?

from B.E.

$$P_B + \rho g h_B = P_H + \rho g h_H$$

$$P_B = P_H + \rho g (h_H - h_B)$$

by standing $\quad h_H - h_B = -0.5\ m$

by Bending $\quad h_H - h_B = 0.4\ m$

from B.E.

$$P_B + \rho g h_B = P_H + \rho g h_H$$

$$P_B = P_H + \rho g (h_H - h_B)$$

by standing $h_H - h_B = -0.5$ m

by Bending $h_H - h_B = 0.4$ m

$$(P_B)_{bending} - (P_B)_{standing} = \rho g (h_H - h_B)_{bending} - \rho g (h_H - h_B)_{stand}$$

$$\Delta P_B = \rho g \left[(h_H - h_B)_{bend.} - (h_H - h_B)_{standing} \right]$$

$$= 1060 (10) [0.4 - (-0.5)]$$

$$= 10600 \times 0.9 = 9540 \text{ Pa}$$

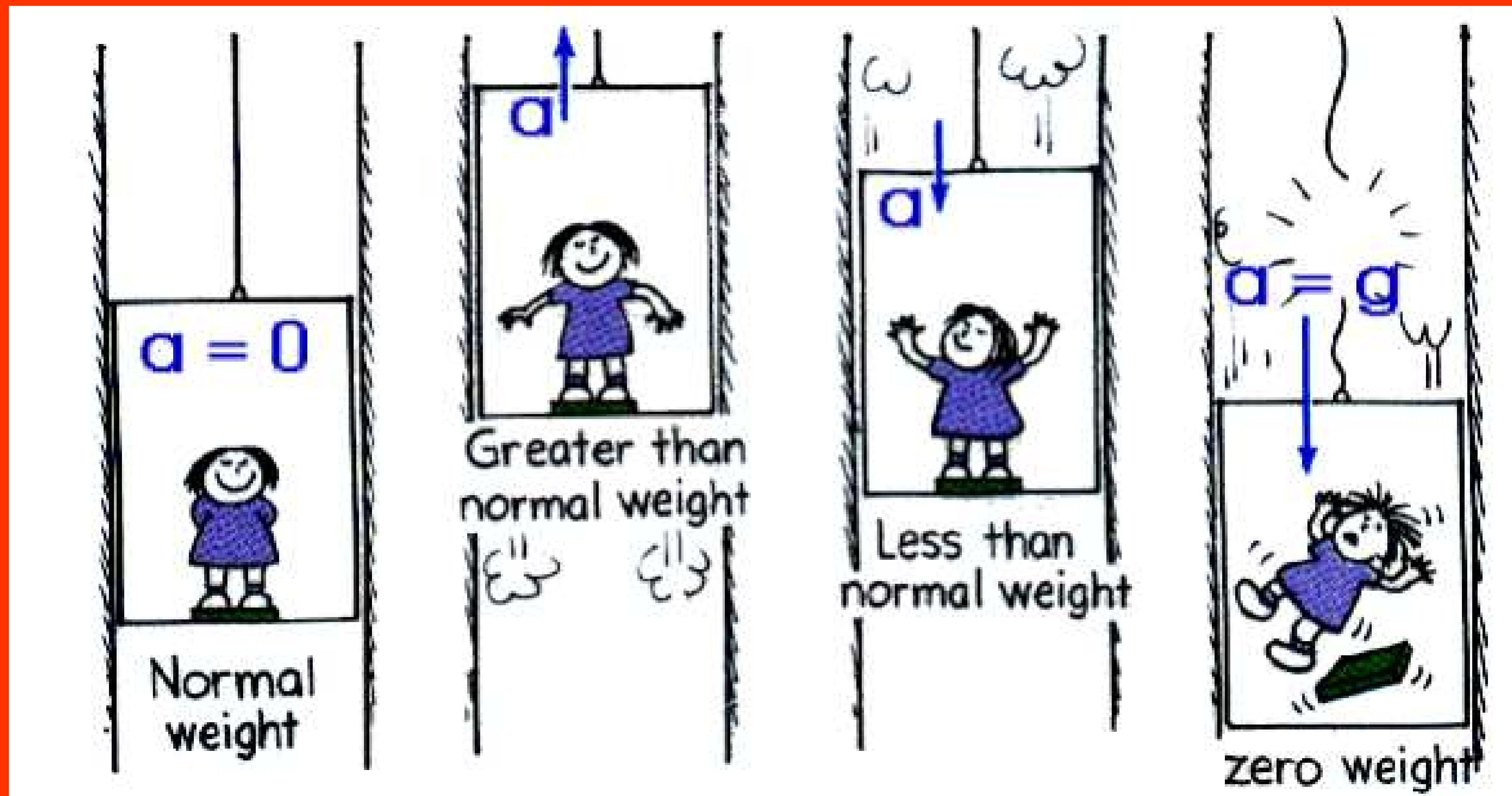
$$\Delta P_B = 9.54 \text{ KPa}.$$

Effect of acceleration on Blood pressure

The question arise:

Is the blood pressure at the organs affected when man under upward or downward acceleration.

When a person in an erect position experiences an upward or downward acceleration, his weight will be different.



Upward acceleration

If a man in an erect position experience upward acceleration a, then his effective weight becomes m(g+a).

Applying Bernoulli's equation to the foot, brain and heart with g replaced by g+a, so we have:

$$p_B = p_H + \rho\,(g+a)(h_H - h_B)$$
$$= p_H - \rho\,(g+a)(h_B - h_H)$$

Fatal upward acceleration when $P_B = 0$

$$0 = p_H - \rho\,(g+a)(h_B - h_H)$$

$$(g+a) = p_H / \rho\,(h_B - h_H)$$

$$(g+a) = \frac{13.3 \times 10^3}{1060\,(0.4)} = 31.4\,ms^{-2} = 3.2\,g$$

So the value of the upward acceleration causing consciousness *is 3.2*
This factor should limit the speed with which a pilot can pull out of dive. A related experience is the feeling of light headache that sometimes occurs when one suddenly stands up

The blood pressure at the foot by the upward acceleration situation

$$p_F = p_H + \rho(g+a)h_H$$

So the blood pressure at the foot increases by increasing the upward acceleration
The blood pressure at the brain decrease by increasing the upward acceleration

Downword acceleration

In an erect position experience downward acceleration leads to change of his effective weight.
Applying Bernoulli's equation to the foot, brain and heart with *g* replaced by g – a, so we have

$$p_B = p_H + \rho\,(g - a)(h_H - h_B)$$

Or

$$p_B = p_H - \rho\,(g - a)(h_B - h_H)$$

Thus the blood pressure at the brain will increase even farther by increasing the downward acceleration. This increase should be controlled and observed, where at certain value of a the blood pressure at the brain may cause an explosion of the arteries in the brain, which is so dangerous. The same calculation for the blood pressure at the foot

$$p_F = p_H + \rho\,(g - a)h_H$$

Which decreases by increasing the downword acceleration

Example

If an elevator accelerates upward at 12 ms^{-2}, what is the average blood pressure in the brain? What is the average blood pressure in the feet? If the elevator accelerates downward with the same acceleration, what is the average blood pressure in the brain and feet? take g =10 ms^{-2}.

① accelerating upward

$$P_B = P_H + \rho(g+a)(h_H - h_B)$$

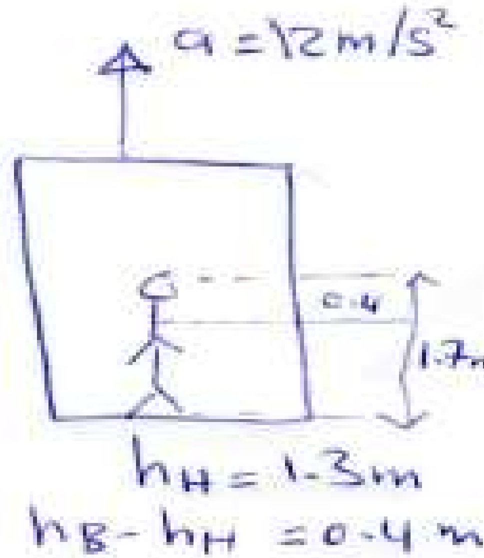
$$P_B = 13.6\times10^3 + 1060(10+12)(-0.4)$$

$$= 13.6\times10^3 + (-9328)$$

$$P_B\uparrow = \boxed{4272\ Pa}$$

نقصان

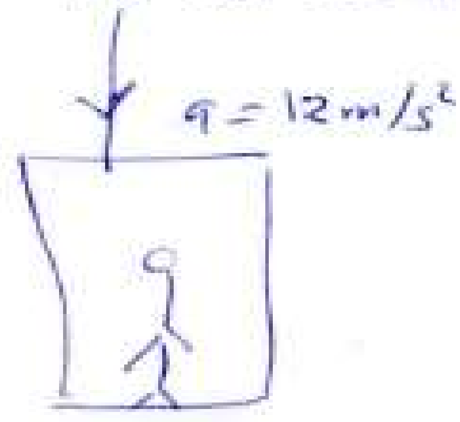
$$P_F\uparrow = P_H + \rho(g+a)h_H$$



② accelerating downward

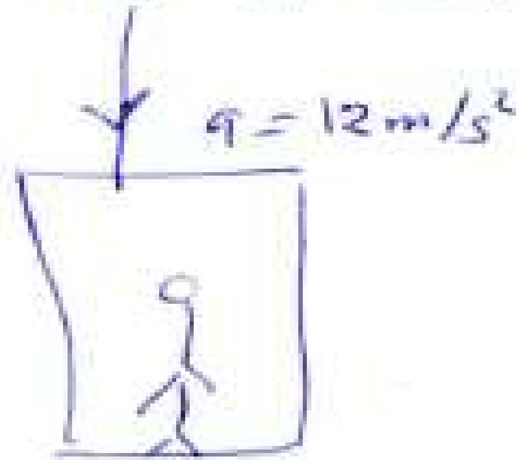
$$\downarrow P_F = P_H + \rho(g-a)h_H$$

$$= 13.6\times10^3 + 1060(10-12)(1.3)$$



$h_H = 1.3\,m$

$h_B - h_H = 0.4\,m$



$$P_B \uparrow = \boxed{4272\ Pa}$$ نقصان

$$P_F \uparrow = P_H + \rho(g+a)h_H$$

② accelerating downward

$$\downarrow P_F = P_H + \rho(g-a)\,h_H$$

$$= 13.6\times10^3 + 1060(10-12)(1.3)$$

$$= 13.6\times10^3 + (-2756)$$

$$\downarrow P_F = 10844\ Pa\ \boxed{= 10.84\ kPa}$$ نقصان

$$\downarrow P_B = P_H + \rho(g-a)(h_H - h_B)$$

$$= 13.6\times10^3 + 1060(10-12)(-0.4)$$

$$= 13600 + 848$$

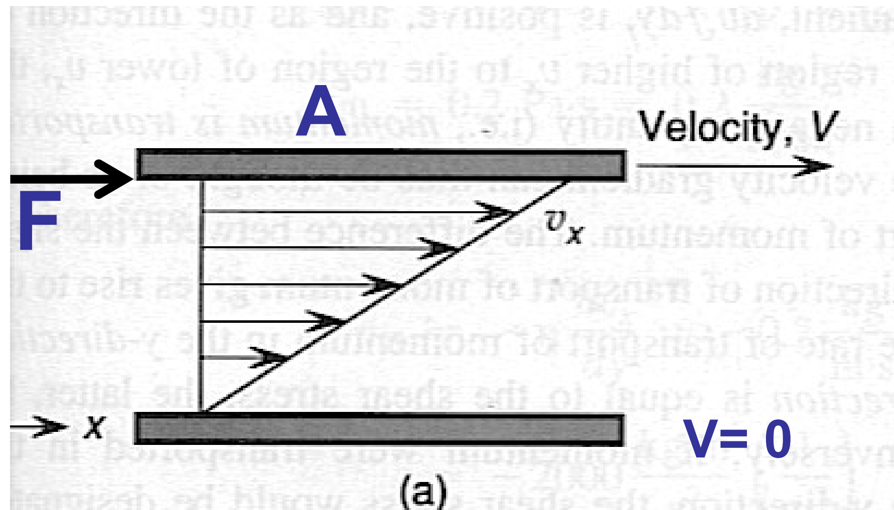
$$\boxed{\downarrow P_B = 14.45\ kPa}$$ زيادة

Viscous Fluid Flow

Resistivity of flow
Frictional forces among the fluid layers and their containers

Viscosity in Gases are different that in Liquids

Viscosity can be defined using two parallel plates separated by a distance Δy and a fluid fills the space between the two plates, one stationary and the other moving at velocity v



It is found that the force required to move the upper plate at constant average speed is proportional directly with the speed gradient and the surface area of the plate and inversely proportional with the separation distance between the plates, which means that:

$$F \propto \Delta v \,,\; F \propto A \text{ and } F \propto \frac{1}{\Delta y} \,, \text{so } F \propto \frac{A\,\Delta v}{\Delta y}$$

$$F = \eta \frac{A\,\Delta v}{\Delta y}$$

The constant of proportion is called the viscosity coefficient, which is represented by the Greek symbol η,

$$\eta = \frac{F/A}{\Delta v/\Delta y}$$

Coefficient of Viscosity

$$\eta = \frac{F/A}{\Delta v/\Delta y}$$

The viscosity can be defines as the ratio between the shearing stress (F / A) to the rate of shearing strain or the gradient of velocity. The dimension of the viscosity coefficient can be deduced as:

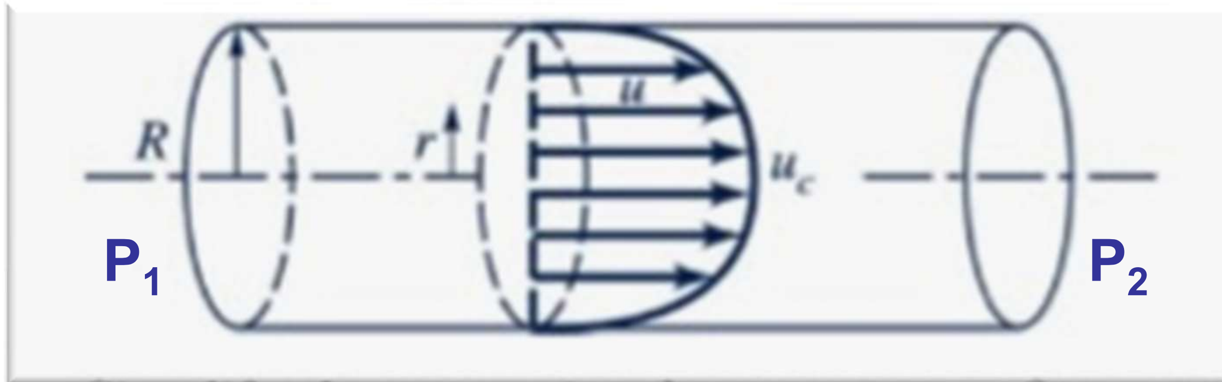
$$[\eta] = \left[\frac{F/A}{\Delta v/\Delta y}\right] = \left[\frac{MLT^{-2}/L^{2}}{LT^{-1}/L}\right] = ML^{-1}T^{-1}$$

The S.I. unit of the viscosity coefficient is Pascal second, where kg.m^{-1}s^{-1} = 1 Pas.s. The Pascal second is rarely used in technical and scientific publications. The common used unit is called poise, where 1 poise= 10^{-1} Pa.s, . l, so the abbreviation centipoises is equal 10^{-3} Pa.s

Temperature C	Castor Oil	Water	Air	Normal blood	Blood Plasma
0	5.3	1.792x10^{-3}	1.71x10^{-5}		
20	0.986	1.005x10^{-3}	1.81x10^{-5}	3.015x10^{-3}	1.81x10^{-3}
37	-	0.695x10^{-3}	1.87x10^{-5}	2.08x10^{-3}	1.257x10^{-3}
40	0.231	0.656x10^{-3}	1.9x10^{-5}		
60	0.08	0.469x10^{-3}	2.00x10^{-5}		
80	0.03	0.357x10^{-3}	2.09x10^{-5}		
100	0.017	0.284x10^{-3}	2.18x10^{-5}		

Laminar Flow in a Tube

Take a cylinder of radius R and length l



$F = \Delta P\, A = (P_1 - P_2)\, A$

Take half of the cylinder
Where the uper surface pass with the center, and the speed of the upper Layer is V_{max} , so that the cross sectional area of the half cylinder is
$A = \pi(R/2)^2$ and its surface area $A_{surf} = 2\pi(R/2)l$

$F = \Delta P\, A = \eta\, A_{surf}\, v_{max} / R$

$\Delta P\, \pi(R/2)^2 = \eta\, 2\pi(R/2)l\, v_{max} / R$

So that

$$V_{max} = \frac{\Delta P\, R^2}{4\eta l}$$

The average speed is half of The maximum speed. So that

$$\bar{V} = \frac{\Delta P\, R^2}{8\eta l}$$

The flow rate is given by

$$Q = A\,\bar{V} = \pi R^2 \frac{\Delta P\, R^2}{8\eta l} = \frac{\pi \Delta P\, R^4}{8\eta l}$$

• Poiseulle's Equation

Volume flow rate, *Q*, related to pressure difference *ΔP*, length *L* and radius R by:

$$Q = \frac{\pi R^4}{8\eta L}\Delta p$$

❑The formula for Q is called ***Poiseuille's law*** **after the physician, Jean Luis Poiseuille.**

❑ High viscosity leads to low flow rate and speed of flow.

❑ The flow rate is proportional to the 4th power of R, which is extremely dependent.

❑This indicates that for blood vessel, any small change in the radius of the vessel results in a considerable change of the flow rate. For example, if the radius of an artery is halved, so the flow rate will be reduced to 1/16

Example 5.9

What is the pressure drop in the blood as it passes through a capillary 5 mm long and $3\mu m$ in radius if the speed of the blood at the center of the capillary is 0.6 ms^{-1}. Take the viscosity of blood to be 2.08 cp. If the radius of the capillary is reduced by 20 %, find the change in the flow rate?

The capillary radius R = $3 \times 10^{-6}\ m$, *and its length* $l = 5 \times 10^{-3}\ m$,

and the ***maximum speed of the blood inside it,*** $v_{max} = 0.6\,\frac{m}{s} = \frac{\Delta P R^2}{4\eta l}$

$$\textit{so the pressure drop } \Delta p = \frac{4\eta l v_{max}}{R^2} = \frac{4\,(2.08\times10^{-3})5\times10^{-3}(0.6)}{9\times10^{-12}} = 2.7\ \text{MPa}$$

When the radius is reduced by 20%, so that $R_2 = 0.8\,R_1$

$$\frac{Q_2}{Q_1} = \frac{R_2^4}{R_1^4} = (0.8)^4 = 0.41$$

Power dissipation

The power dissipated during the flow of a fluid is the rate of energy required to maintain the flow. In general, the power is defined as the net force times the average speed,

$$\mathbb{P} = F\,\bar{V} = \Delta P\, A\, \bar{V} = \Delta P\, Q$$

Measured in Watt

Flow Resistance

The flow resistance can be defined in general as *the ratio of the pressure drop through a segment and the flow rate,* (notice the analogy to electric resistance, the pressure difference like the potential difference, and the flow rate like the electric current).

$$\mathcal{R}_{\mathrm{f}} = \frac{\Delta P}{Q} = \frac{\Delta P}{\pi \Delta P R^4 / 8\eta l} = \frac{8\eta l}{\pi R^4}$$

The unit of flow Resistance is $\mathcal{R}_{\mathrm{f}} = \dfrac{Pa}{m^3/s} = \dfrac{Pa.s}{m^3}$

Which is the viscosity per unit volume or viscosity density

Example 5.11

Compare the flow resistance in a capillary of 5 μm in radius and that in an artery of 5 cm in radius?

$$\frac{\mathcal{R}_f(capillary)}{\mathcal{R}_f(artery)} = \left(\frac{R_a}{R_c}\right)^4 = \left(\frac{5 \times 10^{-2}}{5 \times 10^{-6}}\right)^4 = 10^{16}$$

Example 5.12

A large artery has an inner radius of 4 mm. Blood flows through the artery at the rate of 1 $cm^3 s^{-1}$. Find

a) The average and maximum speed of the blood in the artery

b) The pressure drop in a 10 cm long segment of the artery

c) The flow resistance of blood over the 10 cm segment

d) The power dissipated through the flow

The artery has radius R = $4 \times 10^{-3} m$,

$$the\ flow\ rate\ Q = 10^{-6}\, {m^3}/{s} = A\bar{v} = \pi R^2 \bar{v},$$

$$so\ that\quad \bar{v} = \frac{Q}{\pi R^2} = \frac{10^{-6}}{3.14(16\times10^{-6})} = 0.02\, m/s\ ,then\ \ v_{max} = 0.04\, m/s$$

$$so\ the\ pressure\ drop\ \Delta p = \frac{4\eta l v_{max}}{R^2} = \frac{4\,(2.08\times10^{-3})10\times10^{-2}(0.04)}{16\times10^{-6}} = 2.08\ \text{Pa}$$

$$\mathcal{R}_f = \frac{\Delta P}{Q} = \frac{2.08}{10^{-6}} = 2.08 \times 10^6\ {Pa.s}/{m^3}$$

$$\wp = \Delta PQ = 2.08 \times 10^{-6} = 2.08\ \mu W$$

Turbulent Flow

When flow speed becomes high the streamlines of the flow start to intersect each other; and this flow is called turbulent flow. In such flow, the mechanical energy dissipated is much larger than that in the laminar flow.

Poiseuille's law is applicable for laminar flow.
There is a dimensionless quantity called Reynolds Number (N_R) used to distinguish the type of the flow. Consider a fluid of density ρ and viscosity coefficient η flows with an average velocity v through a tube of radius R, hence the Reynolds number is defined by

$$N_R = \frac{2\rho\bar{v}R}{\eta}$$

It is found experimentally that if

$$N_R < 2000 \qquad \textit{flow is laminar}$$

$$N_R > 3000 \qquad \textit{flow is turbulent}$$

$$2000 < N_R < 3000 \qquad \textit{flow is unstable}$$

Example 5.14

A small human capillary of 100 μm radius has a length of 2 cm, calculate

a) The blood flow resistance across this capillary
b) If the pressure drop across the capillary is 2.3 kPa, what is the flow rate
c) What is the maximum speed of blood through the capillary
d) The power dissipated across the capillary
e) Reynolds's number and then determine the type of flow.

If the total flow resistance of blood through three similar parallel arteries of length 30 cm is $4x10^8$ Pa.sm^{-3}.find

The diameter of each artery

If the power dissipated in pumping the blood in each artery is 0.4 mW, determine if the flow type of blood in the artery, laminar or turbulent? (explain) (take η_{blood} = 3.14 cp, ρ_{blood} = 1100 kgm^{-3}).