Lecture 1

26th Sep. 2019 Thursday

Fundimental Quantities:
- mass
- distance
- time

Derived Quantities:
- speed $\Rightarrow \frac{\text{distance}}{\text{time}}$
- acceleration $\Rightarrow \frac{\text{distance}}{\text{time} \times \text{time}}$
- Work $\Rightarrow$ mass $\cdot \frac{\text{distance}}{\text{time}} \times \frac{\text{distance}}{\text{time}}$

SI unit = mks
↳ meter kilogram second (LMT)

Gaussian system = Cgs
↳ centimeter gram second

British Unit = lb ft s
↳ pound feet second

Q ⇒ a cylinder > height 0.014m
diameter 3.4 µm
mass 400mg
⇒ find the density in Cgs

$$\rho = \frac{\text{mass}}{\text{Volume}} = \frac{0.4g}{\pi r^2 h}$$

$$= \frac{0.4g}{3.14(1.7 \times 10^{-3})^2 (0.14 \times 10^2)}$$

$$= \frac{4 \times 10^{-1}}{3.14 (1.7)^2 \times 10^{-8} (1.4)}$$

$$= \frac{4 \times 10^7}{3.14 \times 1.4 \times 2.9} = \frac{4 \times 10^7}{13}$$

$$= 0.37 \times 10^7 \; g/cm^3$$

Dimentions:

frequency $\frac{1}{T}$ / T^{-1}

Area ⇒ $[A] = L^2$
Volume ⇒ $[V] = L^3$
Speed ⇒ $[v] = \frac{L}{T}$
Acceleration ⇒ $[a] = \frac{L}{T^2}$

Velocity ⇒ $[v] = \frac{L}{T}$ / LT^{-1}
Momentum ⇒ $[p] = \frac{ML}{T}$ / MLT^{-1}
Force ⇒ $[F] = \frac{ML}{T^2}$ / MLT^{-2}
Pressure ⇒ $[P] = \frac{M}{LT^2}$ / $ML^{-1}T^{-2}$

Q ⇒ Example 1.6 (slides)

$$a = [r^n v^m]$$

$$\frac{L}{T^2} = L^n \cdot \frac{L^m}{T^m}$$

$$LT^{-2} = L^n \cdot L^m T^{-m}$$

$$L^1 = L^n \cdot L^m \qquad T^{-2} = T^{-m}$$

$$1 = n + m \qquad -2 = -m$$

$$m = 2$$

$$1 = n + 2$$

$$n = 1$$

Q ⇒ Example 1.7 (slides)

$$f = l\,m\,g$$

$$= m^a g^b l^y$$

$$[f] = [m^{\alpha} g^{b} l^{y}]$$

$$\frac{1}{T} = M^{\alpha} \frac{L^{b}}{T^{2b}} L^{y}$$

$$T^{-1} = M^{\alpha} (LT^{-2})^{b} L^{y}$$

$$L^{0}M^{0}T^{-1} = M^{\alpha} L^{b+y} T^{-2b}$$

$\Rightarrow b+y = 0$

$\Rightarrow \alpha = 0$

$\Rightarrow -2b = -1$

$b = \frac{1}{2}$

$\frac{1}{2} + y = 0$

$y = -\frac{1}{2}$

$$f = m^{0} g^{\frac{1}{2}} l^{-\frac{1}{2}}$$

$$= \sqrt{\frac{g}{l}}$$

Vector & Scalar Quantities

↳ displacement	↳ distance
↳ velocity	↳ speed
↳ acceleration	↳ temperature
↳ force	↳ mass
↳ momentum	↳ energy
	↳ time

• negative vectors mean negative directions not negative numbers

$$\vec{A} + \vec{B} = \vec{C}$$

1st Oct. 2019 Tuesday

Unit Vectors

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}$$

* dimentionless

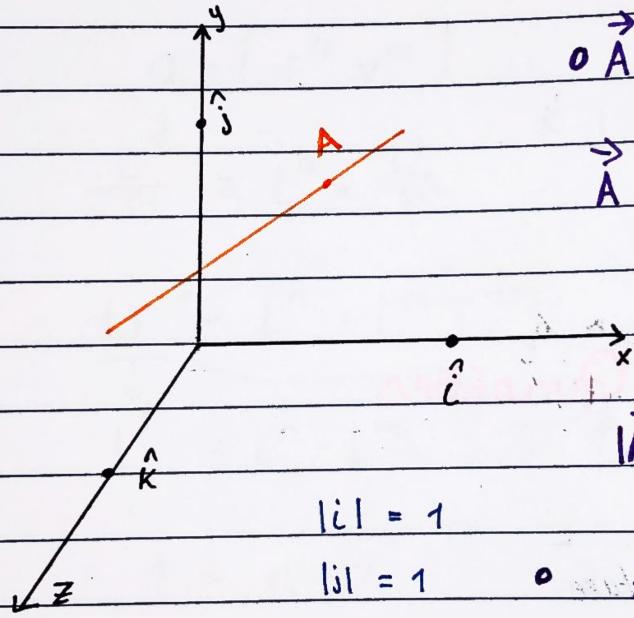
$$\vec{B} = B\hat{b}$$

$$\hat{b} = \frac{\vec{B}}{|\vec{B}|}$$

$$|\hat{a}| = \left|\frac{\vec{A}}{|\vec{A}|}\right|$$

$$= \frac{|A|}{|A|}$$

$$= 1$$



- $\vec{A}\ (A_x, A_y, A_z)$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$A = 2\hat{i} + 5\hat{j} + \hat{k}$$

$$|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

$|i| = 1$

$|j| = 1$

$|k| = 1$

- $\vec{A} \rightarrow (A, \theta)$

$\rightarrow (A_x, A_y, A_z)$

- $\vec{A} + \vec{B} = \vec{C}$

$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

$$= C_x + C_y + C_z$$

Q: $\vec{A} = 4\hat{i} - \hat{k}$
$\vec{B} = 3\hat{i} + \hat{j} - \hat{k}$

- find $\vec{A} - 2\vec{B} = R$ $= R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$

$R_x = A_x - 2B_x \Rightarrow 4 - 2(3) = -2$
$R_y = A_y - 2B_y \Rightarrow 0 - 2(1) = -2$
$R_z = A_z - 2B_z \Rightarrow 1 - 2(-1) = 1$

$$\vec{R} = -2\hat{i} - 2\hat{j} + \hat{k}$$

- find $\hat{r}$

$$\hat{r} = \frac{\vec{R}}{|\vec{R}|}$$

$$= \frac{-2\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{(-2)^2 + (-2)^2 + (1)^2}}$$

$$= \frac{-2\hat{i} - 2\hat{j} + \hat{k}}{3}$$

- find $\hat{b}$

$$\hat{b} = \frac{\vec{B}}{|\vec{B}|}$$

$$= \frac{3\hat{i} + \hat{j} - \hat{k}}{\sqrt{3^2 + 1^2 + (-1^2)}}$$

$$= \frac{3\hat{i} + \hat{j} - \hat{k}}{\sqrt{11}}$$

$$= \frac{3}{\sqrt{11}}\hat{i} + \frac{1}{\sqrt{11}}\hat{j} - \frac{1}{\sqrt{11}}\hat{k}$$

3rd Oct.
2019
Thursday

Multiplying vectors

- Scalar product (Dot Product)

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

max when $\theta = 0 \Rightarrow \vec{A} \cdot \vec{B} = AB$

when $\theta = \frac{\pi}{2} \Rightarrow \vec{A} \cdot \vec{B} = 0$

when $\theta = \pi \Rightarrow \vec{A} \cdot \vec{B} = -AB$

↳ $\hat{i} \cdot \hat{i} = |i||i| \cos 0$

$= 1 \cdot 1 \cdot 1 = 1$

↳ $\hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

↳ $\hat{j} \cdot \hat{k} = 0$

↳ $\vec{A} \cdot \vec{A} = A^2$

↳ $\vec{A} \cdot -\vec{A} = -A^2$

$\vec{A} \cdot \vec{B} = AB \cos\theta$

↳ $(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x + A_yB_y + A_zB_z$

$= AB \cos\theta$

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2} \cdot \sqrt{B_x^2 + B_y^2 + B_z^2}}$$

Q: Three vectors ⇒ $\vec{A} = \hat{i} - \hat{j}$

$\vec{B} = \hat{k}$

$\vec{C} = 3\hat{i} + 2\hat{j}$

1) Find the angle between $\vec{A}$ & $\vec{C} - \vec{B}$

$\vec{H} = \vec{C} - \vec{B}$

$= 3\hat{i} + 2\hat{j} - \hat{k}$

$$\cos\theta = \frac{A_x H_x + A_y H_y + A_z H_z}{\sqrt{A_x^2 + A_y^2 + A_z^2} \cdot \sqrt{H_x^2 + H_y^2 + H_z^2}}$$

$$\cos\theta = \frac{3 - 2}{\sqrt{1+1} \cdot \sqrt{9+4+1}} = \frac{1}{\sqrt{2} \cdot \sqrt{14}} = \frac{1}{\sqrt{28}}$$

$\theta = 79.12$

2) Find the angles that the vector 'H' makes with the Cartesian axises

x-axis ⇒ $\vec{H} \cdot \vec{i} = |\vec{H}|\,|\hat{i}| \cos\theta_x$

$(3\hat{i} + 2\hat{j} - \hat{k}) \cdot \hat{i} = \sqrt{14} \cdot 1 \cdot \cos\theta_x$

$3 = \sqrt{14} \cos\theta_x$

$$\cos\theta = \frac{3}{\sqrt{14}}$$

$\theta = 36.7$

y-axis ⇒ $\vec{H} \cdot \vec{j} = |\vec{H}|\,|\hat{j}| \cos\theta_y$

$(3\hat{i} + 2\hat{j} - \hat{k}) \cdot \hat{j} = \sqrt{14} \cdot 1 \cdot \cos\theta_y$

$2 = \sqrt{14} \cos\theta_y$

$$\cos\theta_y = \frac{2}{\sqrt{14}}$$

$\theta = 57.69$

z-axis ⇒

$$\cos\theta_z = \frac{\vec{H} \cdot \hat{k}}{|H|\,|k|}$$

$$= \frac{-1}{\sqrt{14}}$$

$\theta = 105.5$

$\cos\theta_x = \frac{A_x}{|A|}$, $\cos\theta_y = \frac{A_y}{|A|}$, $\cos\theta_z = \frac{A_z}{|A|}$

$\hookrightarrow \cos\theta_x^2 + \cos\theta_y^2 + \cos\theta_z^2$

$= \frac{A_x^2 + A_y^2 + A_z^2}{A^2}$

- Vector Product (cross Product)

$$\vec{A} \times \vec{B} = \vec{C} \neq \vec{B} \times \vec{A}$$

$|\vec{C}| = AB\sin\theta$ (as magnitude)

Torque (τ) $\Rightarrow$ a measure of the force that can cause an object to rotate about an axis

$$\tau = \vec{r} \times \vec{F}$$

$$|\tau| = rF\sin\theta$$

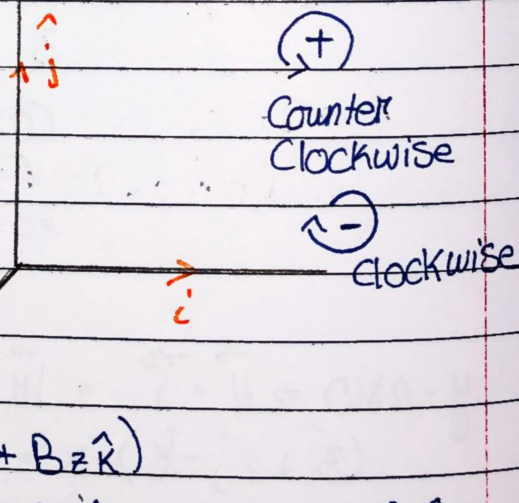
$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$

$\theta = 0$, $\sin(0) = 0$

$\hat{i} \times \hat{k} = -\hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$



$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_y\hat{k} - A_xB_z\hat{j} - A_yB_x\hat{k} + A_yB_z\hat{i} + A_zB_x\hat{j} + A_zB_y\hat{i}$

$= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

$\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k} \Rightarrow |\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$

$$\begin{array}{ccc} + & - & + \\ \left| \begin{matrix} i & j & K \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{matrix} \right| \end{array}$$

$\hat{i} = (A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$

Q: $\vec{A} = 2\hat{i} - \hat{j}$

$\vec{B} = 5\hat{k} - \hat{i}$

$\vec{C} = 3\hat{j} - 2\hat{k}$

1) Find the unit vector in the direction of $\vec{A} \times \vec{C}$

$\vec{S} = \vec{A} \times \vec{C}$

$$= \begin{array}{ccc} + & - & + \\ \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ 0 & 3 & -2 \end{matrix} \right| \end{array}$$

$= 2\hat{i} + 4\hat{j} + 6\hat{k}$

$\hat{S} = \dfrac{\vec{S}}{|S|}$

$= \dfrac{2i + 4j + 6k}{\sqrt{56}}$

2) Find the angle between $\vec{B} - \vec{A}$ & $\vec{C} \times \vec{B}$

$\vec{R} = \vec{B} - \vec{A}$

$= (5\hat{k} - \hat{i}) - (2\hat{i} - \hat{j})$

$= 5\hat{k} - 3\hat{i} + \hat{j}$

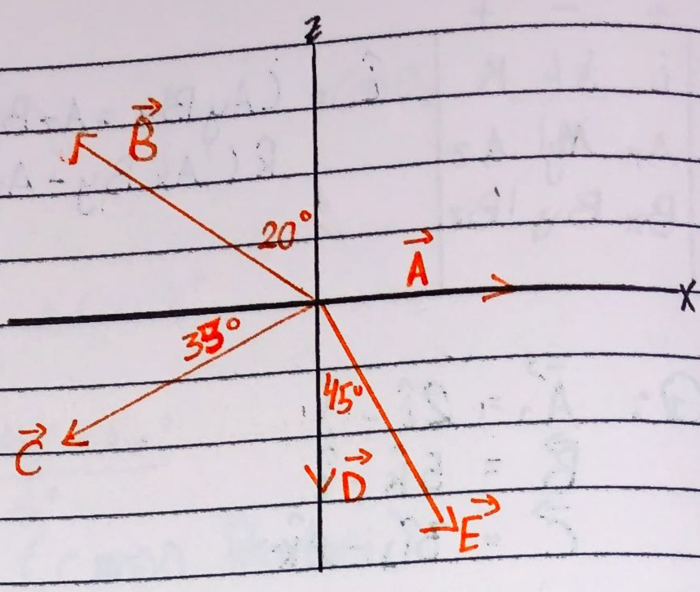
$\vec{L} = \vec{C} \times \vec{B}$

$$\begin{array}{ccc} + & - & + \\ \left| \begin{matrix} i & j & K \\ 0 & 3 & -2 \\ -1 & 0 & 5 \end{matrix} \right| \end{array} = 15i + 2j + 3k$$

$\cos\theta = \dfrac{R_xL_x + R_yL_y + R_zL_z}{|R|\ |L|}$

$= \dfrac{-45 + 2 + 15}{\sqrt{35}\ \sqrt{238}} = \dfrac{-28}{\sqrt{35}\ \sqrt{238}} \therefore \theta = 107.87$

Q: $|\vec{A}| = 4m$
$|\vec{B}| = \frac{1}{2} m$
$|\vec{C}| = 6\ m$
$|\vec{D}| = |\vec{E}| = 3m$



1) Find $\vec{A} - 2\vec{C}$

$\vec{A} = 4\hat{i}$ $\qquad |C_x| = -12 \cos 35 = -9.8$

$\vec{C} = -9.8\hat{i} - 6.8\hat{k}$ $\qquad |C_z| = -12 \sin 35 = -6.8$

$2C = 2(-9.8\hat{i}) - 2(6.8\hat{k})$
$= -19.6i - 13.6\hat{k}$

$A - 2C$
$= 4i - (-19.6\hat{i}) - 13.6\hat{k}$
$= 23.6\hat{i} - 13.6\hat{k}$

2) Find $\vec{A} \times (\underbrace{\vec{D} + \vec{E}}_{J})$

$\vec{D} = -3\hat{k}$ $\qquad \vec{J} = -3\hat{k} + (-2.6\hat{k}) + 1.6\hat{i}$

$|E_z| = -3 \sin 45 = -2.6$ $\qquad = -5.6\hat{k} + 1.6i$

$|E_x| = 3 \cos 45 = 1.6$

$\vec{A} \times \vec{J} = (4\hat{i}) \times (-5.6\hat{k} + 1.6\hat{i})$

$$\begin{array}{ccc} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ 1.6 & 0 & -5.6 \end{array}$$

$0i + 22.4\hat{j} + 0\hat{k}$

3) Find the unit vector in the direction of $\frac{\vec{A} + \vec{C}}{W}$ and $\frac{\vec{E} \times \vec{B}}{P}$

$\vec{A} = 4\hat{i}$

$\vec{C} = -9.8\hat{i} - 6.8\hat{k}$

$W = \vec{A} + \vec{C} = -5.8\hat{i} - 6.8\hat{k}$

$\hat{w} = \frac{\vec{W}}{|W|} = \frac{-5.8\hat{i} - 6.8\hat{k}}{\sqrt{5.8^2 + 6.8^2}}$

$= -\frac{5.8}{9.02}\hat{i} - \frac{6.8}{9.02}\hat{k}$

$\vec{E} \times \vec{B} = EB\sin\theta$

$P = 3 \times 6 \sin 155$

$= 7.6\hat{j}$

$\hat{p} = \frac{\vec{P}}{|P|} = \frac{7.6\hat{j}}{\sqrt{7.6^2}}$

$= \frac{7.6\hat{j}}{7.6}$

$= \hat{j}$