Lecture 3

17th Oct. 2019 Thursday

Stress & Strain

① Stress ⇒ measure of force felt by material

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

σ symbol

SI unit is pascals $1Pa = 1N/m^2$

↳ ① tensile stress
↳ ② Compression stress
↳ ③ Shear stress

Hooke's law → stress is proportional to strain ← within its elastic limit

② Strain ⇒ measure of deformation

$$\text{strain} = \frac{\Delta L}{L}$$

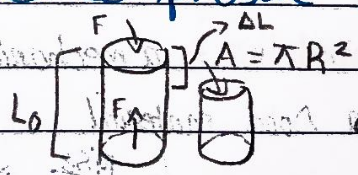
ε symbol

- dimensionless
- unitless

$\Delta L = L_{new} - L_0$ $\Delta L > 0$

① tension stress

② Compresive stress



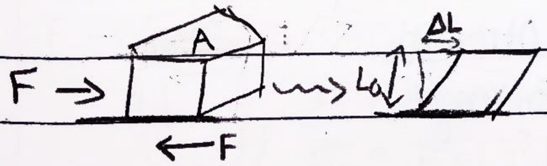
$\sigma = \frac{F}{A_\perp}$

$\varepsilon = \frac{-\Delta L}{L_0}$

$\Delta L = L_{new} - L_0$

$\Delta L < 0$

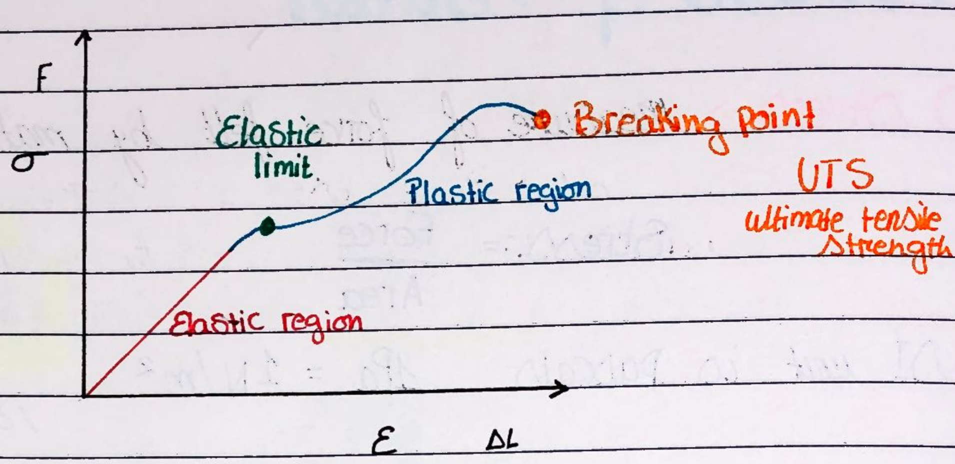
③ Shear stress



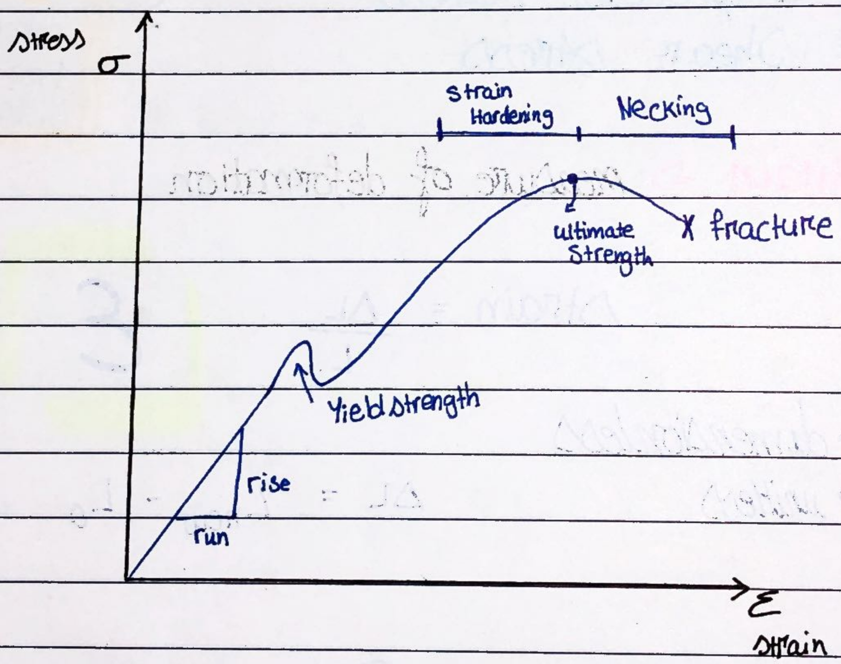
$\sigma_{sh} = \frac{F}{A_{\parallel}}$ $\varepsilon_{sh} = \frac{\Delta L}{L_0}$ $G = \frac{F/A}{\Delta L / L_0}$

22nd Oct. 2019 Tuesday

Materials Behavior Under Stress



→ Elastic limit ⇒ the maximum stress that the material can withstand just before the fracture



→ Young's modulus ⇒ a mechanical property that measures the stiffness of a solid material

↳ $\frac{\text{rise}}{\text{run}}$ ⇒ slope

Saftey Factor (S) = $\frac{\text{ultimate strength}}{\text{applied strength}}$

Linear Region (Elastic Region)

Hooke's law $F = -Kx$

Young's modulus ⇒ the ratio of stress over strain in the region which Hooke's Law is obeyed for the material

$$Y = \frac{\sigma}{\varepsilon}$$

$$\Rightarrow Y = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{F \cdot L_0}{\Delta L \cdot A}$$

$$Y = \frac{K L_0}{A}$$

$$\Rightarrow K = \frac{YA}{L_0}$$

Hooke's
↓
$F \propto \Delta L$

$F = K \Delta L$

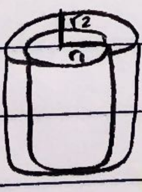
↖ spring constant (when it has the same diameter)

$$\therefore F = \frac{YA}{L_0} \Delta L$$

Question: What is the difference between the spring constant and Young's modulus?

Young's modulus is a characteristic of the material while the spring constant is not a characteristic of the material

Q: Example 7 ⇒



$r_1 = 0.02m$

$r_2 = 0.023m$

$\sigma = 5 \times 10^7 Nm^{-2}$

$F = ?$

$\sigma = \frac{F}{A} \Rightarrow F = \sigma A$

$= \sigma \pi (r_2^2 - r_1^2)$

$= 5 \times 10^7 \pi (0.023^2 - 0.02^2)$

$= 20263.27 N$

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Q: Example 8 ⇒ $L_0 = 1.2m$

$\varepsilon = 1.3 \times 10^{-4}$

$\Delta L = ?$

$\varepsilon = \frac{\Delta L}{L_0}$

$$\therefore \Delta L = \varepsilon \cdot L_0$$
$$= 1.3 \times 10^{-4} \cdot 1.2$$
$$= 1.56 \times 10^{-4} \text{ m}$$

Shear Modulus

$$S = \frac{F/A}{\Delta x/h}$$

shear stress

shear strain

Bulk Modulus

$$B = -\frac{\Delta F/A}{\Delta V/V} = -\frac{\Delta P}{\Delta V/V}$$

change in pressure

volume strain

$B = \frac{Y}{3}$

$\frac{1}{B} = K$ (compressibility)

deriving
↳

- rho $\rho = \frac{m}{V}$

$\frac{\Delta \rho}{\Delta V} = \frac{-m}{V^2}$

$\Delta \rho = \frac{-m}{V^2} \Delta V$

$$\frac{\Delta \rho}{\rho} = \frac{\frac{-\cancel{m}}{V^2} \Delta V}{\frac{\cancel{m}}{\cancel{V}}}$$

$$= -\frac{\Delta V}{V}$$

- $E = \frac{1}{2} F \Delta L$

$$\frac{\frac{F}{A}}{\frac{\Delta L}{L_0}} = \frac{\sigma}{\varepsilon_0} = Y$$

$$\Rightarrow F = \frac{Y \Delta L A}{L_0}$$

$$E = \frac{1}{2} \frac{YA}{L_0} \Delta L^2$$

- $\sigma_{max} = \frac{F_{max}}{A}$

$$\sigma_{max} = Y \frac{\Delta L}{L_0} \quad \Rightarrow \quad \Delta L_{max} = \frac{\sigma_{max} L_0}{Y}$$

$$E_{max} = \frac{1}{2} \frac{\cancel{Y}A}{\cancel{L_0}} \Delta \frac{\sigma^2_{max} \cancel{L_0^2} L_0}{\cancel{Y^2} Y}$$

$$E_{max} = \frac{A L_0 \sigma^2_{max}}{2Y} \quad (J)$$

-> Energy density => the energy per unit volume

$$U_{max} = \frac{E_{max}}{V} = \frac{E_{max}}{AL_0} = \frac{\sigma^2_{max}}{2Y}$$