Lecture 5

Fluid Mechanics

- Fluid ⇒ a continous, amorphous substance whose molecules move freely past one another and has the ability to flow & assume the shape of its container; liquid or gas

↳ fluids are inable to support a shear stress in static equilibrium

↳ in fluids stress is a function of rate of strain

◦ Characteristics of Fluids

$F = ma$

$$\underbrace{F}_{\substack{\text{Force} \\ N}} = \underbrace{\Delta P}_{\substack{\text{Pressure} \\ N/m^2}} \underbrace{A}_{\substack{\text{Area} \\ m^2}}$$

Bernculli equation

$\frac{N/m^2}{\text{Pascal}}$

$$P = \frac{F \rightarrow \text{force}}{A \rightarrow \text{Area}} \qquad \frac{mg}{A}$$

$$P = \frac{W \rightarrow \text{Energy}}{\Delta V \rightarrow \text{volume}}$$

$$\underbrace{\Delta m}_{\text{unit mass}} = \overset{\text{density}}{\rho} \underbrace{\Delta V}_{\text{unit volume}}$$

- Kinetic Energy Density ⇒ the amount of kinetic energy stored in a given system or region of space per unit volume

$$KE = \frac{1}{2} \Delta m v^2$$
$$= \frac{1}{2} \rho \Delta V v^2$$

$$KE_{density} = \frac{1}{2} \rho v^2$$

$$\frac{KE}{V} = \frac{\frac{1}{2} \rho \cancel{V} v^2}{\cancel{V}}$$

- Potential Energy Density ⇒ the amount of potential energy per unit volume

$$PE_{density} = \rho g h$$

$$\frac{PE}{\Delta V} = \frac{\Delta m g h}{\Delta V} = \frac{\rho \Delta V g h}{\Delta V}$$

- Compressibility & Incompressibility
  - ↳ if density in a fluid is constant through the flow then its incompressable
  - ↳ if density isn't fixed, then its compressable

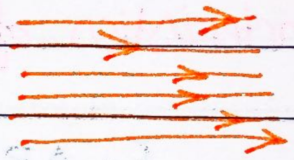
- Viscous & Nonviscous

-Viscosity ⇒ frictional forces originated inside the fluids and is a resistance to flow

- Types of flow

-Streamline ⇒ a line that describes the flow of the fluid and its direction

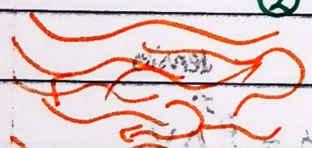
① Laminar flow

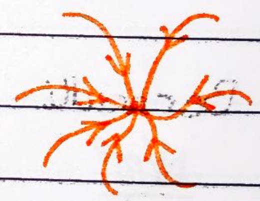


↳ speed of the flow is low

↳ streamlines are parallel

② Turbulent flow





↳ speed of the flow is high

↳ Streamlines intersect with each other

③ Transition flow

↳ between laminar & turbulent

↳ unsteady / unpredictable flow

◦ Fluid Flow & the Continuity Equation

$$Q_{in} = Q_{out}$$

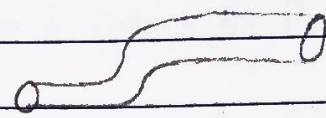
$$Q = \frac{\Delta V}{\Delta t} = \frac{A\,\Delta x}{\Delta t} = A\vec{v}$$

$$A_1\vec{v}_1 = A_2\vec{v}_2$$

$$\cancel{\pi} r_1^2\vec{v}_1 = \cancel{\pi} r_2^2\vec{v}_2$$

$$r_1^2\vec{v}_1 = r_2^2\vec{v}_2$$

◦ Bernoulli's Equation



↳ where $\vec{v}\downarrow$ $P\uparrow$

↳ The work done on a fluid as it flows from one place to another is equal to the change in its mechanical energy

* ↳ for incompressable fluids; density is constant

↳ for nonviscous fluids; no energy loss

$$F = (P_1 - P_2)A$$

$$F = \Delta P A$$

$$\underset{\text{work}}{W} = F \cdot \underset{\text{displacement}}{\Delta x}$$

$$= (P_1 - P_2)\underbrace{A\Delta x}_{\Delta V}$$

$$W = \Delta P \Delta V \longrightarrow ME$$

$$\Delta KE = \frac{1}{2}\rho\Delta V\vec{v}_2^{\,2} - \frac{1}{2}\rho\Delta V v_1^2$$

$$\Delta PE = \rho\Delta V g h_2 - \rho\Delta V g h_1$$

$$(P_1 - P_2)\cancel{\Delta V} = \frac{1}{2}\rho\cancel{\Delta V}\vec{v}_2^{\,2} - \frac{1}{2}\rho\cancel{\Delta V}\vec{v}_1^{\,2} + \rho\cancel{\Delta V}gh_2 - \rho\cancel{\Delta V}gh_1$$

move P_1s together P_2s together

$$P_1 + \frac{1}{2}\rho\vec{v}_1^{\,2} + \rho g h_1 = P_2 + \frac{1}{2}\rho\vec{v}_2^{\,2} + \rho g h_2$$

$$\therefore\ P + \frac{1}{2}\rho\vec{v}^{\,2} + \rho g h = \text{Constant}$$

↳ Bernoulli's equation

$P + ME$ is the same everywhere in the flow

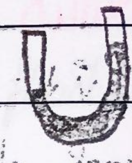
↳ if two variables are missing use equation of continuity

• Application of Bernoulli's Equation

↳ **Static Consequence** ⇒ zero speed of flow

fluids settle

$$KE = 0$$



$$P + \rho g h = \text{constant}$$

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

$$P_a + \rho g h_1 = P_2 + \rho g h_2$$

$$P_2 = P_a + \rho g (h_1 - h_2)$$

$(P - P_a)$ ⇒ gauge pressure

P: absolute Pressure

P_a: atmospheric Pressure

↳ **Horizontal Flow Consequence** ⇒ flow of the fluid is horizontal

$PE = c$

$$P + \frac{1}{2}\rho \vec{v}^2 = \text{constant}$$

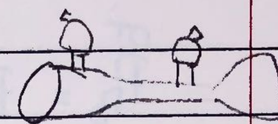
$$P_1 + \frac{1}{2}\rho \vec{v}_1^2 = P_2 + \frac{1}{2}\rho \vec{v}_2^2$$

$$P_{out} + \frac{1}{2}\rho \vec{v}_{out}^2 = P_{in} + \frac{1}{2}\rho \vec{v}_{in}^2$$

$$P_{out} - P_{in} = \frac{1}{2}\rho (\bar{v}_{in}^2 - \bar{v}_{out}^2)$$

- Venturi Tube

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$



$$V_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}}$$

- Mass Flow Rate / Volume Flow Rate

$$\text{mass of flowrate} = \frac{\text{mass of fluid}}{\text{time taken to collect the fluid}}$$

$$\text{time} = \frac{\text{mass}}{\text{mass flow rate}}$$

- The Role of Gravity on Blood circulation

↳ focuses on Brain, Heart, Feet

$P_B = P_H = P_F$ (in the reclining position)

$P_F = P_H + \rho g h_H = P_B + \rho g h_H$ (in the standing position)

$P_B = P_H + \rho g (h_H - h_{(B)})$

standing

$P_B < P_H$

$P_F > P_H$

$P_F = P_H + \rho g (h_H - h_B)$

- Effect of Acceleration on Blood Pressure

Upwards acceleration

$P_B = P_H + \rho(g+a)(h_H - h_B)$

or $P_B = P_H - \rho(g+a)(h_B - h_H)$

$P_F = P_H + \rho(g+a)\, h_H$

Blood Pressure @ Foot ↑ when upward acceleration ↑

Downward Acceleration

$$P_B = P_H + \rho (g-a)(h_H - h_B)$$

or

$$P_B = P_H - \rho (g-a)(h_B - h_H)$$

Viscous Fluid Flow

- viscousity ⇒ resistance to flow

$$F \propto \Delta V$$

$$F \propto A$$

$$F \propto \frac{1}{\Delta y} \therefore F \propto \frac{A \Delta V}{\Delta y}$$

$$F = \eta \frac{A \Delta V}{\Delta y}$$

viscousity coefficient (Pa·s)

distance

$$\eta = \frac{F/A}{\frac{\Delta x}{\Delta y} \cdot \frac{1}{\Delta t}}$$

$$= \frac{\text{Shear stress}}{\text{rate of Shear strain}}$$

↳ 1 Poise $= 10^{-1}$ Pa·s

↳ 1 cp $= 10^{-2}$ Poise $= 10^{-3}$ Pa·s

↳ $\eta = ML^{-1}T^{-1}$

Laminar Flow in a Tube

↳ Surface area ⇒ $2\pi RL$

↳ Cross sectional area ⇒ πR^2

• $V_{max} = 2 \cdot \vec{V}$

$$F = \Delta P \, A_{cross}$$

↳ $F = (P_1 - P_2) \pi r^2$

$$F = \frac{A_{surf} \cdot V_{max}}{R}$$

$$F = \eta \frac{A_{surf} \cdot V_{max}}{R}$$

$$V_{max} = \frac{\Delta P R^2}{4 \eta L}$$

$$\bar{V} = \frac{\Delta P R^2}{8 \eta L}$$

Flow Rate (Poiseuille's law)

$$Q = AV = \pi R^2 \bar{V}$$

$$\therefore Q = \frac{\pi \Delta P R^4}{8 \eta L}$$

↳ high viscosity = $\downarrow Q \downarrow \bar{v}$
↳ R is dependent

Power Dissipation

$$\mathbb{P} = \frac{\text{work}}{\text{time}} = \frac{F \Delta x}{\Delta t} \rightarrow \bar{v} = F\bar{V}$$

watt

$$\mathbb{P} = \Delta P A \bar{V}$$
$$= \Delta P Q$$

$$\mathbb{P} = Q^2 R_f$$
$$\text{or} = \frac{\Delta P^2}{R_f}$$

Flow Resistance

$$R_f = \frac{\Delta P}{Q} \quad \text{or} \quad R_f = \frac{8 \eta L}{\pi R^4}$$

Pa.s/m³

$$= \frac{\cancel{\Delta P}}{\frac{\pi \cancel{\Delta P} R^4}{8 \eta L}} = \frac{8 \eta L}{\pi R^4}$$

Blood Vessels connections

Series

↳ Q is constant
↳ P is distributed

$\Delta P_{tot} = \Delta P_1 + \Delta P_2 + \Delta P_3$

$Q_{tot} = Q_1 = Q_2 = Q_3$

$R_f = R_1 + R_2 + R_3$

Parallel

↳ Q is distributed
↳ P is constant

$Q_{tot} = Q_1 + Q_2 + Q_3$

$\Delta P_{tot} = \Delta P_1 = \Delta P_2 = \Delta P_3$

$$\frac{1}{R_{ftot}} = \frac{1}{R_{f1}} + \frac{1}{R_{f2}} + \frac{1}{R_{f3}}$$

Turbulent Flow

$$N_R = \frac{2 \rho \bar{V} R}{\eta}$$

$N_R < 2000$ Laminar flow
$N_R > 3000$ Turbulent flow
$2000 < N_R < 3000$ Unstable flow