Lecture 6

Biolectricity

Coulombs Law

$$F_e = K_e \frac{|q_1||q_2|}{r^2}$$

Coulombs electric force — F_e

Coulombs Constant — K_e

Charge — q_1, q_2

distance — r^2

$$K_e = \frac{1}{4\pi \varepsilon_0}$$

$K_e = 9 \times 10^9 \ N \cdot m^2 / C^2$

$\varepsilon_0 = 8.85 \times 10^{-11} \ C^2 / N \cdot m^2$

- Current ⇒ the rate at which the charge flows through a surface

$$I = \frac{Q}{t}$$

Current — I

A → ampere

Charge — Q

C → Coulomb

time — t

$$V = \frac{I}{n \cdot e \cdot A}$$

drift velocity — V

Current — I

Cross sectional area — A

charge of nutron & electron — e

$$J = \frac{I}{A}$$

current density — J

current — I

C.S. Area — A

• Resistance & Ohm's Law

$$V = IR$$

Current

Resistance $\Omega \rightarrow$ ohm

voltage difference → volt

↳ when talked about with temperature:

$$R_T = R_0(1+\alpha(T-T_0))$$

$$\rho_T = \rho_0(1+\alpha(T-T_0))$$

where α is the temperature coefficient of resistance

length of wire

$$\text{Resistance} \rightarrow R = \rho\frac{L}{A} \leftarrow \text{C.S. Area}$$

resistivity (changes for diff. substances) (given)

• Electric Power

- Power ⇒ work done per unit time

$$P = I\Delta V \quad (IR) \qquad P = I^2 r \qquad P = \frac{V^2}{R}$$

$\frac{Q}{t}$

↳ used to find Power in a load Resistor

↳ used to find power in internal Resistor

↳ used to find Load Resistor

• Electromotive force (emf) (battery)

$$\varepsilon = V_R + V_r \quad \therefore \quad I = \frac{\varepsilon}{R+r}$$

load $= IR$

internal $= Ir$

$$V = \varepsilon - Ir$$

↳ Terminal voltage

Potential difference $\Delta V = \varepsilon - Ir$

∘ Simple Resisive Circuits

↳ Series

- $R_{eq} = R_1 + R_2 + R_3$

↳ Parallel

- $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
- Same Potential difference (ΔV)

∘ The Capacitor → combination of two conductors

↳ called plates

capacitance of Capacitor $C = \frac{Q}{\Delta V}$ ← Charge; ← potential difference

F → Farad

∘ Parallel Plate Capacitor

capacitance $C = \frac{K \varepsilon_0 A}{d}$ ← distance

K → dielectric constant (sometimes absent)

ε_0 → Epsilon

A → Surface Area

- Epsilon → the permitivity of the medium

∘ RC Circuits

↳ Current is not constant

↳ series of resistors & capacitors

∘ Charging a Capacitor

↳ $t<0$ capacitor doesn't push back against current

Current is located at resistor

↳ the voltages accross the resistor and the capacitor will change with time

$\frac{q}{t} \leftarrow I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$

$V_C(t) = \mathcal{E}(1 - e^{-t/RC})$

$Q = 0$ @ $t = 0$

$Q = C\mathcal{E}$ max Q $t \to \infty$

$I_{max} = \mathcal{E}/R$ @ $t = 0$

$I_{min} = \mathcal{E}/R$ $t \to \infty$

$\tau = RC$

↳ time constant

$Q = C\mathcal{E}$

↳ max charge

$I_0 = \frac{\mathcal{E}}{R}$

max charging current

$q(t) = Q(1 - e^{-t/RC})$

$I(t) = I_0 e^{-t/RC}$

↳ Charge and current as a function of time

Discharging a Capacitor

↳ $t=0$ capacitor begins to discharge through the resistor

$$\frac{q(t)}{C} + I(t)R = 0$$

↳ $\boxed{q(t) = Qe^{-t/RC}}$

↳ $\boxed{I(t) = -I_0 e^{-t/RC}}$

$I_0 = Q/RC$

- negative sign = discharging current direction is opposite of the one charging

$V_C(t) = V_0 e^{-t/RC}$

@ $t = RC$ → $V_C(t) = \frac{V_0}{e}$

Nerve Conduction

Nernst equation → Potential difference across the membrane

Thermal energy

$$V = \pm 2.3 \frac{K_B T}{q} \log\left(\frac{C_o}{C_i}\right)$$

concentration of ions inside & outside

electronic Charge

J/K 37°C K

$$\hookrightarrow \pm 2.3 \frac{1.38\times10^{-23} \cdot 310K}{1.6\times10^{-19}} \log\left(\frac{C_o}{C_i}\right)$$

C

$$V = \pm 61.4\times10^{-3} \log\left(\frac{C_o}{C_i}\right)$$

V

Resistance

$$R_a = \rho \frac{L}{\pi r^2}$$ ← of axoplasm — Cross-sectional area

Ω, Ω·m

$$R_m = \frac{\rho_m}{2\pi r l}$$ ← of membran — Surface area

Ω·m², Ω

Space parameter λ ⇒ how impulse signals travel along the axon before the leakage is complete

↳ $R_m = R_a$

usually 0.05 - 0.7 cm

$$\lambda = \sqrt{\frac{\rho_m r}{2\rho_a}}$$

Capacitance

$K\varepsilon_0$

$$C = \frac{\varepsilon A}{d}$$

C for unmyelinated axon $= 10^{-2}\ F/m^2$

C for myelinated axon $= 5\times10^{-5}\ F/m^2$

		Unmyelinated	Myelinated
R	ρ_a	$2\ \Omega \cdot m$	$2\ \Omega \cdot m$
wall resistivity	$\rho_{UA} \longrightarrow$	$0.2\ \Omega \cdot m^2$	ρ_{MA} $40\ \Omega \cdot m^2$
wall $\frac{C}{A}$	$\frac{C}{A}$	$10^{-2}\ F/m^2$	$5 \times 10^{-6}\ F/m^2$

Propagation speed

$$V(t) = V_0 \left(1 - e^{-t/R_a C}\right)$$

↳ resting potential

Charging time $\tau = R_a \times C$

$$v = \frac{r}{2 C_m \rho_a x}$$

x → length of each unit

C_m: membrane Capacitance

ρ_a: resistivity $2\ \Omega \cdot m$

$$v = \frac{\lambda}{C_m \rho_m x}$$

↳ the wider the axon, the faster the axonal speed of propagation