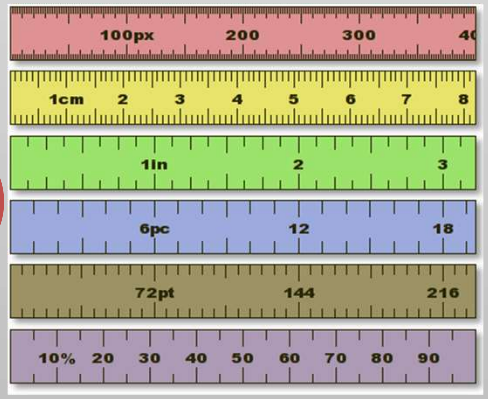
Lecture 1
Quantities and Dimensions

Type Quantities

- **Choose three basic quantities (Fundamentals):**
  - **LENGTH**
  - **MASS**
  - **TIME**
- **Define other units in terms of these (Derivatives)**
- ***Speed , density, force, work, etc..***
- ***speed=Length/time,***
- ***density= mass/ (length)^3***
- ***force= mass x acceleration = mass x length/(time)^2***
- ***Work = force x distance = mass x (length)^2/(time)^2***

Unit for 3 Basic Quantities

- **Many possible choices for units of Length, Mass, Time (Fundamental Units)**
- **In 1960, standards bodies control and define Système Internationale (SI) unit as, or mks**
  - **LENGTH: Meter (m)**
  - **MASS: Kilogram (kg)**
  - **TIME: Second (s)**



Fundamental and Derivative Quantities and SI Units

	Length	**meter**	**m**
F	**Mass**	**kilogram**	**kg**
	Time	**second**	**s**
	speed	**Meter/second**	**m/s**
D	**Force**	**Newton (N)**	**Kg m/s^2**
	density	**Kg/m^3**	**Kg/m^3**
	work	**Joule**	**Kgm^2/s^2**

Approximate values of some measured lengths

Distance from earth to most remote known star	1×10^{26} meters
Distance from Earth to Andromeda galaxy	2×10^{22} m
One light year	9×10^{15} m
Orbit radius of Earth about sun	2×10^{11} m
Mean distance from Earth to moon	4×10^{8} m
Length of a football field	9×10^{1} m
Length of a housefly	5×10^{-3} m
Size of smallest dust particles	1×10^{-4} m
Size of most living cells	1×10^{-5} m
Diameter of hydrogen atom	1×10^{-10} m
Diameter of atomic nucleus	1×10^{-14} m
Diameter of a proton	1×10^{-15} m

Approximate values of some masses

Observable Universe	1×10^{52} kilograms
Earth	6×10^{24} kilograms
Shark	1×10^{2} kilograms
Human	7×10^{1} kilograms
Mosquito	1×10^{-5} kilograms
Bacterium	1×10^{-15} kilograms
Hydrogen Atom	2×10^{-27} kilograms
Electron	9×10^{-31} kilograms

Prefixes for SI Units

- 3,000 m = 3 x1,000 m
  = 3 x 10^3 m = 3 km
- 1,000,000,000 = 10^9 = 1G
- 1,000,000 = 10^6 = 1M
- 1,000 = 10^3 = 1k

- 141 kg = ? g
- 1 GB = ? Byte = ? MB

10^x	Prefix	Symbol
x=18	exa	E
15	peta	P
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deca	da

Prefixes for SI Units

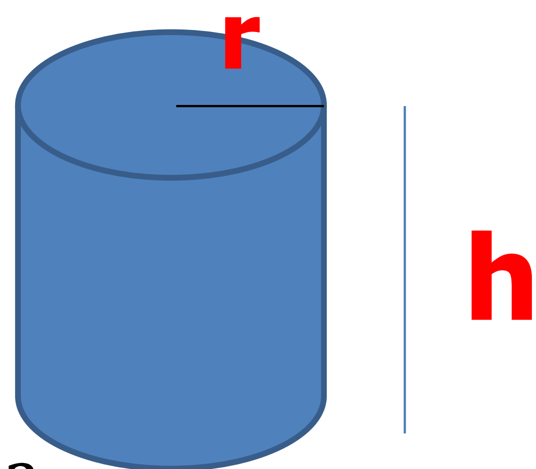
10^x	Prefix	Symbol
x=-1	deci	d
-2	centi	c
-3	milli	m
-6	micro	μ
-9	nano	n
-12	pico	p
-15	femto	f
-18	atto	a

Other Unit System

- U.S. customary system: foot, slug, second
- Cgs system: cm, gram, second
- We will use SI units in this course, but it is useful to know conversions between systems.
  - 1 mile = 1609 m = 1.609 km , 1 ft = 0.3048 m = 30.48 cm
  - 1 m = 39.37 in. = 3.281 ft , 1 in. = 0.0254 m = 2.54 cm
  - 1 lb = 0.465 kg 1 oz = 28.35 g 1 slug = 14.59 kg
  - 1 day = 24 hours = 24 * 60 minutes = 24 * 60 * 60 sec

Unit Conversion

A cylinder of 12 cm in height and its diamer is 40 mm, calculate The density of its material in SI unit, if its mass is 870 g?



$$\text{The density} = \frac{mass}{volume} = \frac{m}{\pi r^2 h}$$

$$= \frac{870\times10^{-3}\ kg}{3.14\times(20\times10^{-3})^2 m^2\ (12\times10^{-2})m}$$

$$\cong 6 \times 10^3\ kg/m^3$$

Dimensions, Units and Equations

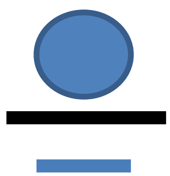
- Quantities have dimensions:
  - **Length – L, Mass – M, and Time - T**
- Quantities have units: Length – m, Mass – kg, Time – s
- To refer to the dimension of a quantity, use square brackets, e.g. $[F]$ means dimensions of force.

Quantity	Area	Volume	Speed	Acceleration
Dimension	$[A] = L^2$	$[V] = L^3$	$[v] = L/T$	$[a] = L/T^2$
SI Units	m^2	m^3	m/s	m/s^2

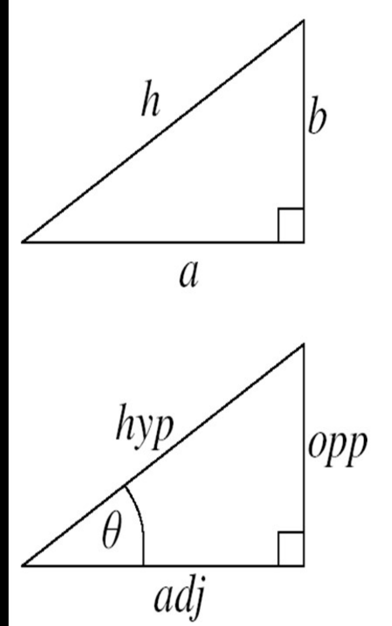
Dimensionless quantity

is a quantity that has no unit and it is a ratio between two similar quantities, to which no physical dimension is assigned. It is also known as a bare number or pure number or a quantity of dimension one [1]

π



$$Specific\ Gravity = \frac{density\ of\ the\ object}{density\ of\ water} = \frac{\rho_{object}}{\rho_{H_2O}}$$



Pythagoras's Theorem

$$a^2 + b^2 = h^2$$

Trigonometric Ratios

$$\sin(\theta) = \frac{opp}{hyp}$$

$$\cos(\theta) = \frac{adj}{hyp}$$

$$\tan(\theta) = \frac{opp}{adj}$$

Quantity	Description	Dimension	Units
Velocity	$Displacement / Time$	$[v] = [LT^{-1}]$	m/sec
Acceleration	$Displacement / Time^2$	$[a] = [LT^{-2}]$	m/sec2
Momentum	$Mass \times Velocity$	$[p] = [MLT^{-1}]$	Kg m/sec
Force	$Momentum / Time$	$[F] = [MLT^{-2}]$	Newton
Pressure	$Force / Area$	$[P] = [ML^{-1}T^{-2}]$	Poise

Why Dimensional Analysis is Important

- ❑ Help to understand Physics , even if you are not Physicst
- ❑ Check your equation or Formula (Correct or not?!!!)
- ❑ Derive relation between physical quantities in
- ❑ physical phenomena
  - ✓ Equations must be dimensionally consistent?????
    - ❖ Means that each term in the equation have the same dimension

$$A + B = C + D$$

A dimension equation is said to be consistent, only and only if dimensions of equations are same on both sides. Also, remember that if an equation is dimensionally correct it doesn't mean it is a

Check the consistency of the equation

$x = x_0 + v_0t + (1/2)$ at

where x and x_0 are distances, t is time, v is velocity and a is an acceleration of the body.

Now to check if the above equation is dimensionally correct, we have to prove that dimensions of physical quantities are same on both sides. Also, we have to keep in mind that quantities can only be added or subtracted if their dimensions are same.

x = distance = [L]

x_0= distance = [L]

v_0t = velocity × time = $[LT^{-1}] \times [T] = [L]$

at = acceleration × time = $[LT^{-2}] \times [T] = [LT^{-1}]$

Since dimensions of at is not equals to the dimension of the other terms in the, so the equation is said to be inconsistent and dimensionally incorrect.

Example

Check whether the given equation is dimensionally correct.

$W = 1/2\ mv^2 - mgh$

where W stands for work done, m means mass, g stands for gravity, v for velocity and h for height.

To check the above equation as dimensionally correct, we first write dimensions of all the physical quantities mentioned in the equation.

$W = \text{Force} \times \text{Displacement} = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

$1/2\ mv^2 = \text{Kinetic Energy} = [M] \times [L^2T^{-2}] = [ML^2T^{-2}]$

$mgh = \text{Potential Energy} = [M] \times [LT^{-2}] \times [L] = [ML^2T^{-2}]$

Since all the dimensions on left and right sides are equal it is a dimensionally correct equation.

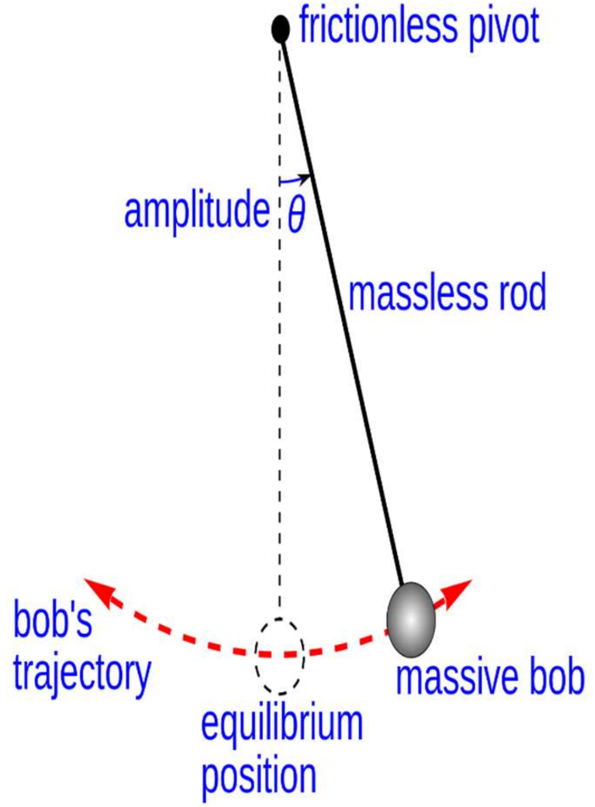
Use of DA

Derive relations between physical quantities involved in physical phenomena

The method of dimension analysis can also help in finding out relations between physical quantities. If we know how physical quantities depend on each other, we can find the relation between them easily by equating dimension on both sides.

Example

Suppose a bob is hanging from a ceiling (simple Pendulum) and time period of oscillations depends on 'length "l" of the thread, mass "m" of the bob and gravity "g"'. In order to find a relation between time and other physical quantities, we proceed as follows:



Let's time depends on powers x, y and z of length l, mass m and gravity g of the bob. Then the equation becomes

$T = k\, l^x m^y g^z$, where k is a proportionality constant

Writing dimensions on both sides, we get

Arranging powers accordingly we get

$[[M^0L^0T] = k\, [L]^x[M]^y[LT^{-2}]^z$

$M^0L^0T] = k\, [M^yL^{x+z}T^{-2z}]$

Equating powers on both sides, we get three equations

$y = 0$, $x + z = 0$, $-2z = 1$, Solving the linear equations we get

$x = 1/2$, $y = 0$ and $z = -1/2$

Substituting the values of x, y and z in the equation we have derived the following relation:

$$T = k\, L^{1/2}M^0g^{-1/2} = k\sqrt{\frac{l}{g}}$$

Example 1.6

Suppose we are told that the acceleration **a** of a particle moving with uniform speed **v** in a circle of radius r is proportional to some power of r, say r^n, and some power of **v**, say $\mathbf{v}^m$ How can we determine the values of n and m?

Example 1.7

If the frequency (f) of a simple harmonic motion of a pendulum can be written as,

$$f = m^{\alpha} g^{\beta} l^{\gamma}$$

where m is the mass of the oscillating pendulum, g is the acceleration due gravity, and l is the length of the pendulum. Find α, β, and γ.

Vector and Scalar Quantities

- **Vectors: magnitude + direction**
  - **Displacement**
  - **Velocity (magnitude and direction!)**
  - **Acceleration**
  - **Force**
  - **Momentum**

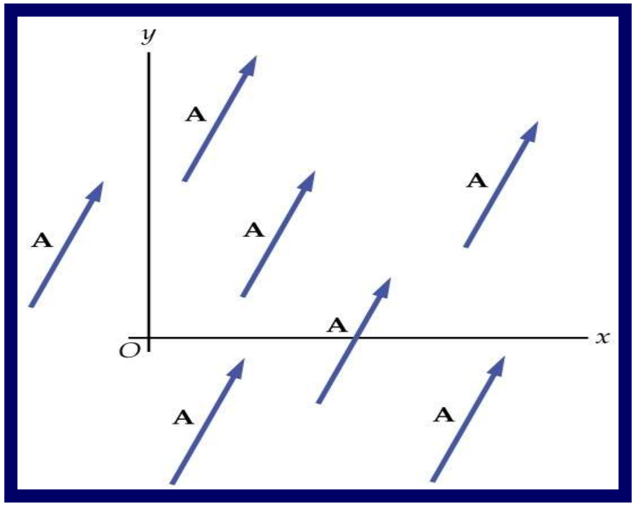
- Scalars: **magnitude**
  - Distance
  - Speed (magnitude of velocity)
  - Temperature
  - Mass
  - Energy
  - Time

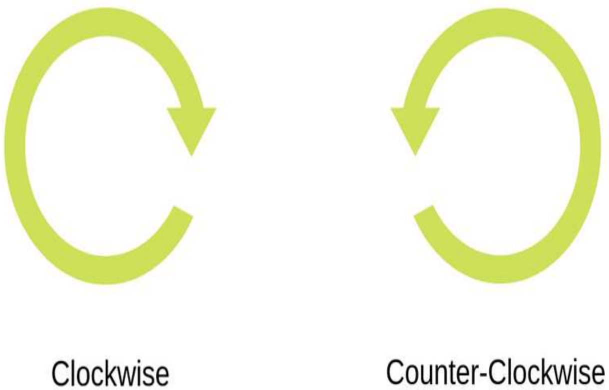
To describe a vector we need more information than to describe a scalar! Therefore vectors are more complex!

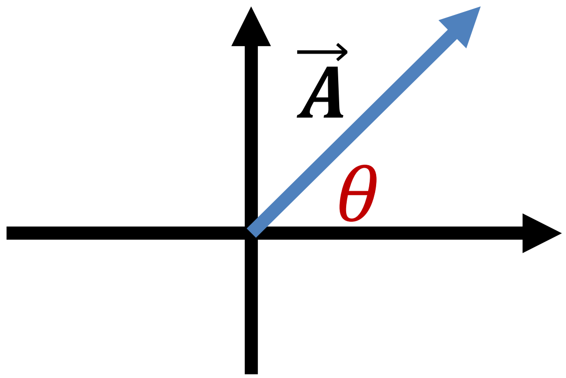
Important Notation

- To **describe vectors** we will use:
  - The bold font: Vector A is **A**
  - Or an arrow above the vector: $\vec{A}$
  - In the pictures, we will always show vectors as arrows
  - Arrows point the direction
  - To describe the magnitude of a vector we will use absolute value sign: $|\vec{A}|$ or just A,
  - The vector can be represented

    By its magnitude A, and its direction θ, (A, θ), where θ is the direction that the vector makes counterclockwise with the positive x- axis

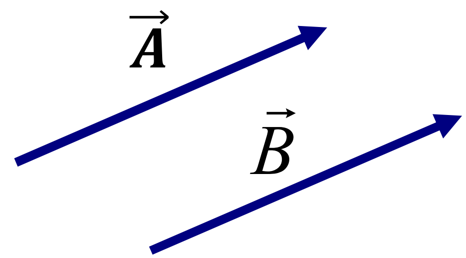


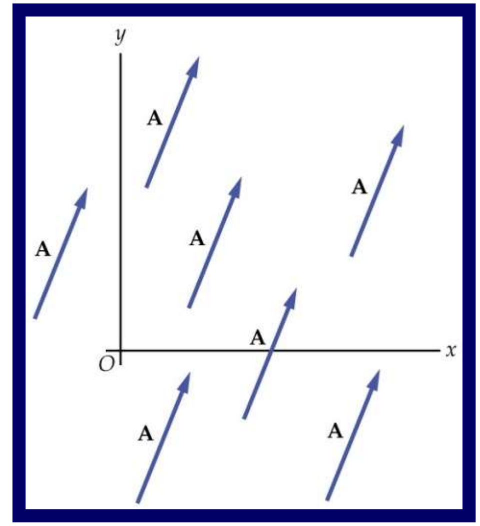




Properties of Vectors

- **Equality of Two Vectors**
  - Two vectors are equal if they have the same magnitude and the same direction
- **Movement of vectors in a diagram**
  - Any vector can be moved parallel to itself without being affected

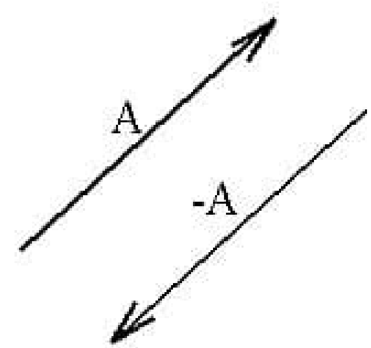




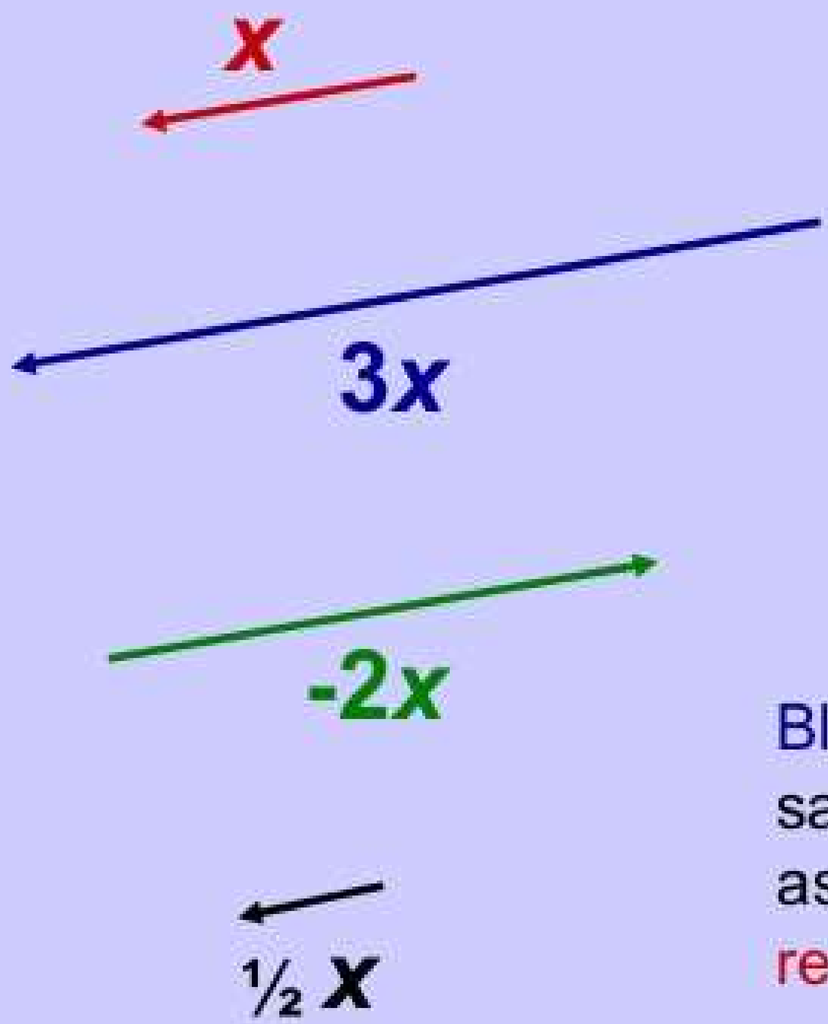
❑ Negative Vectors

- Two vectors are **negative** if they have the same magnitude but are 180° apart (opposite directions)

$$\vec{\mathbf{A}} = -\vec{\mathbf{B}};\ \vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$$



Scalar Multiplication

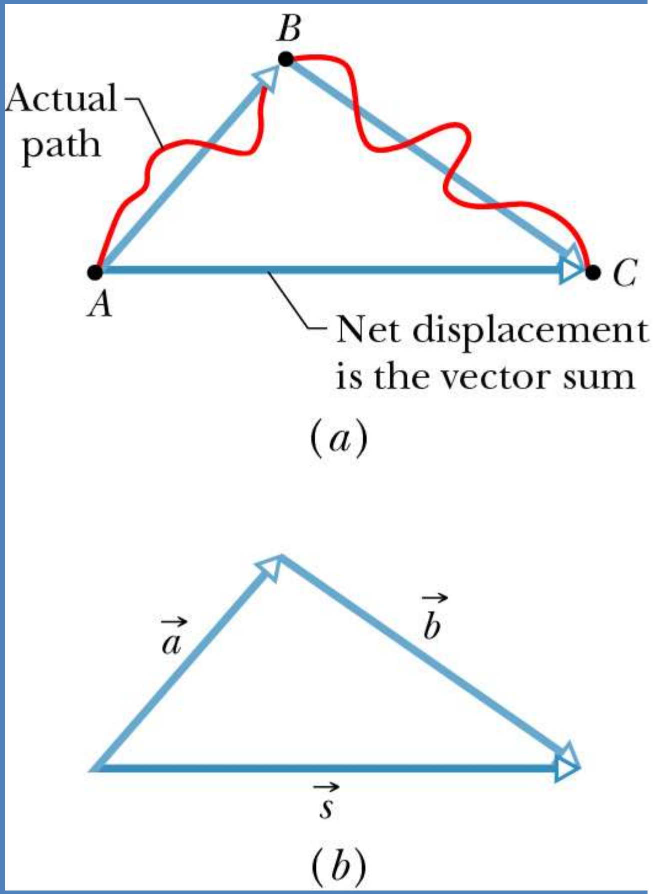


Scalar multiplication means multiplying a vector by a real number, such as 8.6. The result is a parallel vector of a different length. If the scalar is positive, the direction doesn't change. If it's negative, the direction is exactly opposite.

Blue is 3 times longer than red in the same direction. Black is half as long as red. Green is twice as long as red in the opposite direction.

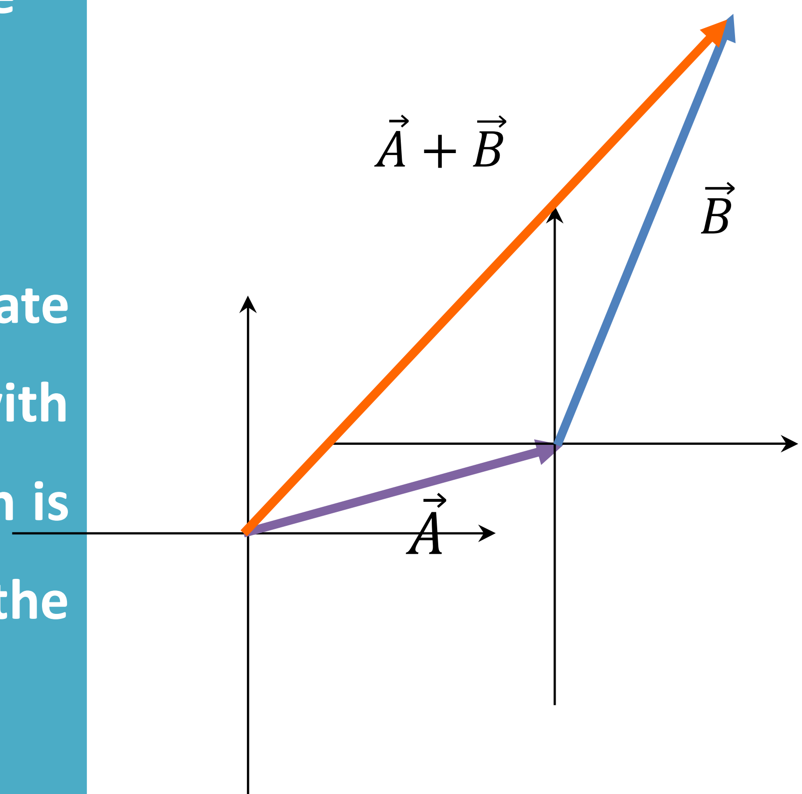
Adding Vectors (New Vector)

- When adding vectors, their directions must be taken into account
- Units must be the same
- Geometric Methods
  - Use scale drawings
- Algebraic Methods
  - More convenient



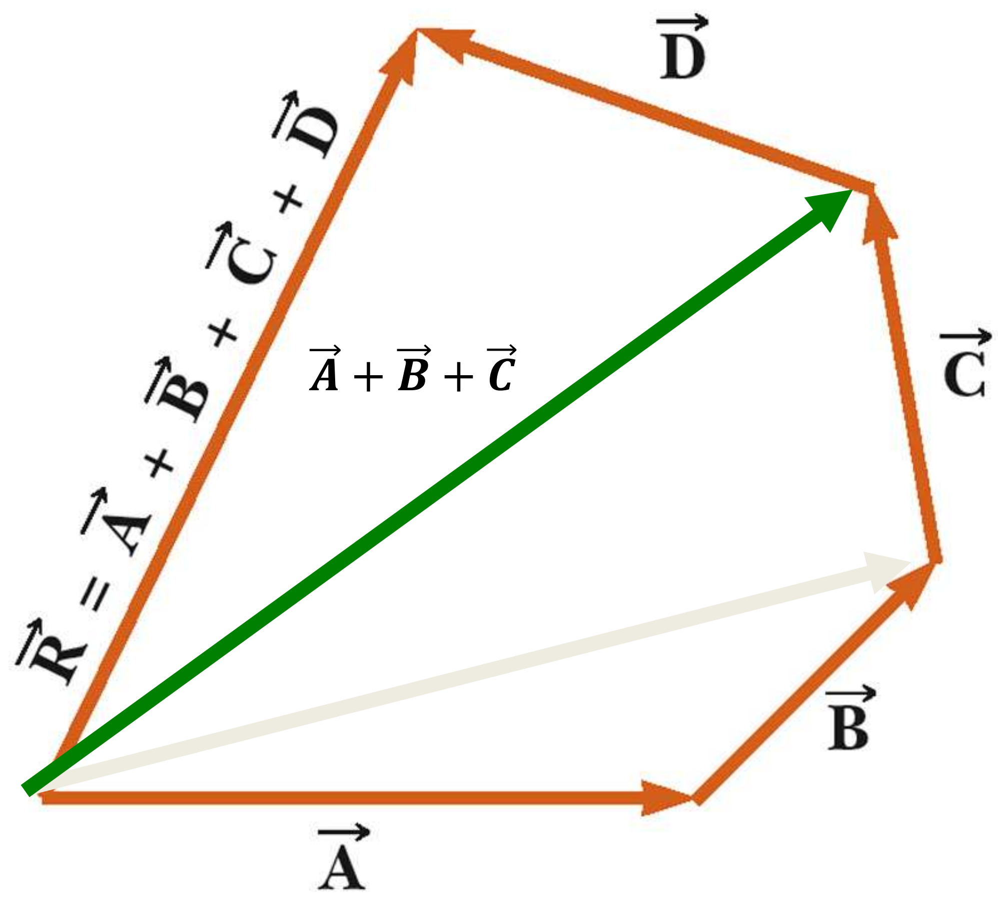
Adding Vectors Geometrically (Triangle Method)

- Draw the first vector $\vec{A}$ with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector $\vec{B}$ with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{A}$ and parallel to the coordinate system used for $\vec{A}$: “tip-to-tail”.
- The resultant is drawn from the origin of $\vec{A}$ to the end of the last vector $\vec{B}$



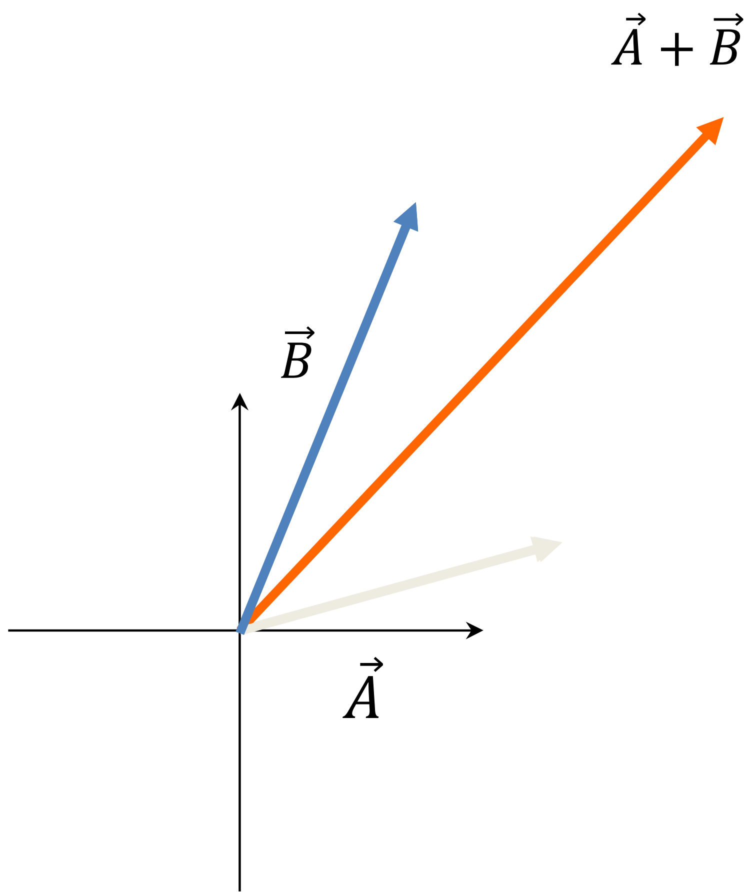
Adding Vectors Graphically

- **When you have many vectors, just keep repeating the process until all are included**
- **The resultant is still drawn from the origin of the first vector to the end of the last vector**



Adding vectors geometrically (polygon method)

- **Draw the first vector $\vec{A}$ with the appropriate length and in the direction specified, with respect to a coordinate system**
- **Draw the next vector $\vec{B}$ with the appropriate length and in the direction specified, with respect to the same coordinate system**
- **Draw a parallelogram**
- **The resultant is drawn as a diagonal from the origin**



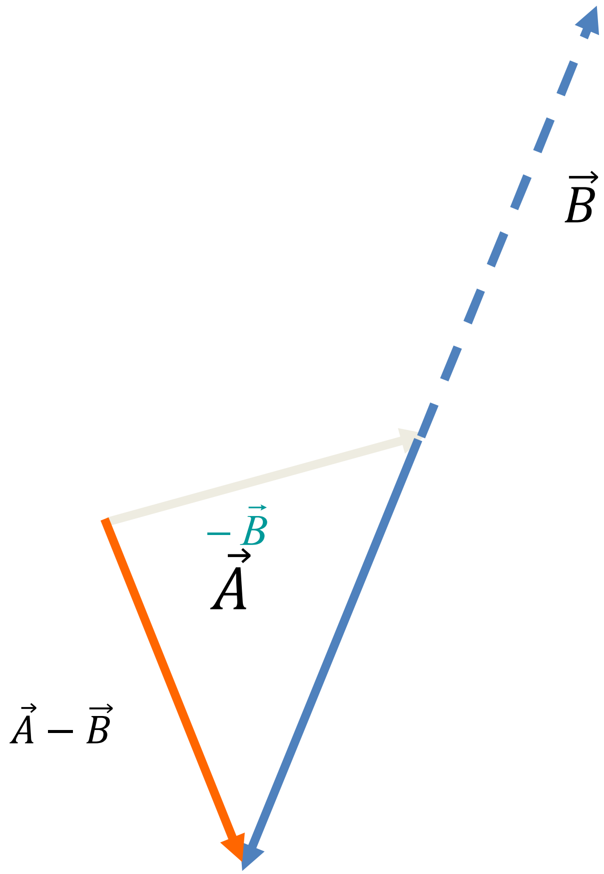
$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

Vector Subtraction

- Special case of vector addition
  - Add the negative of the subtracted vector

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$$

- Continue with standard vector addition procedure

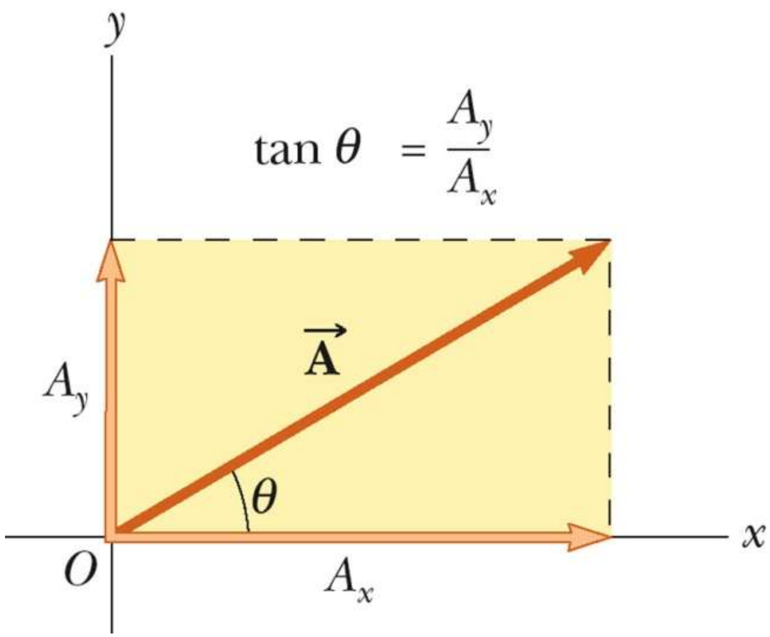


Vector Analysis
Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components** These are the projections of the vector along the x- and y-axes

A_x is the x-component
A_y is the y-component

$$\cos\theta = \frac{A_x}{A} \qquad \sin\theta = \frac{A_y}{A}$$



$$\begin{cases} A_x = A\cos(\theta) \\ A_y = A\sin(\theta) \end{cases}$$

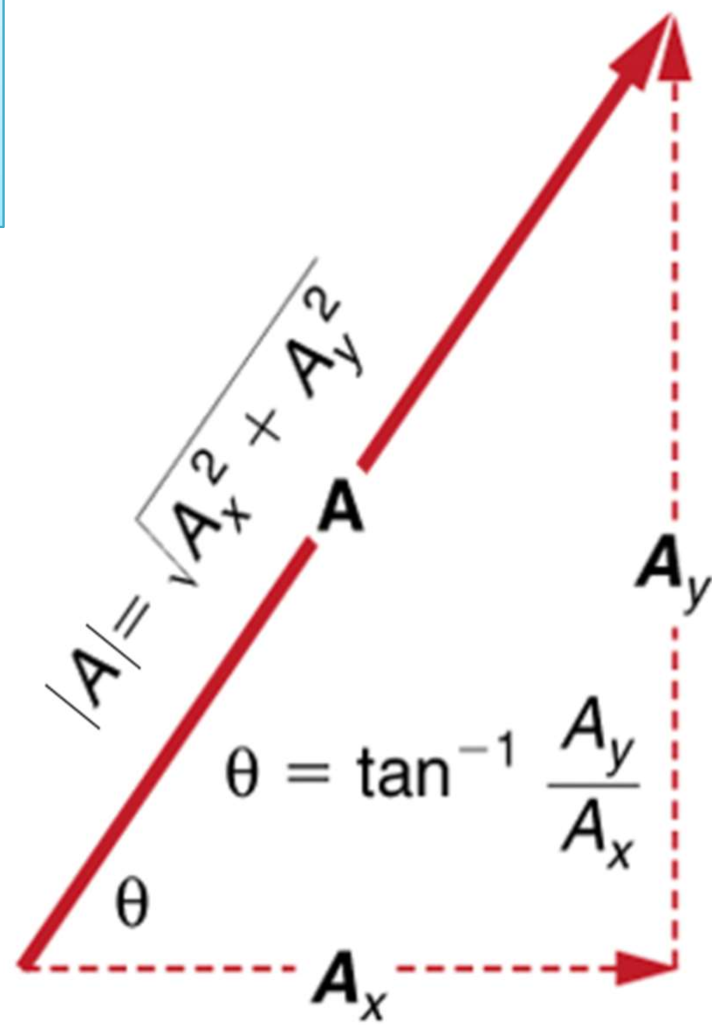
$$\begin{cases} \left|\vec{A}\right| = \sqrt{(A_x)^2 + (A_y)^2} \\ \tan(\theta) = \frac{A_y}{A_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \end{cases}$$

Components of a Vector

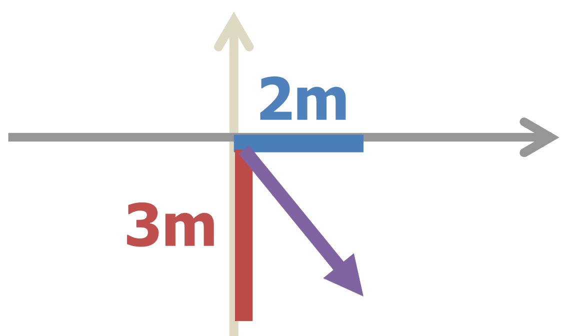
If we have a vector (4 m, 60°), *find the x and y component*

$A_x = A\cos\theta = 4\cos 60 = 4\,(0.5) = 2m$

$A_y = A\sin 60 = 4(\frac{\sqrt{3}}{2}) = 2\sqrt{3}\ m$



If we have a vector (2m, -3m), find its magnitude and direction

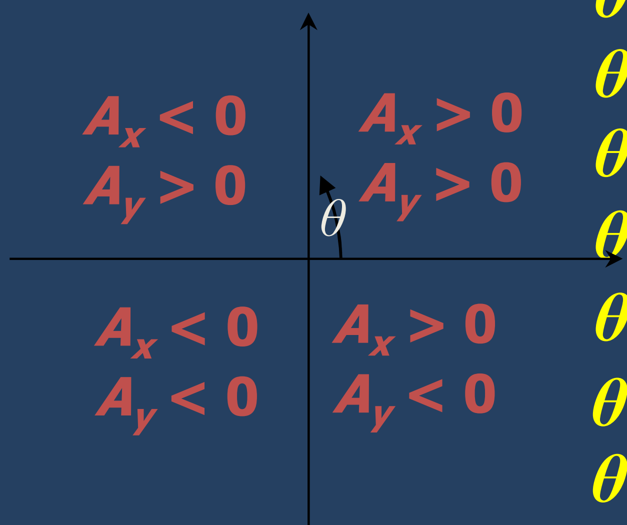


$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{4+9} = \sqrt{13}m$$

$$\theta = tan^{-1}(\frac{A_y}{A_x}) = tan^{-1}(\frac{-3}{2}) = -56.3^{\circ} = 303.7^{\circ}$$

Components of a Vector

- The previous equations are valid *only if ϑ is measured with respect to the x-axis*
- The components can be positive or negative and will have the same units as the original vector



$\theta=0,\ A_x=A>0,\ A_y=0$

$\theta=45^\circ,\ A_x=A\cos 45^\circ>0,\ A_y=A\sin 45^\circ>0$

$\theta=90^\circ,\ A_x=0,\ A_y=A>0$

$\theta=135^\circ,\ A_x=A\cos 135^\circ<0,\ A_y=A\sin 135^\circ>0$

$\theta=180^\circ,\ A_x=-A<0,\ A_y=0$

$\theta=225^\circ,\ A_x=A\cos 225^\circ<0,\ A_y=A\sin 225^\circ<0$

$\theta=270^\circ,\ A_x=0,\ A_y=-A<0$

$\theta=315^\circ,\ A_x=A\cos 315^\circ<0,\ A_y=A\sin 315^\circ<0$

Addition and subtraction of Vectors

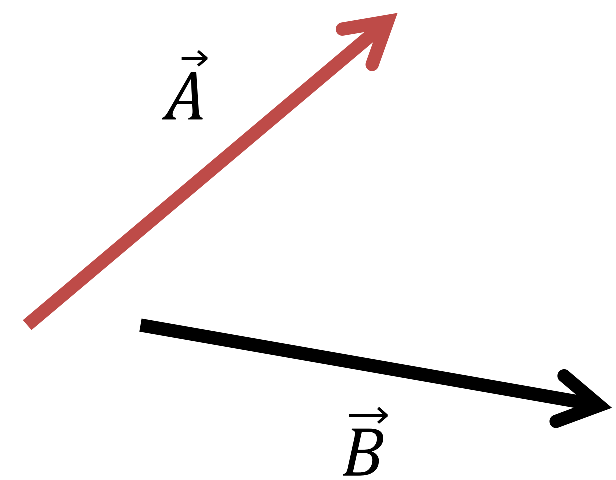
$\vec{A}$: (A_x, A_y) $\vec{B}$: (B_x, B_y)

$\vec{A} \mp \vec{B} = \vec{R}$, where $\vec{R}$: (R_x, R_y)

$R_x = A_x \mp B_x$,

$R_y = A_y \mp B_y$

$$\boldsymbol{R} = \sqrt{\boldsymbol{R_x^2} + \boldsymbol{R_y^2}}$$



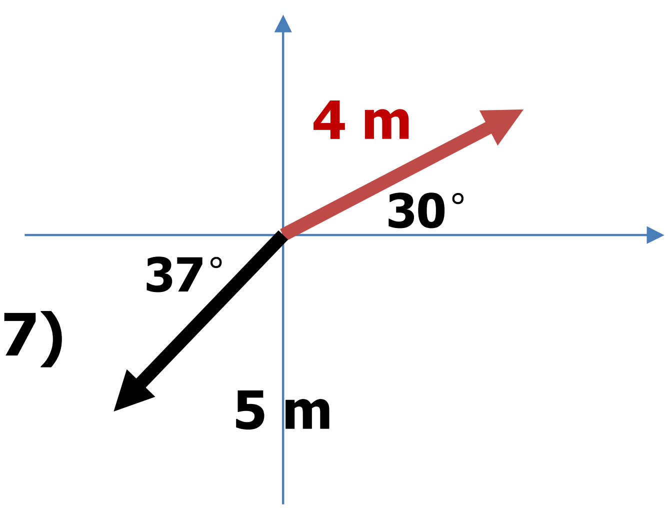
Two vectors $\vec{A}$: (4 m, 30°) and $\vec{B}$: (5 m, 217°), find $\vec{A} - \vec{B}$

$\vec{A}$: (4 m, 30°)= (4cos30, 4sin30)

$\vec{B}$: (5 m, 217°) =(- 5cos37, -5sin37)

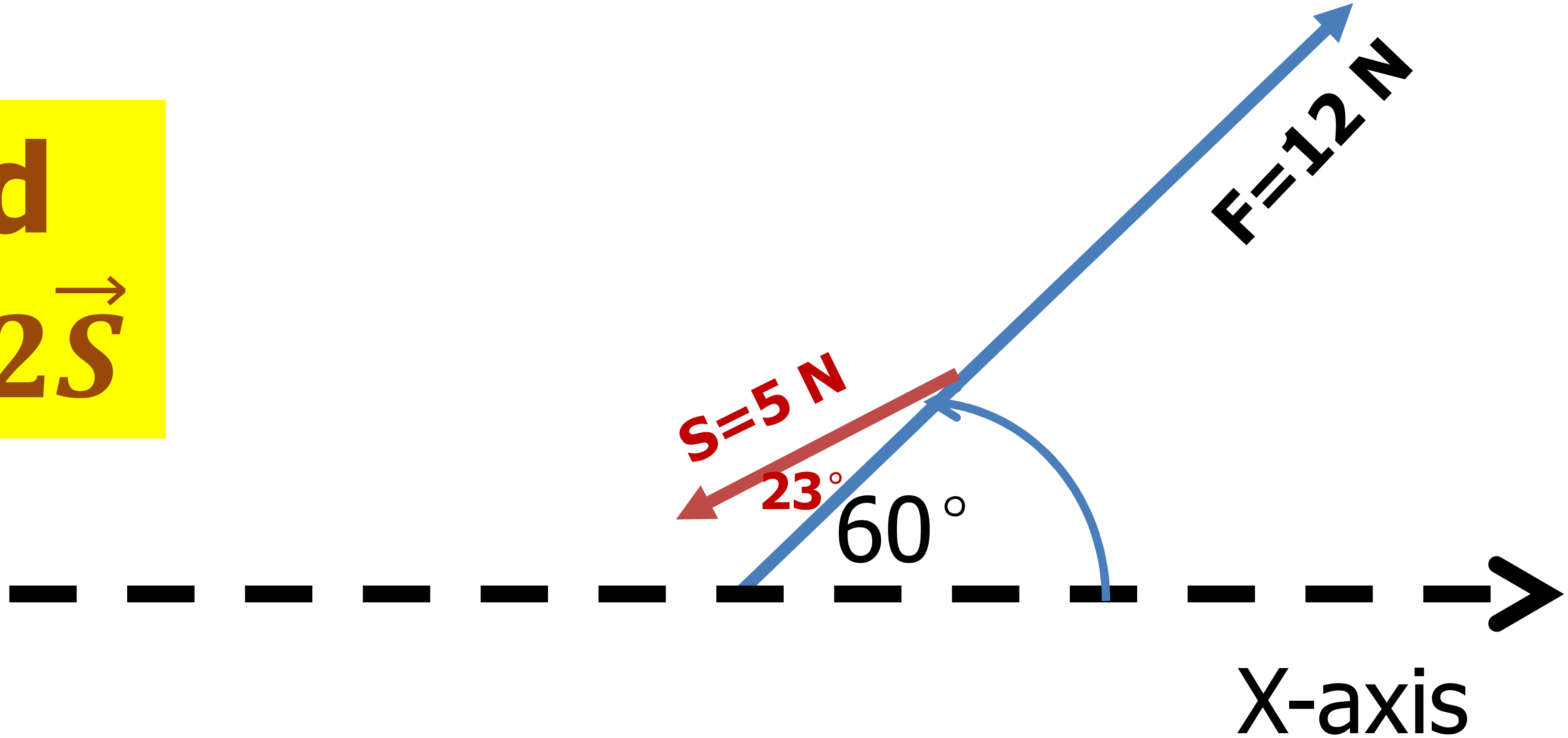
$\vec{A} - \vec{B} = \vec{R}$: ($R_x$, R_y)

= (4cos30+5cos37, 4sin30+ 5sin37)



Find

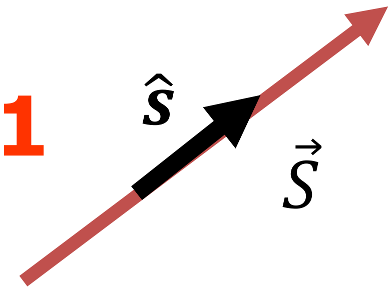
$$\vec{F} + 2\vec{S}$$



Unit Vector

A **unit vector** is a vector with a magnitude of one unit. Any vector has an associated unit vector in the same direction but having a magnitude of one. Any vector can be expressed as a scalar multiple of its unit vector.

$$\hat{s} = \frac{\vec{s}}{s}, \text{ and } |\hat{s}| = \frac{|\vec{s}|}{s} = 1$$

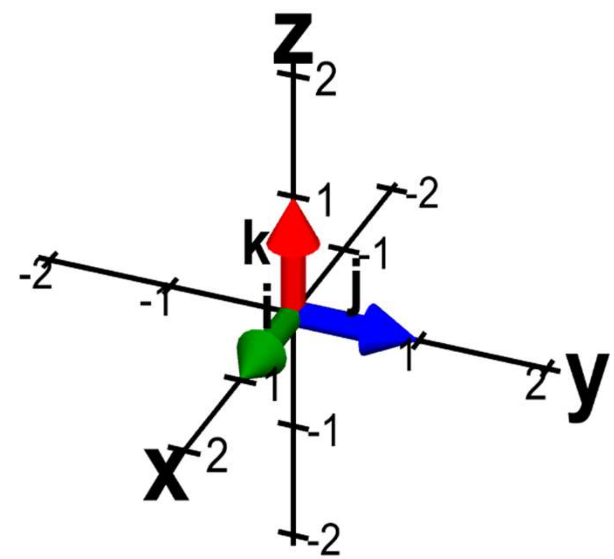


- Unit vectors *i*-hat, *j*-hat, *k*-hat

  $\hat{i} \rightarrow x \quad \hat{j} \rightarrow y \quad \hat{k} \rightarrow z$

- Unit vectors used to specify direction
- Unit vectors have a magnitude of 1
- Then $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Example : Operations with Vectors

- Vector **A** is described algebraically as **(-3, 5),** while vector **B** is **(4, -2).** Find the value of **magnitude** and **direction** of the sum (**C**) of the vectors **A** and **B**.

$$\vec{A} = -3\hat{\imath} + 5\hat{\jmath} \qquad \vec{B} = 4\hat{\imath} - 2\hat{\jmath}$$

$$\vec{C} = \vec{A} + \vec{B} = (-3 + 4)\hat{\imath} + (5 - 2)\hat{\jmath} = 1\hat{\imath} + 3\hat{\jmath}$$

$$C_x = 1 \qquad C_y = 3$$

$$C = (C_x{}^2 + C_y{}^2)^{1/2} = (1^2 + 3^2)^{1/2} = 3.16$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1} 3 = 71.56^\circ$$

Example

If we have two vectors $\vec{C} = 2i - 3j + 5k$ and $\vec{D} = -i + 5j - k.$ find
a) $\vec{C} + 2\vec{D}$, b) the unit vector in the direction of $\vec{D} + 2\vec{C}$

a)

$$\vec{C} + 2\vec{D} = \vec{M}$$

where $\vec{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$

$$M_x = C_x + 2D_x = 2 + (-2) = 0$$

$$M_y = C_y + 2D_y = -3 + (10) = 7$$

$$M_z = C_z + 2D_z = 5 + (-2) = 3$$

$$\therefore \vec{C} + 2\vec{D} = \boxed{\vec{M} = 7\hat{j} + 3\hat{k}}$$

b)

$D + 2\vec{C} = \vec{F}$, $\vec{F} = (-\hat{i} + 4\hat{i}) + (5\hat{j} - 6\hat{j}) + (-\hat{k} + 10\hat{k})$

$\vec{F} = -5\hat{i} - \hat{j} + 9\hat{k}$, $F = \sqrt{25 + 1 + 81} = \sqrt{107}$

unit vector $\hat{f} = \dfrac{\vec{F}}{F} = \dfrac{1}{\sqrt{107}}(-5\hat{i} - \hat{j} + 9\hat{k})$

Adding Vectors Algebraically

- Consider two vectors

$$\vec{\boldsymbol{A}} = \boldsymbol{A_x}\hat{\boldsymbol{\imath}} + \boldsymbol{A_y}\hat{\boldsymbol{\jmath}} + \boldsymbol{A_z}\hat{\boldsymbol{k}}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + \boldsymbol{B_z}\hat{\boldsymbol{k}}$$

- Then

$$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \left(\boldsymbol{A_x}\hat{\boldsymbol{\imath}} + \boldsymbol{A_y}\hat{\boldsymbol{\jmath}} + \boldsymbol{A_z}\hat{\boldsymbol{k}}\right) + \left(\boldsymbol{B_x}\hat{\boldsymbol{\imath}} + \boldsymbol{B_y}\hat{\boldsymbol{\jmath}} + \boldsymbol{B_z}\hat{\boldsymbol{k}}\right)$$
$$= (\boldsymbol{A_x} + \boldsymbol{B_x})\hat{\boldsymbol{\imath}} + (\boldsymbol{A_y} + \boldsymbol{B_y})\hat{\boldsymbol{\jmath}} + \widehat{(\boldsymbol{A_z} + \boldsymbol{B_z})\boldsymbol{k}}$$

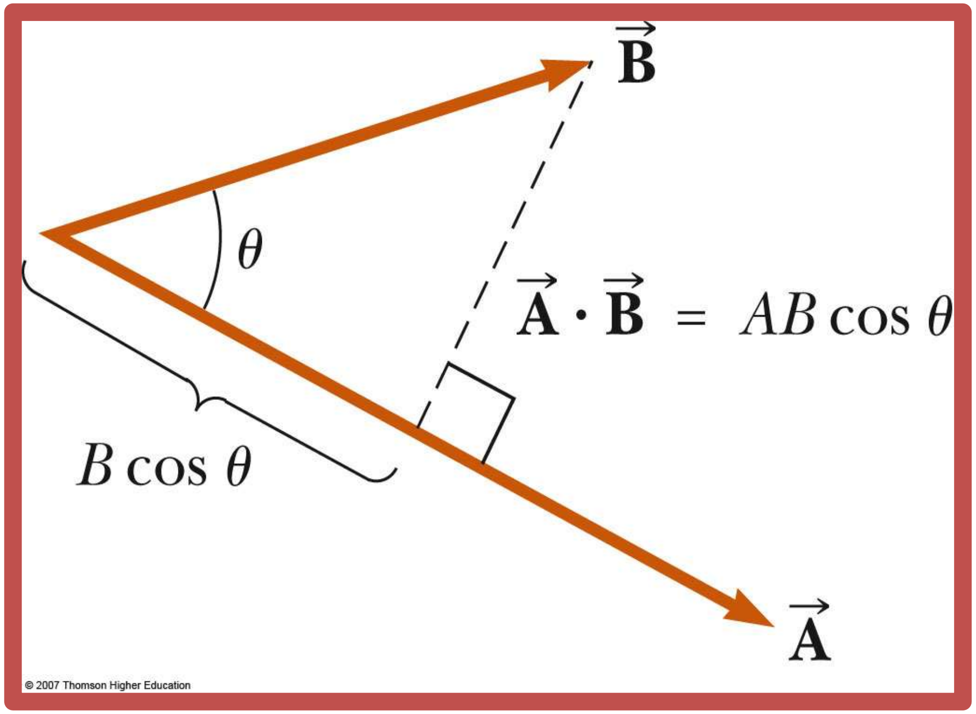
- If $\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + \widehat{(\boldsymbol{A_z} + \boldsymbol{B_z})\boldsymbol{k}}$
- so $C_x = A_x + B_x \qquad C_y = A_y + B_y$

$$C_z = (\boldsymbol{A_z} + \boldsymbol{B_z})$$

Scalar Product of Two Vectors

The scalar product, or dot product, is a mathematical operation used to determine the component of a given force in the direction of the displacement

- **The scalar product of two vectors is written as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$**
  - **It is also called the dot product**
- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\,\cos\,\theta$
  - **θ is the angle *between* *A* and *B***
- **Applied to work, this means**



$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

Properties of Scalar Products

If	Then
$\vec{A}$ and $\vec{B}$ are $\perp$	$\vec{A}\cdot\vec{B}=0$
$\vec{A}$ and $\vec{B}$ are parallel	$\vec{A}\cdot\vec{B}=AB$
$\vec{A}\cdot\vec{B}=0$	Either $\vec{A}=0$ or $\vec{B}=0$ or $\vec{A}\perp\vec{B}$

Furthermore,

$\vec{A}\cdot\vec{A}=A^2$ Because $\vec{A}$ is parallel to itself

$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$ Commutative rule of multiplication

$(\vec{A}+\vec{B})\cdot\vec{C}=\vec{A}\cdot\vec{C}+\vec{B}\cdot\vec{C}$ Distributive rule of multiplication

For unit vectors

$\hat{\imath}.\hat{\imath}=\hat{\jmath}.\hat{\jmath}=\hat{k}.\hat{k}=1$, each unit vector parallel with itself

$\hat{\imath}.\hat{\jmath}=\hat{\jmath}.\hat{k}=\hat{\imath}.\hat{k}=0$, as they are perpendicular to each other

Dot Product

- The dot product says something about how parallel two vectors are.
- The dot product (scalar product) of two vectors can be thought of as the projection of one onto the direction of the other.

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$
$$\vec{A} \cdot \hat{\imath} = A\cos\theta = A_x$$

- Components

$$\boldsymbol{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

Derivation

- How do we show that $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
- Start with $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

  $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

- Then
$$\begin{aligned}\vec{A} \cdot \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= A_x \hat{i} \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) + A_y \hat{j} \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) + A_z \hat{k} \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})\end{aligned}$$
- But
$$\hat{i} \cdot \hat{j} = 0; \;\; \hat{i} \cdot \hat{k} = 0; \;\; \hat{j} \cdot \hat{k} = 0$$
$$\hat{i} \cdot \hat{i} = 1; \;\; \hat{j} \cdot \hat{j} = 1; \;\; \hat{k} \cdot \hat{k} = 1$$
- So
$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_x \hat{i} \cdot B_x \hat{i} + A_y \hat{j} \cdot B_y \hat{j} + A_z \hat{k} \cdot B_z \hat{k} \\ &= A_x B_x + A_y B_y + A_z B_z\end{aligned}$$

Example 1.17

Consider the two vectors **a** and **b**. Suppose **a** has modulus 4 units, **b** has modulus 5 units, and the angle between them is 60° . Find the scalar product of the two vectors.

Example 1.19

Suppose we wish to test whether or not the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, where

$\mathbf{a} = 3i + 2j - k$ and $\mathbf{b} = i - 2j - k$

Given the vectors: $\mathbf{A} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = 5\mathbf{i} + 5\mathbf{j}$, find:

a. The dot product $\mathbf{A} \cdot \mathbf{B}$.

b. The projection of **A** onto **B**.

c. The angle between **A** and **B**.

d. A vector of magnitude 2 in the XY plane perpendicular to **B**.

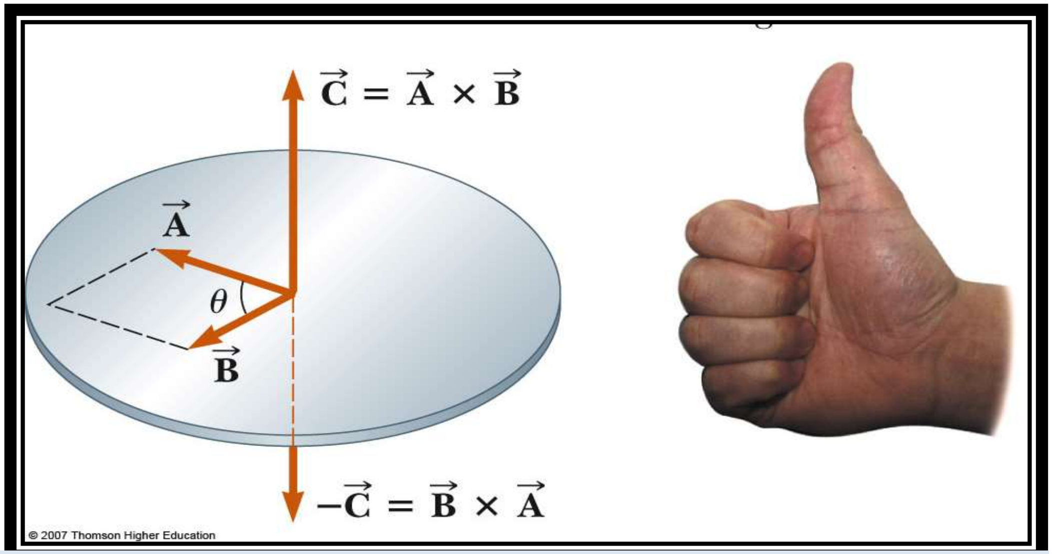
e. Find the angles that the vector A+B makes with the X,Y and Z axises

Cross Product

The vector product or cross product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them. The direction of the product vector is perpendicular to the plane containing the two vectors, in accordance with the right hand screw rule or right hand thumb rule

$$\vec{C} = \vec{A} \times \vec{B}$$

$$|\vec{C}| = |\vec{A} \times \vec{B}| = AB \sin\theta$$



- **Direction: C perpendicular to both A and B First practice**

$$\vec{A} \times \vec{B} = \vec{B} \times \vec{A}\ ?$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

The cross product of two vectors results in a new vector.

- θ is smaller angle between the vectors
- Cross product of any parallel vectors = zero
- Cross product is maximum for perpendicular vectors

Cross Product

$$\vec{C} = \vec{A} \times \vec{B}$$

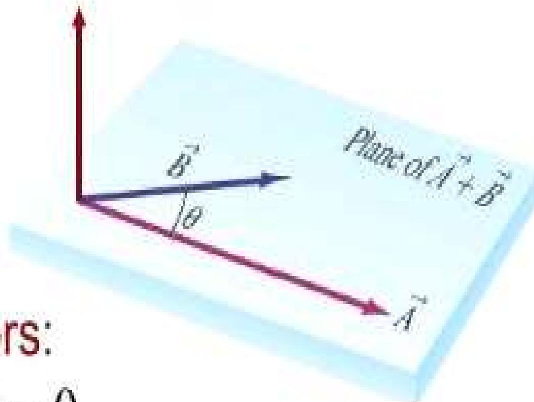
- Cross product:

$$\mathbf{A} \times \mathbf{B} = \mathbf{C} \Rightarrow C = AB\sin\theta;\ \ \mathbf{C} \perp \mathbf{A};\ \ \mathbf{C} \perp \mathbf{B}$$

- Properties:

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$



- Parallel and orthogonal vectors:

$$\mathbf{A} \parallel \mathbf{B} \Rightarrow \sin\theta = 0 \Rightarrow \mathbf{A} \times \mathbf{B} = 0$$

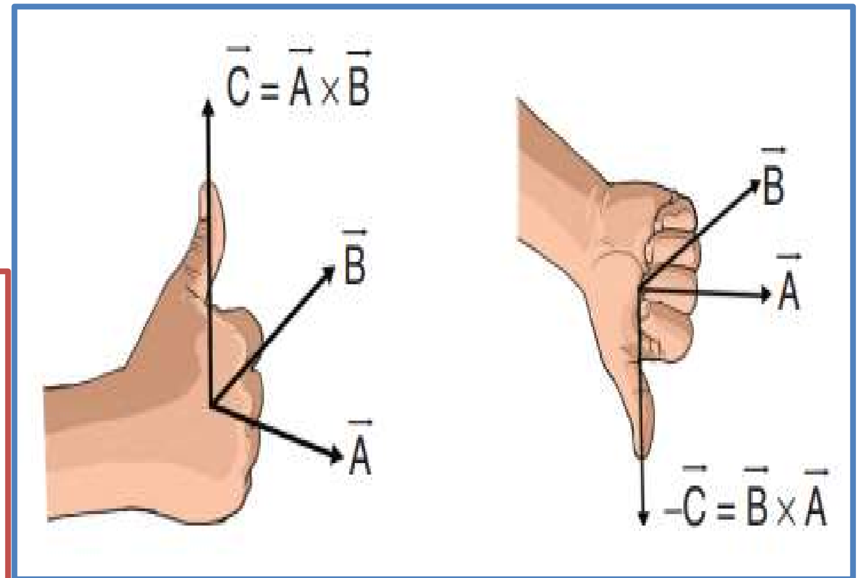
$$\mathbf{A} \perp \mathbf{B} \Rightarrow \sin\theta = 1 \Rightarrow |\mathbf{A} \times \mathbf{B}| = AB$$

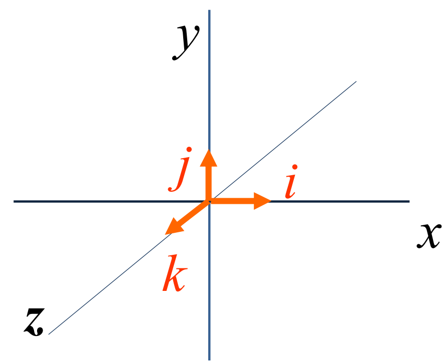
- Cross product of a vector with itself:

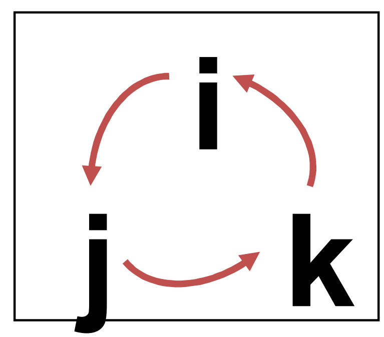
$$\mathbf{A} \times \mathbf{A} = 0$$

$$\hat{i} \times \hat{j} = \hat{k};\ \ \hat{i} \times \hat{k} = -\hat{j};\ \ \hat{j} \times \hat{k} = \hat{i}$$

$$\hat{i} \times \hat{i} = 0;\ \ \hat{j} \times \hat{j} = 0;\ \ \hat{k} \times \hat{k} = 0$$







More about Cross Product

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

- How do we show that $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$
- Start with

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

- Then

$$\begin{aligned}\vec{A} \times \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath} \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) + A_y\hat{\jmath} \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) + A_z\hat{k} \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})\end{aligned}$$

- But

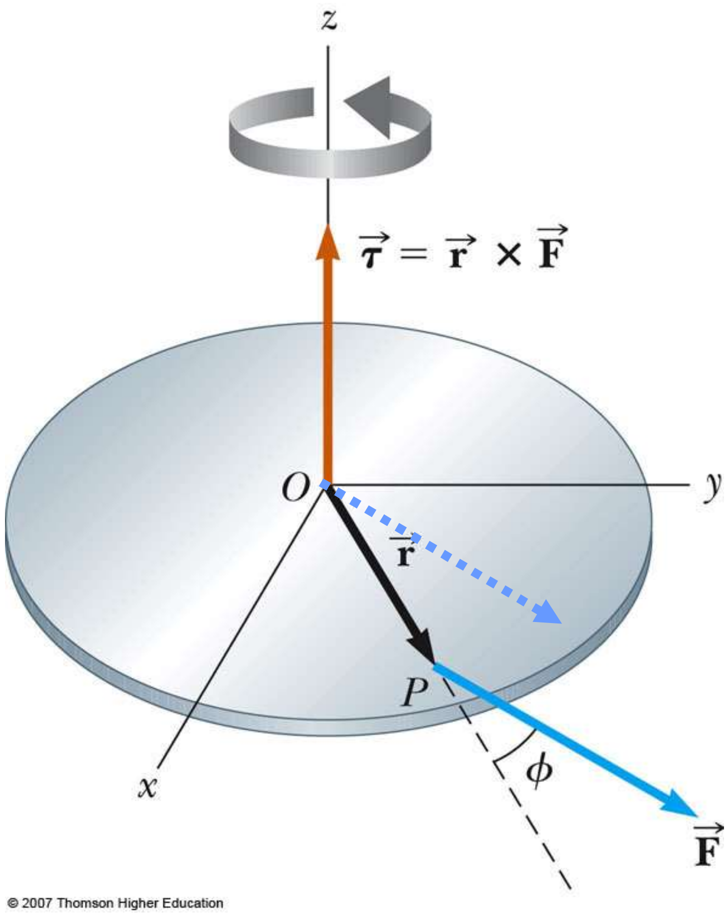
$$\hat{\imath} \times \hat{\jmath} = \hat{k};\ \ \hat{\imath} \times \hat{k} = -\hat{\jmath};\ \ \hat{\jmath} \times \hat{k} = \hat{\imath}$$
$$\hat{\imath} \times \hat{\imath} = 0;\ \ \hat{\jmath} \times \hat{\jmath} = 0;\ \ \hat{k} \times \hat{k} = 0$$

- So

$$\vec{A} \times \vec{B} = A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k} + A_y\hat{\jmath} \cdot B_x\hat{\imath} + A_y\hat{\jmath} \cdot B_z\hat{k}$$
$$+A_z\hat{k} \cdot B_x\hat{\imath} + A_z\hat{k} \cdot B_y\hat{\jmath}$$

Torque as a Cross Product

$$\vec{\tau} = \vec{r} \times \vec{F}$$



- The torque is the cross product of a force vector with the position vector to its point of application

$$|\tau| = rF\sin\theta = r_\perp F = r F_\perp$$

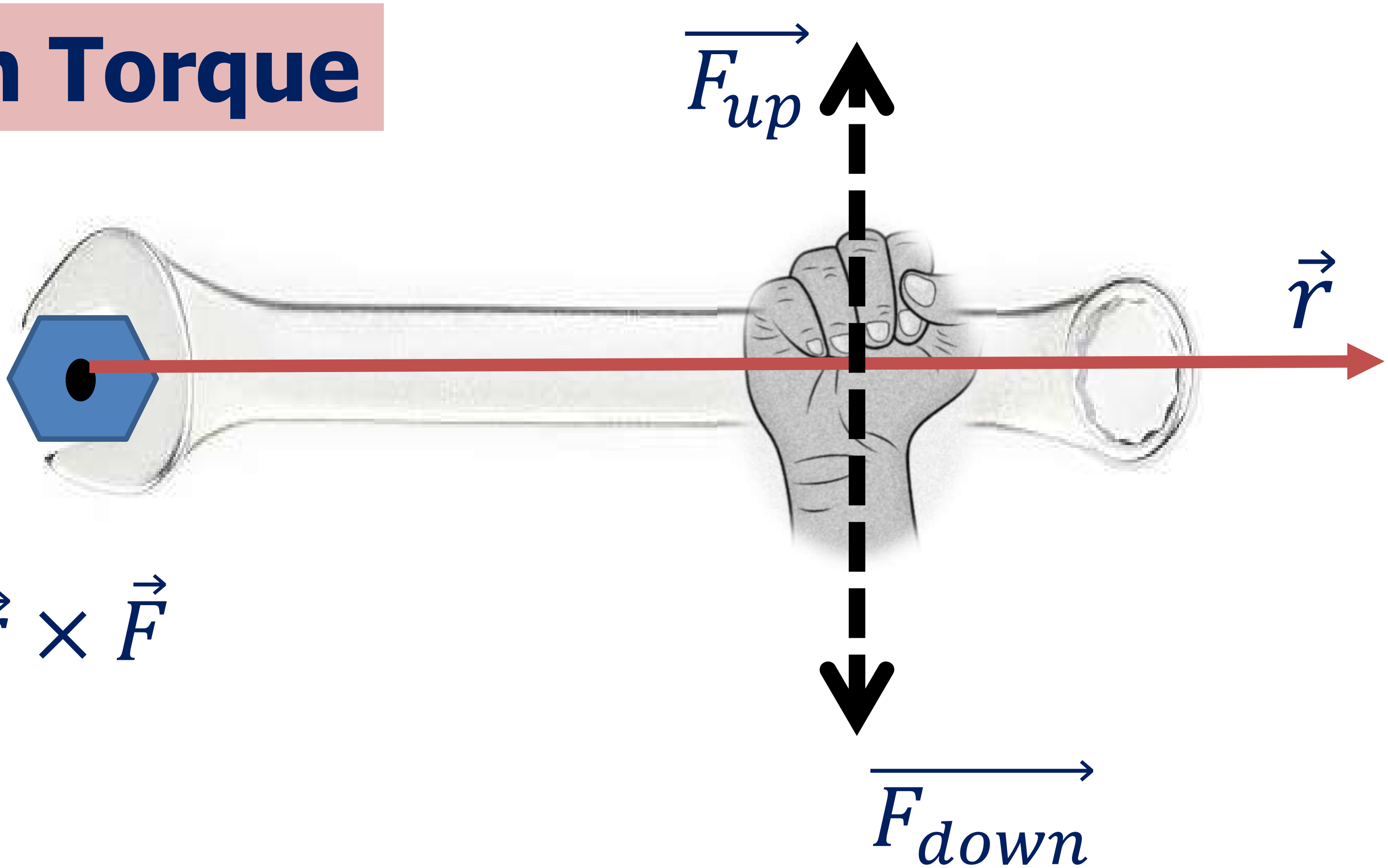
- The torque vector is perpendicular to the plane formed by the position vector and the force vector (e.g., imagine drawing them tail-to-tail)
- Right Hand Rule: curl fingers from r to F, thumb points along torque.

$$\vec{\tau}_{net} = \sum_{all\ i} \vec{\tau}_i = \sum_{all\ i} \vec{r}_i \times \vec{F}_i \ \text{(vector sum)}$$

Superposition:

- **Can have multiple forces applied at multiple points.**
- **Direction of τ_{net} is angular acceleration axis**

Wrench Torque



$$\vec{\tau} = \vec{r} \times \vec{F}$$

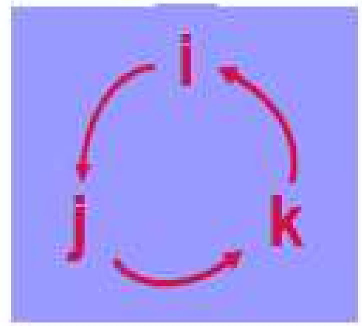
$\overrightarrow{F_{up}}$ cause a counterclockwise rotation, then the torque is positive

$\overrightarrow{F_{down}}$ cause a clockwise rotation, then the torque is negative

Calculating Cross Products

Find: $\vec{A}\times\vec{B}$ **Where:** $\vec{A}=2\hat{i}+3\hat{j}$ $\vec{B}=-\hat{i}+2\hat{j}$

Solution:
$$\vec{A}\times\vec{B}=(2\hat{i}+3\hat{j})\times(-\hat{i}+2\hat{j})$$
$$=2\hat{i}\times(-\hat{i})+2\hat{i}\times2\hat{j}+3\hat{j}\times(-\hat{i})+3\hat{j}\times2\hat{j}$$
$$=0+4\hat{i}\times\hat{j}-3\hat{j}\times\hat{i}+0=4\hat{k}+3\hat{k}=7\hat{k}$$



Calculate torque given a force and its location

$$\vec{F}=(2\hat{i}+3\hat{j})N \qquad \vec{r}=(4\hat{i}+5\hat{j})m$$

Solution:
$$\vec{\tau}=\vec{r}\times\vec{F}=(4\hat{i}+5\hat{j})\times(2\hat{i}+3\hat{j})$$
$$=4\hat{i}\times2\hat{i}+4\hat{i}\times3\hat{j}+5\hat{j}\times2\hat{i}+5\hat{j}\times3\hat{j}$$
$$=0+4\hat{i}\times3\hat{j}+5\hat{j}\times2\hat{i}+0=12\hat{k}-10\hat{k}=2\hat{k}\ (\mathrm{Nm})$$

Determinent Solution

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{\imath} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} + (-\hat{\jmath}) \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$\begin{aligned} \vec{A} \times \vec{B} \quad = \quad & (A_y B_z - A_z B_y)\,\hat{\imath} \quad + \\ & (A_z B_x - A_x B_z)\,\hat{\jmath} \quad + \\ & (A_x B_y - A_y B_x)\,\hat{k} \end{aligned}$$

Two vectors u= i+2j+k, and v= -j+2k, find uxv

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} - \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix}$$

$$= i\,(2 \times 2 - 1 \times (-1)) - j\,(1 \times 2 - 1 \times 0) + k\,(1 \times (-1) - 2 \times 0)$$

$$= 5i - 2j - 1k = (5, -2, -1)^T$$

Given the vectors $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{B} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ find $\mathbf{A} \times \mathbf{B}$ and $|\mathbf{A} \times \mathbf{B}|$.

Answer 1:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 4 \\ 3 & 1 & -2 \end{vmatrix}$$

$$= \mathbf{i}(4-4) + \mathbf{j}(12+2) + \mathbf{k}(1+6)$$

$$= 14\mathbf{j} + 7\mathbf{k}$$

$$|\mathbf{A} \times \mathbf{B}| = \sqrt{196+49} = \sqrt{245} = 15.7$$

24. For the following three vectors $\mathbf{A} = -i + 2j$, $\mathbf{B} = i - 2k$, *and* $\mathbf{C} = -2j + 2k$ find,

1. The angle between the vectors $\mathbf{D} = \mathbf{A} + \mathbf{B}$, *and* $\mathbf{T} = \mathbf{A} - \mathbf{C}$

2. find the vectors $\mathbf{E} = \mathbf{A} \times \mathbf{C}$, *and* $\mathbf{F} = \mathbf{B} \times \mathbf{C}$, and the unit vectors $\hat{E}$

25. When two vectors A and B are drawn from a common point, the angle between them is θ . Show that the magnitude of their vector sum is given by $\sqrt{A^2 + B^2 + 2AB\cos\theta}$. If **A** and **B** have the same magnitude, under what condition will their vector sum have the same have the magnitude of the vector **A** or **B**?

Two vectors $\vec{A}$ and $\vec{B}$ have the magnitudes of 2 and 5 respectively, and the magnitude of their vector product is 6. Find the magnitude of their scalar product.

A) A sailboat sails 2 km east, then 4 km southeast, and then an additional unknown displacement. Its final position is 6 km directly east of the starting point. Find the third **unknown displacement** of the journey.

B) Two vectors $\vec{A} = \hat{i} - 3\hat{k}$, $\vec{B} = 3\hat{j} - \hat{k}$, find the **unit vector** in the direction of $\vec{B} \times \vec{A}$

If $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$, $\vec{B} = \hat{i} + \hat{j}$, find

a) a third vector $\vec{C}$ has the magnitude of $\vec{A}$ and make an angle 90° to the vector $\vec{B}$.

b) The angle between $(\vec{A} \times \vec{B})$ *and* $(\vec{A} - \vec{C})$

Three vectors A= 2m, B= 4m and C= 3m. Vector A lies in the x-z plane make an angle 30° with the z-axis, Vector B lies in the y-z plane make an angle 60° with the y-axis and Vector C lies parallel to the y-axis. Find

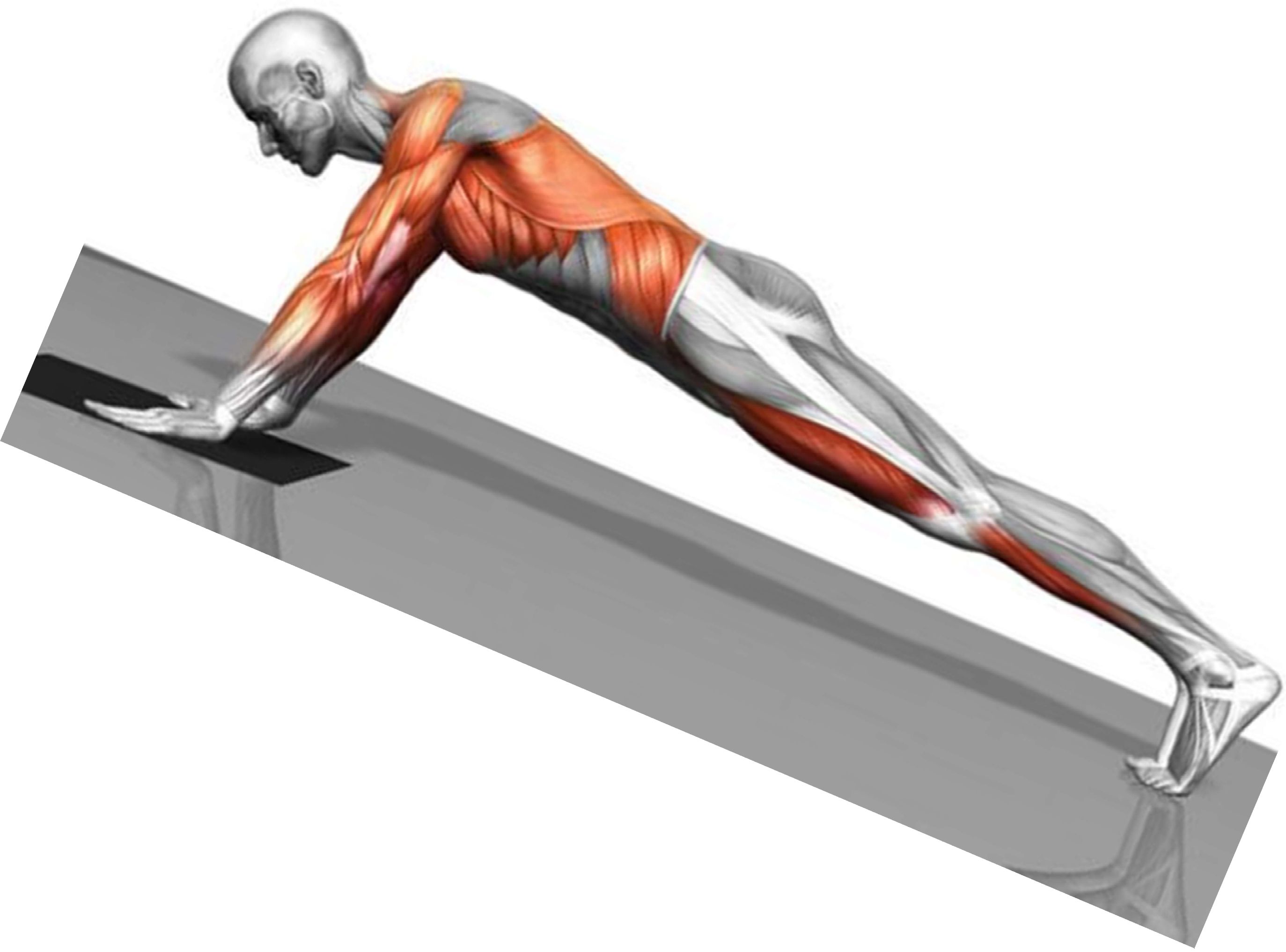
a) $\vec{B} \cdot (\vec{A} \times \vec{C})$

b) The angle that the vector $\vec{A} - \vec{C}$ make with the z-axis

Vector $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and vector $\vec{B} = -2\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$, Find a third vector in the 3D-space has double of magnitude and perpendicular to the vector ($\vec{A} - \vec{B}$) and make an angle 60° with the Y- axis.

Two vectors $\vec{A}$ and $\vec{B}$ have the same length and located in the xz plane are drawn from a common point. The Vector $\vec{B}$ is 60° above vector $\vec{A}$. If their vector sum equal 4 m, what is the magnitude of each of them. Find the unit vector in the direction perpendicular to $\vec{A}$ and $\vec{A} + \vec{B}$.

Chapter 2
Biostatic



MUSCULOSKELETAL SYSTEM:

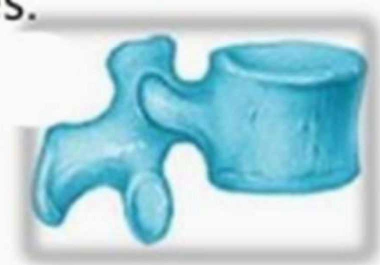
The combination of bones, joints, muscles and related connective tissues are known as the musculoskeletal system.

Joints:

Where the bones come together (and with the power of the muscles) give a variety of range of motion (R.O.M). Joints, (joint, classifications) consist of 360 joints.

Bones:

The framework of the body is the skeleton (Bones). Bones of the skeletal system consist of 206 bones.



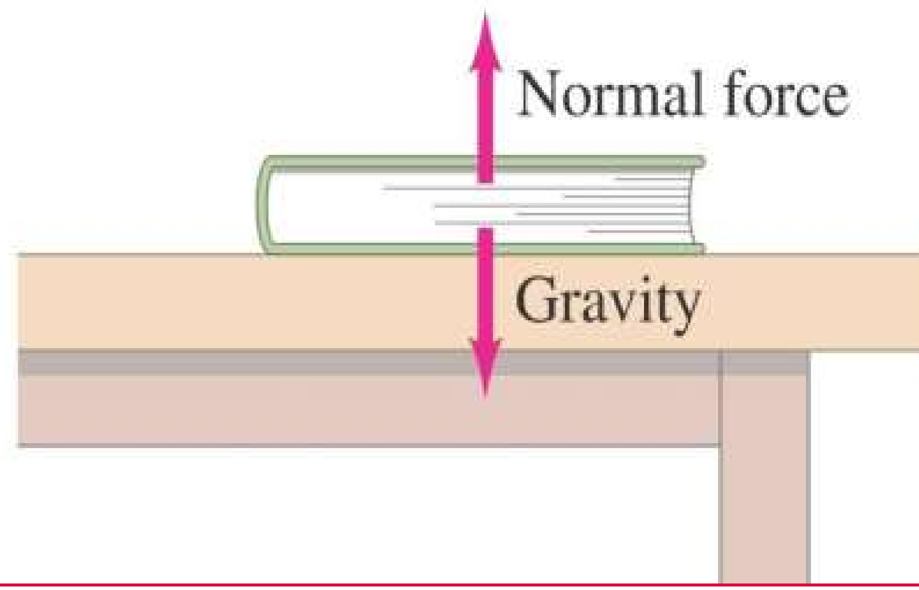
Muscles:

The body's motive power. (muscular system) consist of 640 muscles.

Equilibrim (Statics)

Object at rest, net force is zero

- Example: Book on a table, two forces acting on it
  - Gravitational force
  - Normal force
- Book at rest, acceleration zero, total force is zero
- Gravitational force and normal force balance each other
- The book is in equilibrium ("balance" in Latin)



Static Equilibrium (No Motion)

- Static equilibrium conditions

  1. No Translational Force

  Resultant external force must equal zero

$$\vec{F}_{net} = \sum \vec{F}_{ext} = 0$$

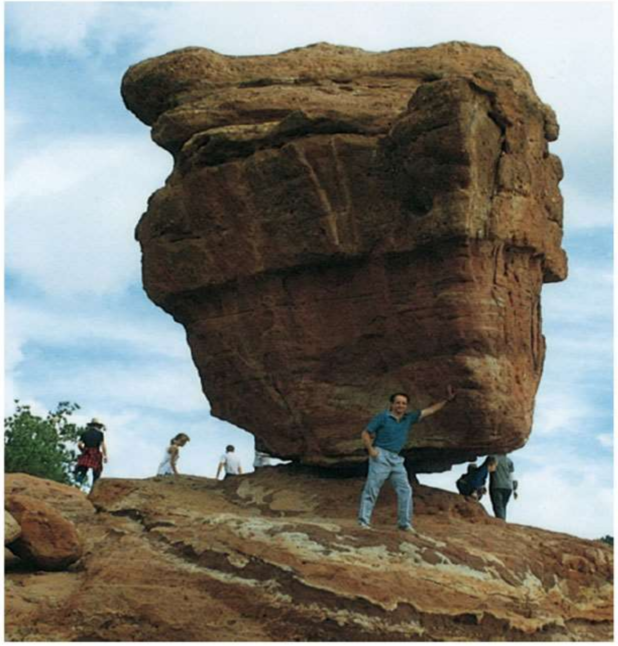
$$\sum F_x = \sum F_y = \sum F_z = 0$$

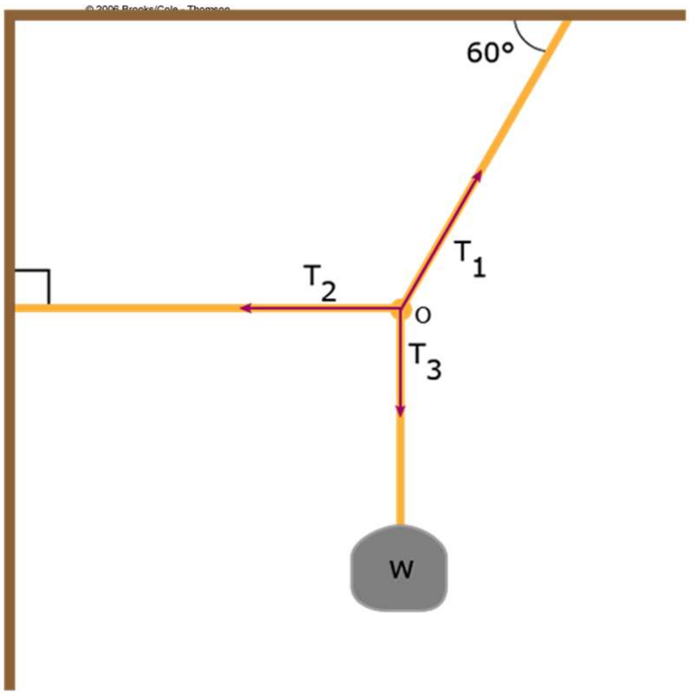
$$\sum F_x = T_1 \cos\theta - T_2 = 0$$

$$\sum F_y = T_1 \sin\theta - T_3 = 0$$

Upper Forces balance Down forces

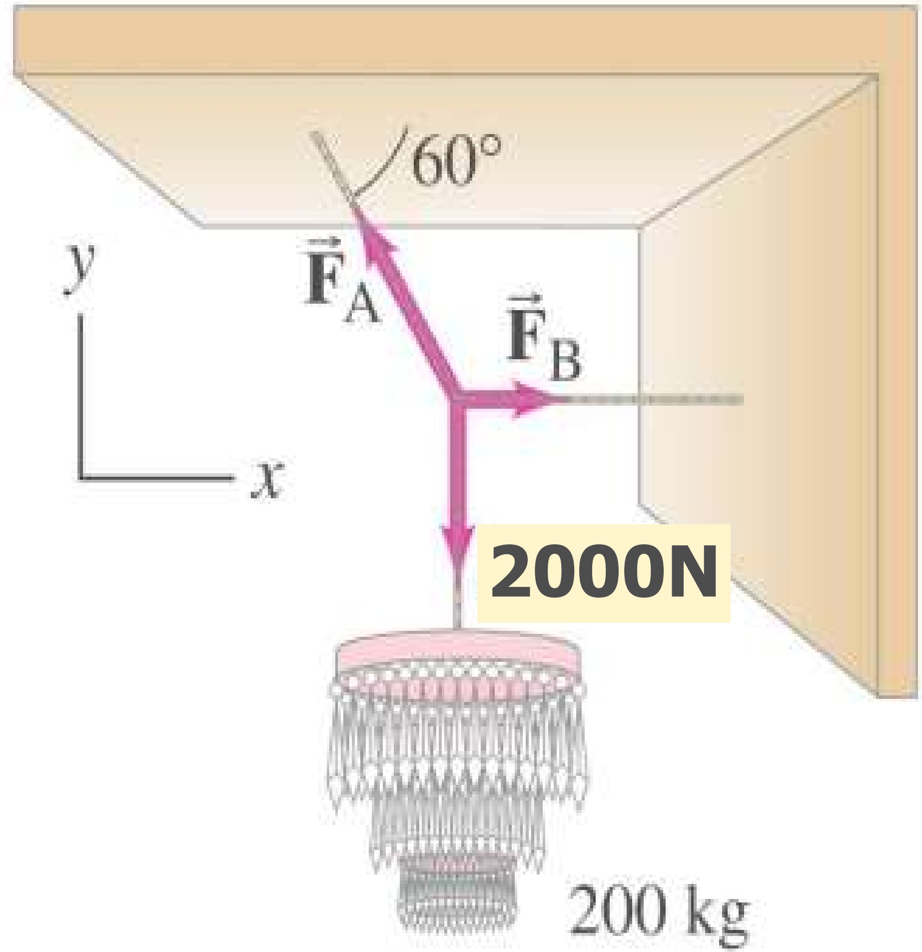
Forces to the right balance Forces to the left





Example

Calculate the tensions $\vec{F}_A$ and $\vec{F}_B$ in the two cords that are connected to the vertical cord supporting the 200-kg chandelier shown. Ignore the mass of the cords.



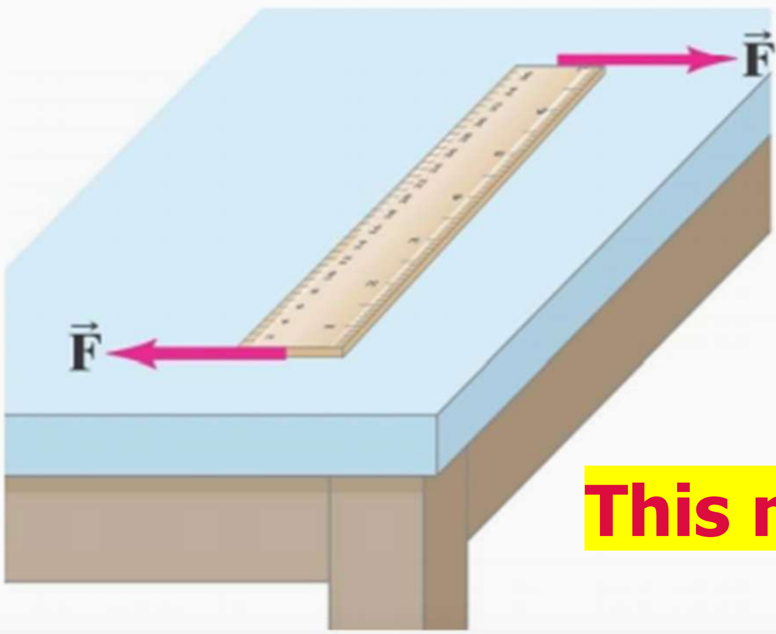
Conditions for Equilibrium

- For an extended object to be in equilibrium, a second condition must be satisfied

2. This second condition involves that No rotational motion of the extended object

The net external torque on the object must equal zero

The second condition of equilibrium:



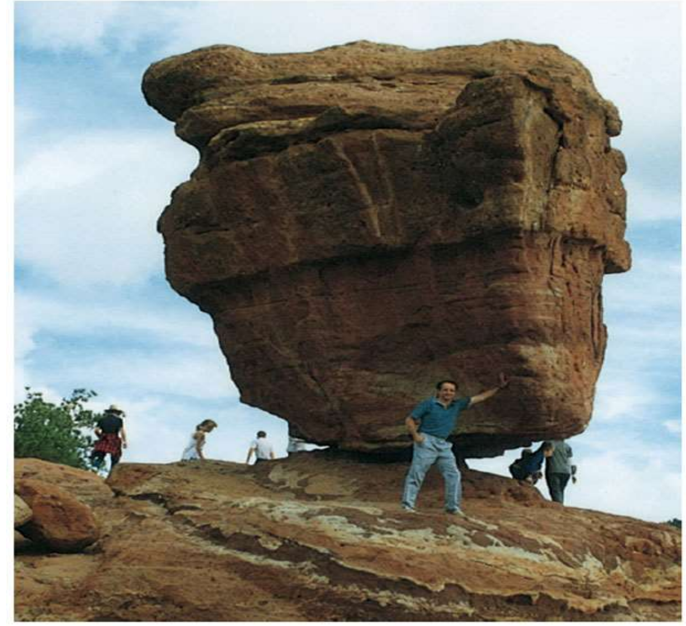
$$\Sigma\tau = 0$$

$$\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = I\vec{\alpha} = 0$$

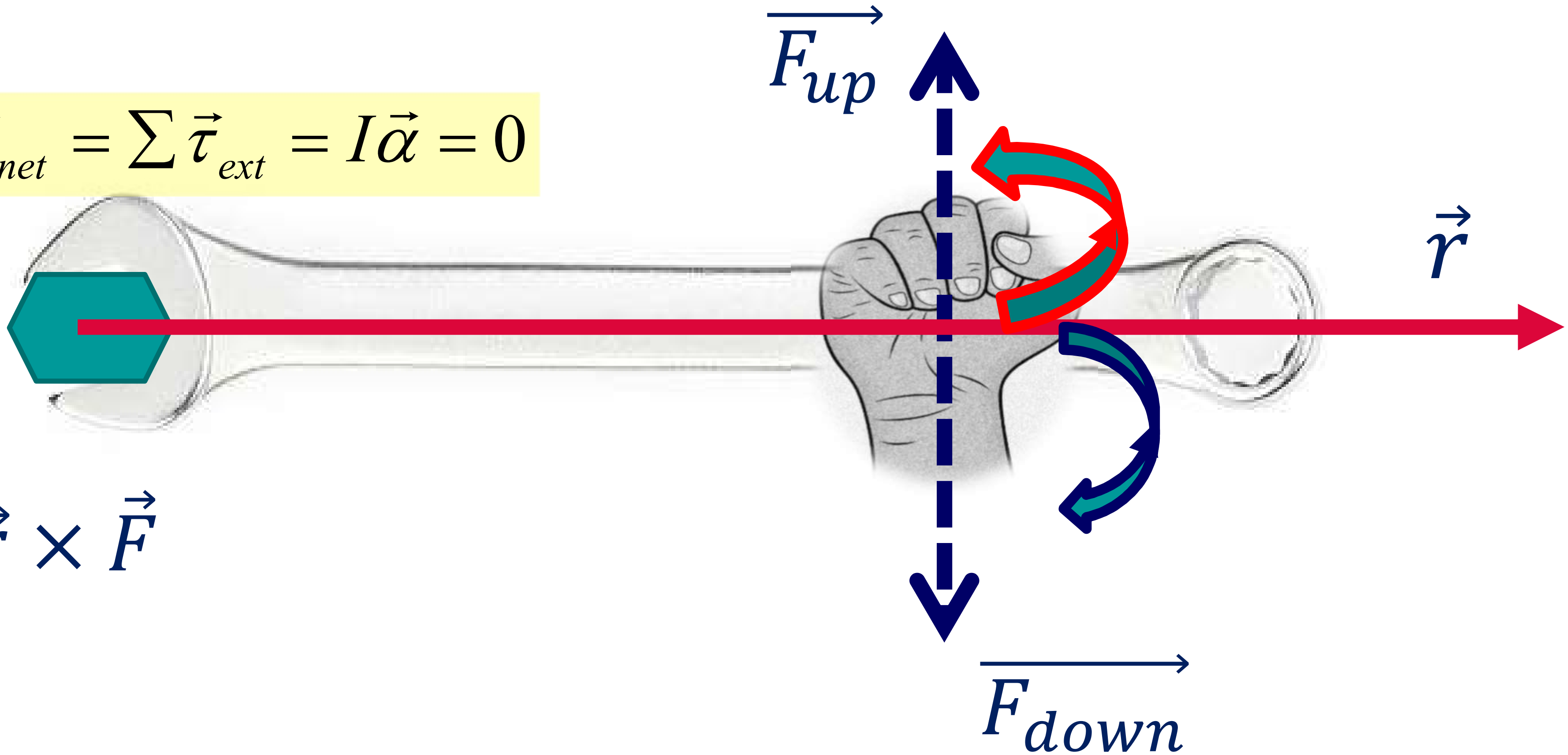
This must be true for any axis of rotation

$\overrightarrow{F_{up}}$ cause a counterclockwise rotation, then the torque is positive

$\overrightarrow{F_{down}}$ cause a clockwise rotation, then the torque is negative



$$\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = I\vec{\alpha} = 0$$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

2^{nd} Conditions for Equilibrium

- The net torque equals zero
  - This means that the torque causes counterclockwise rotation balances with the torque that cause clockwise rotation

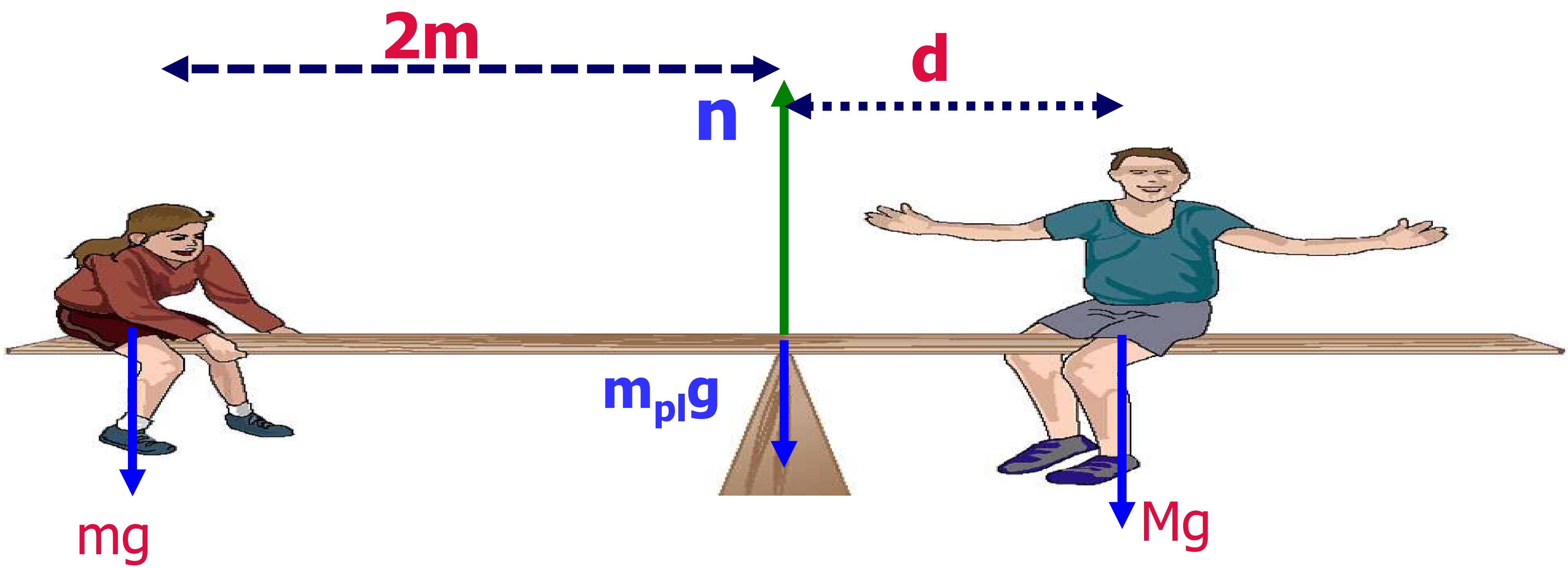
$$\sum \vec{\tau} = 0$$

$$\vec{\boldsymbol{\tau}}_{net} = \sum \vec{\boldsymbol{\tau}}_{ext} = \mathbf{0}$$

$$\boldsymbol{\tau}_{C.C.W} = \boldsymbol{\tau}_{C.W}$$

A seesaw consisting of a uniform board of mass m_{pl} and length L supports a father and daughter with masses M and m, respectively. The father is a distance d from the center, and the daughter is a distance 2 m from the center.

- A) Find the magnitude of the upward force n exerted by the support on the board.
- B) Find where the father should sit to balance the system at rest.



A) Find the magnitude of the upward force **n** exerted by the support on the board.

B) Find where the father should sit to balance the system at rest.

$$F_{net,y} = n - mg - Mg - m_{pl}g = 0$$

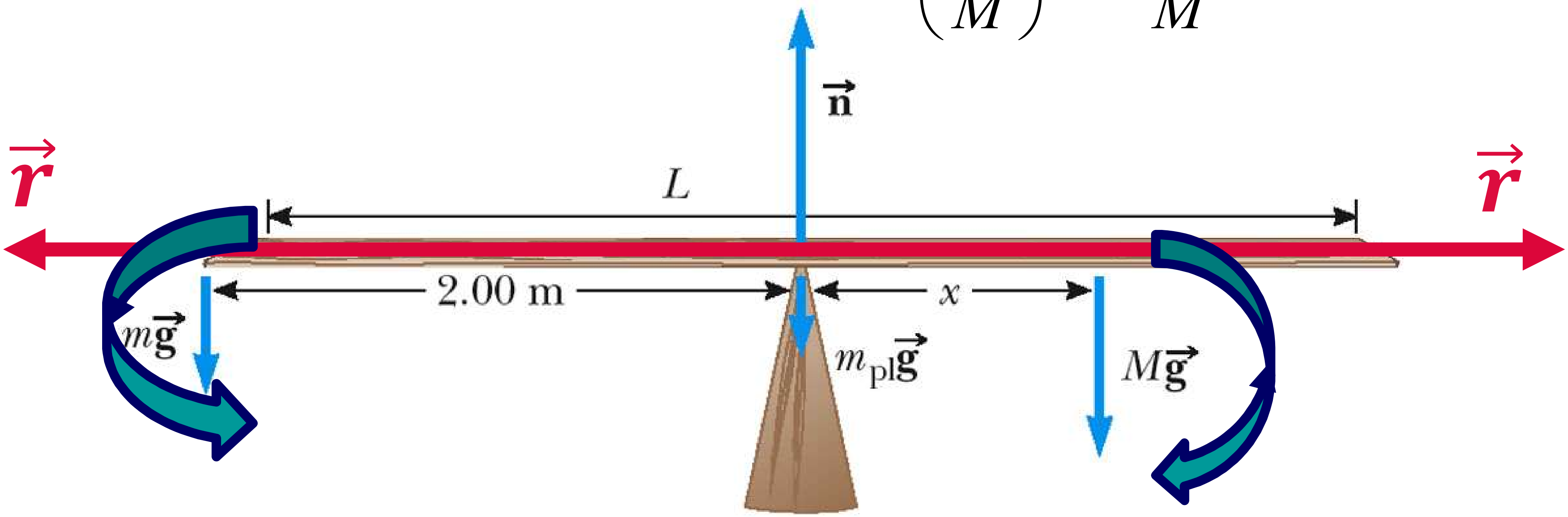
$$n = mg + Mg + m_{pl}g$$

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$

$$= mgd - Mgx + 0 + 0 = 0$$

$$mgd = Mgx$$

$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M} < 2.00\ \text{m}$$



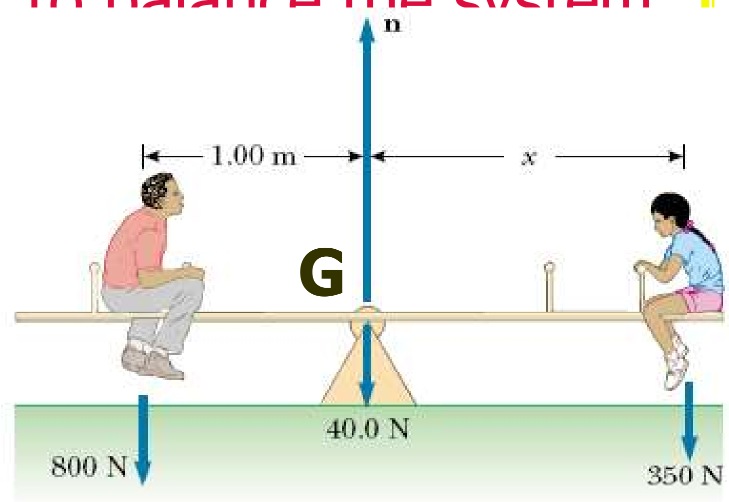
Axis of Rotation

- The net torque is about an axis through any point in the xy plane
- Does it matter which axis you choose for calculating torques?
- NO. The choice of an axis is arbitrary
- If an object is in translational equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis
- We should be smart to choose a rotation axis to simplify problems

EXAMPLE 1

A uniform 40.0-N board supports a father and daughter weighing 800 N and 350 N, respectively. If the support is under the center of gravity of the board and if the father is 1.00 m from the center,

(b) Determine where the child should sit to balance the system



(b) Take an axis perpendicular to the page through the center of gravity **G** as the axis for our torque

$\Sigma \tau = 0$ yields

$$n(1.00\text{ m}) - (40.0\text{ N})(1.00\text{ m}) - (350\text{ N})(1.00\text{ m} + x) = 0$$

$$\boxed{x = 2.29\text{ m.}}$$

Center of Gravity

- The torque due to the gravitational force on an object of mass M is the force Mg acting at the center of gravity of the object
- If g is uniform over the object, then the center of gravity of the object coincides with its center of mass
- If the object is homogeneous and symmetrical, the center of gravity coincides with its geometric center

Where is the Center of Mass ?

- Assume m_1 = 1 kg, m_2 = 3 kg, and x_1 = 1 m, x_2 = 5 m, where is the center of mass of these two objects?

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

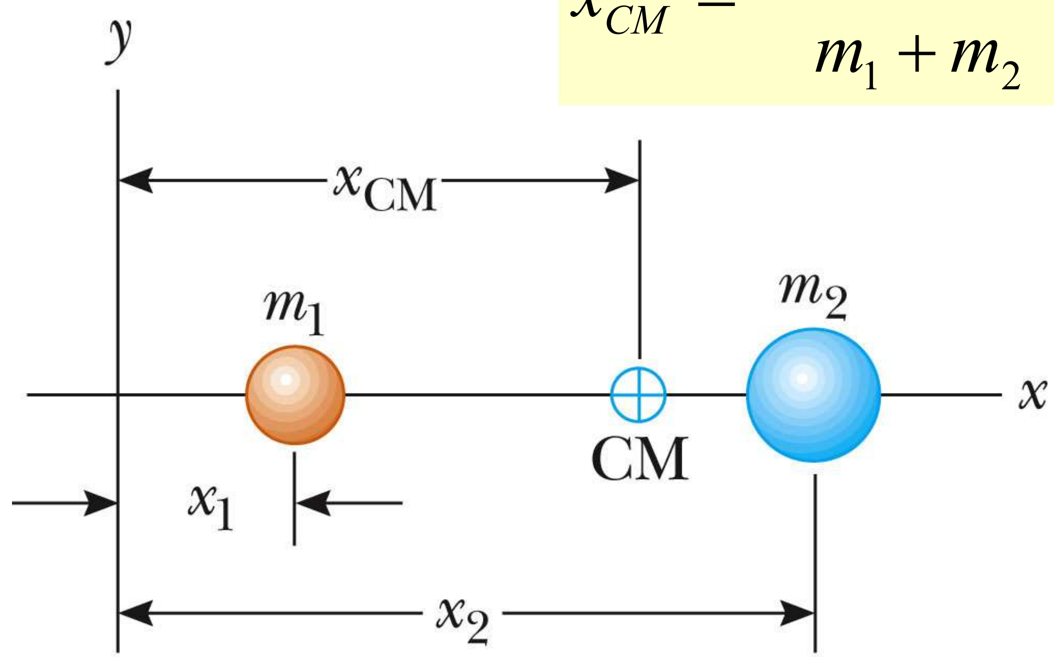
A) x_{CM} = 1 m

B) x_{CM} = 2 m

C) x_{CM} = 3 m

D) x_{CM} = 4 m

E) x_{CM} = 5 m



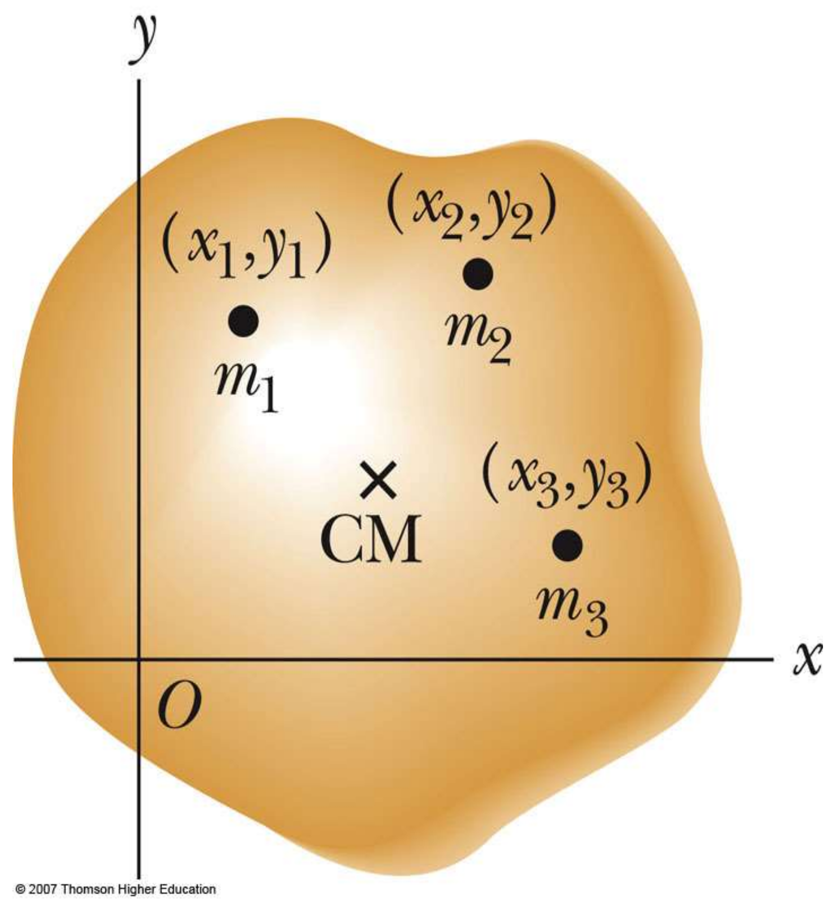
Center of Mass (CM)

- An object can be divided into many small particles
  - Each particle will have a specific mass and specific coordinates
- The x coordinate of the center of mass will be

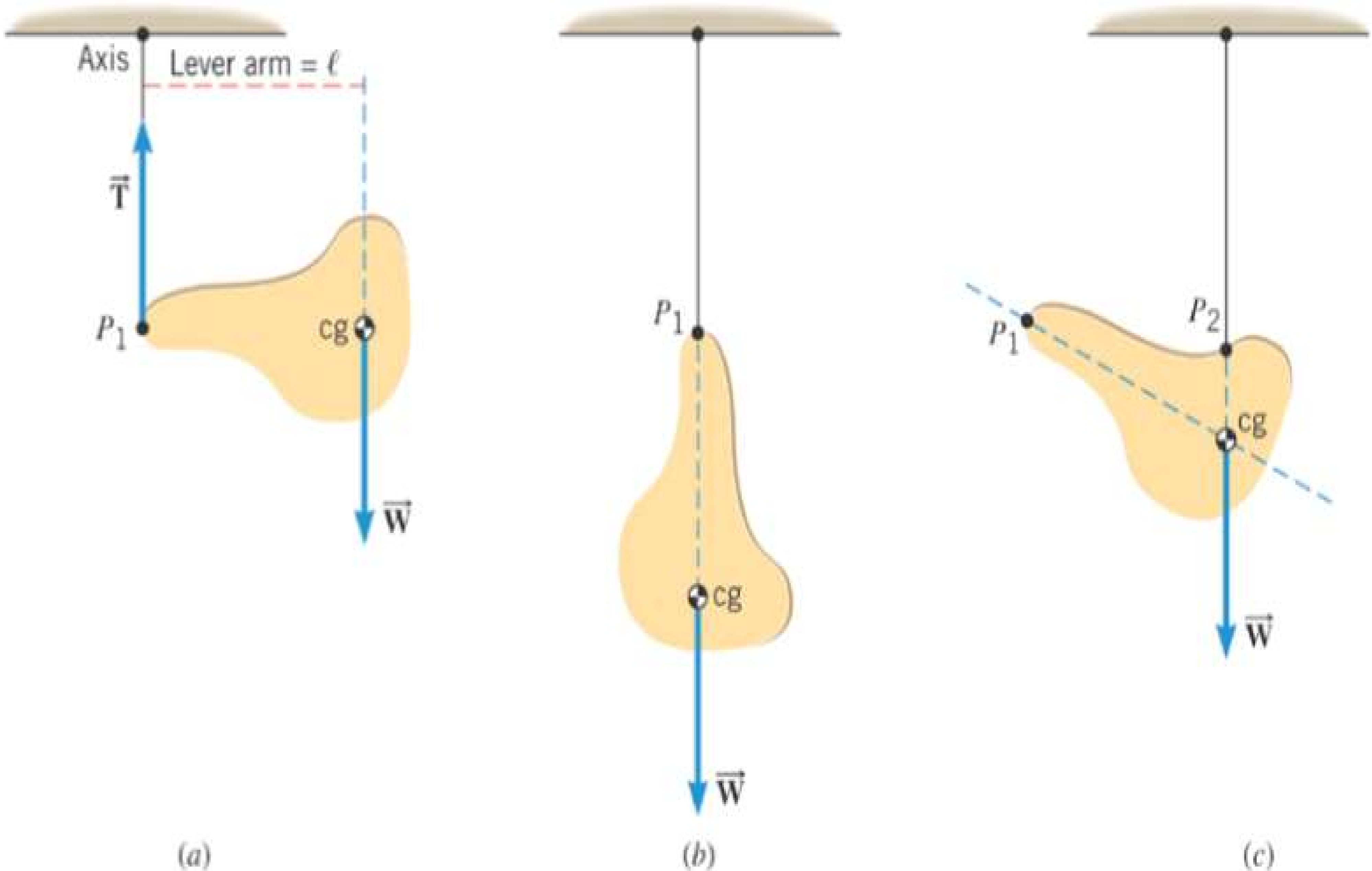
$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

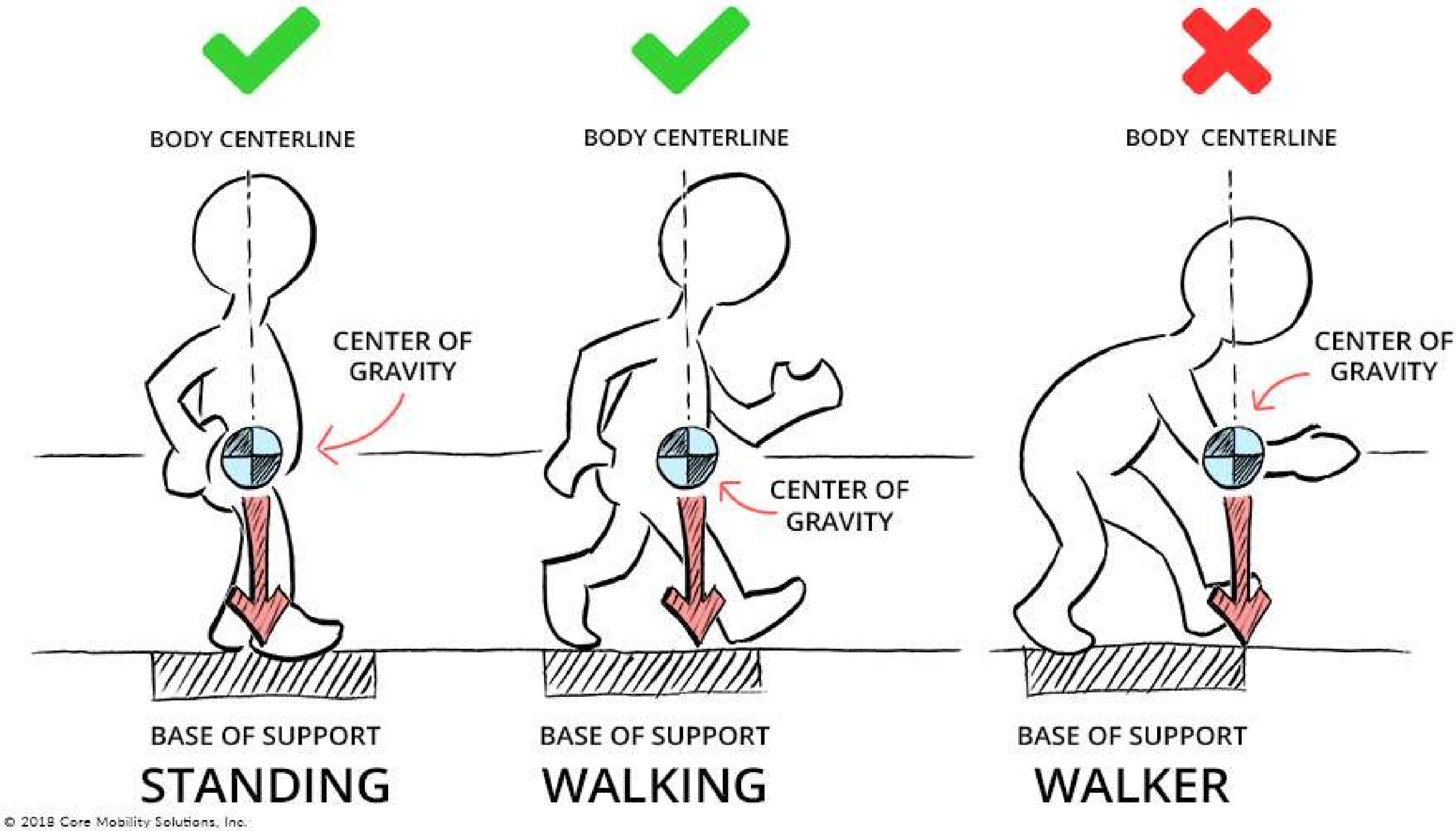
- Similar expressions can be found for the y coordinates

$$Y_{CM} = \frac{\sum_i m_i Y_i}{\sum_i m_i}$$



Center of Gravity for Irregular shape




BODY CENTERLINE
CENTER OF GRAVITY
BASE OF SUPPORT
STANDING
BODY CENTERLINE
CENTER OF GRAVITY
BASE OF SUPPORT
WALKING
BODY CENTERLINE
CENTER OF GRAVITY
BASE OF SUPPORT
WALKER
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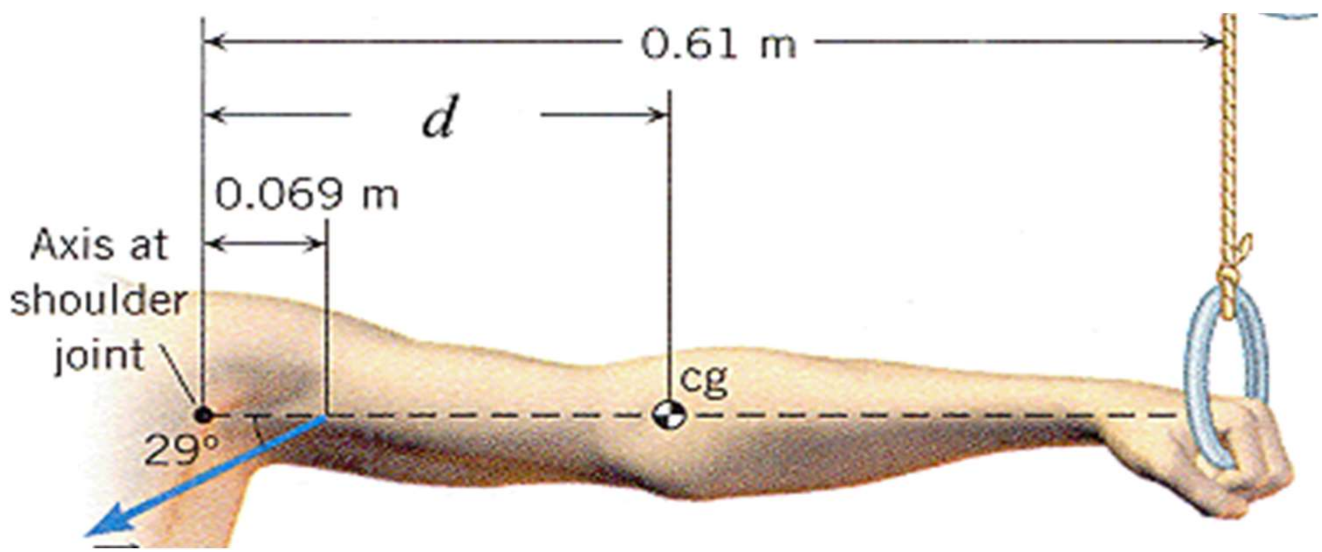

0.61 m
d
0.069 m
Axis at shoulder joint
cg
29°

Problem-Solving Strategy 1

- Draw sketch, decide what is in or out the system
- Draw a free body diagram (FBD)
- Show and label all external forces acting on the object
- Indicate the locations of all the forces
- Establish a convenient coordinate system
- Find the components of the forces along the two axes
- Apply the first condition for equilibrium
- Be careful of signs

$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

Problem-Solving Strategy 2

- Choose a convenient axis for calculating the net torque on the object
  - Remember the choice of the axis is arbitrary
- Choose an origin that simplifies the calculations as much as possible
  - A force that acts along a line passing through the origin produces a zero torque
- Be careful of sign with respect to rotational axis
  - positive if force tends to rotate object in CCW
  - negative if force tends to rotate object in CW
  - zero if force is on the rotational axis
- Apply the second condition for equilibrium $\tau_{net,z} = \sum \tau_{ext,z} = 0$

Problem-Solving Strategy 3

- The two conditions of equilibrium will give a system of equations
- Solve the equations simultaneously
- Make sure your results are consistent with your free body diagram
- If the solution gives a negative for a force, it is in the opposite direction to what you drew in the free body diagram
- Check your results to confirm

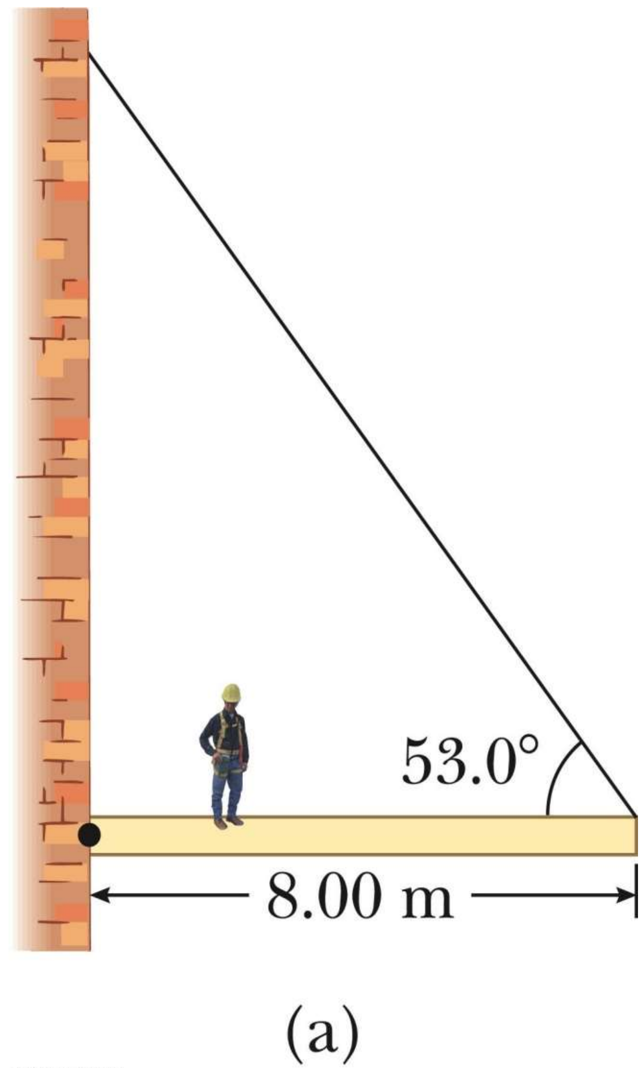
$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

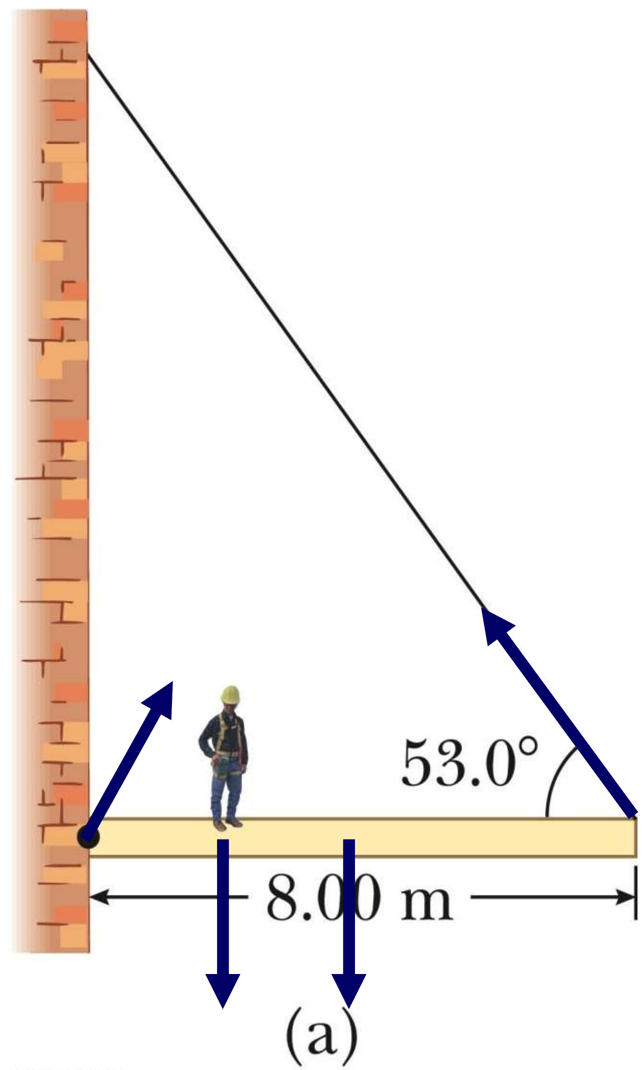
Horizontal Beam Example

- A uniform horizontal beam with a length of l = 8.00 m and a weight of W_b = 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of $\phi = 53°$ with the beam. A person of weight W_p = 600 N stands a distance d = 2.00 m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



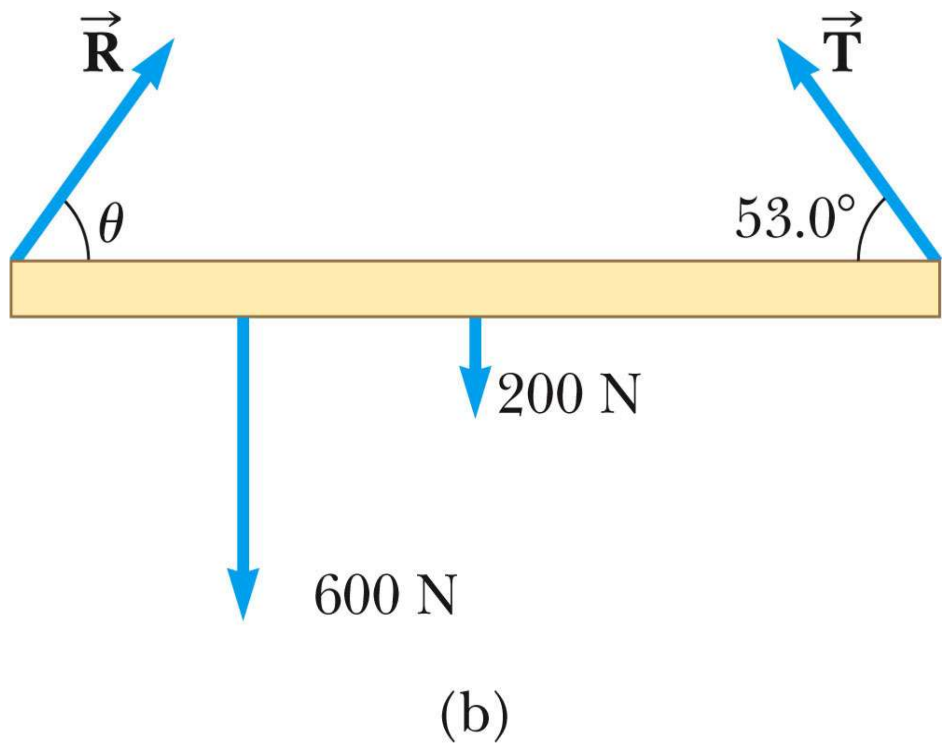
Horizontal Beam Example

- The beam is uniform
  - So the center of gravity is at the geometric center of the beam
- The person is standing on the beam
- What are the tension in the cable and the force exerted by the wall on the beam?



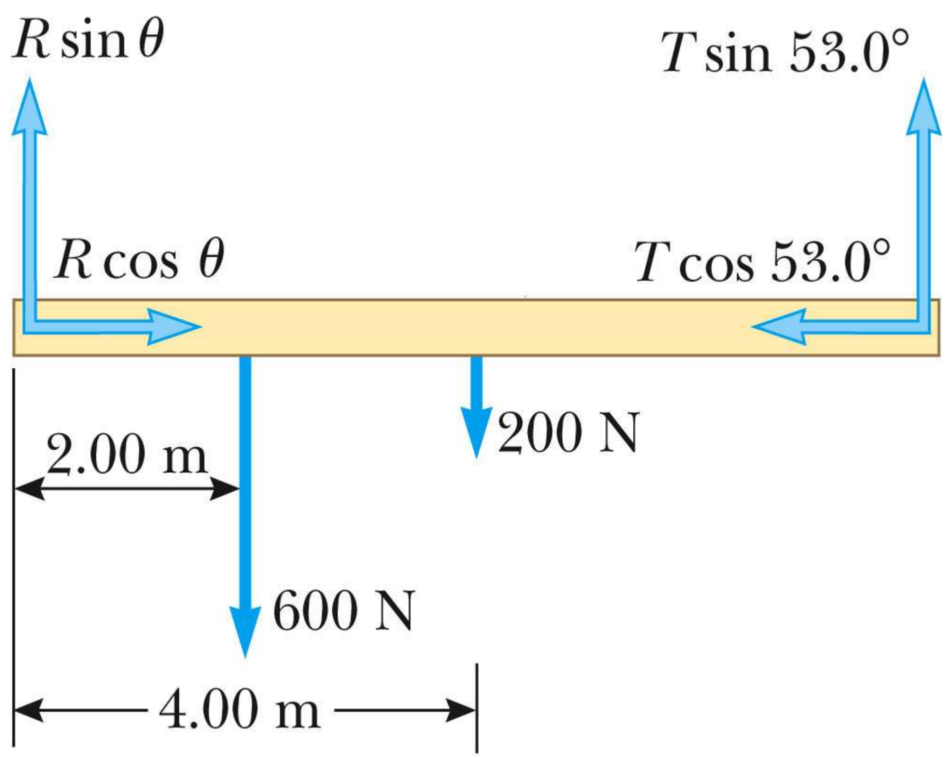
Horizontal Beam Example, 2

- Analyze
  - Draw a free body diagram
  - Use the pivot in the problem (at the wall) as the pivot
    - This will generally be easiest
  - Note there are three unknowns (T, R, θ)



Horizontal Beam Example, 3

- The forces can be resolved into components in the free body diagram
- Apply the two conditions of equilibrium to obtain three equations
- Solve for the unknowns



(c)

$$\sum \tau_z = (T\sin\phi)(l) - W_p d - W_b\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W_p d + W_b\left(\frac{l}{2}\right)}{l\sin\phi} = \frac{(600\,N)(2m) + (200\,N)(4m)}{(8m)\sin 53^\circ} = 313\,N$$

$$\sum F_x = Rx - T\cos 57 = 0$$

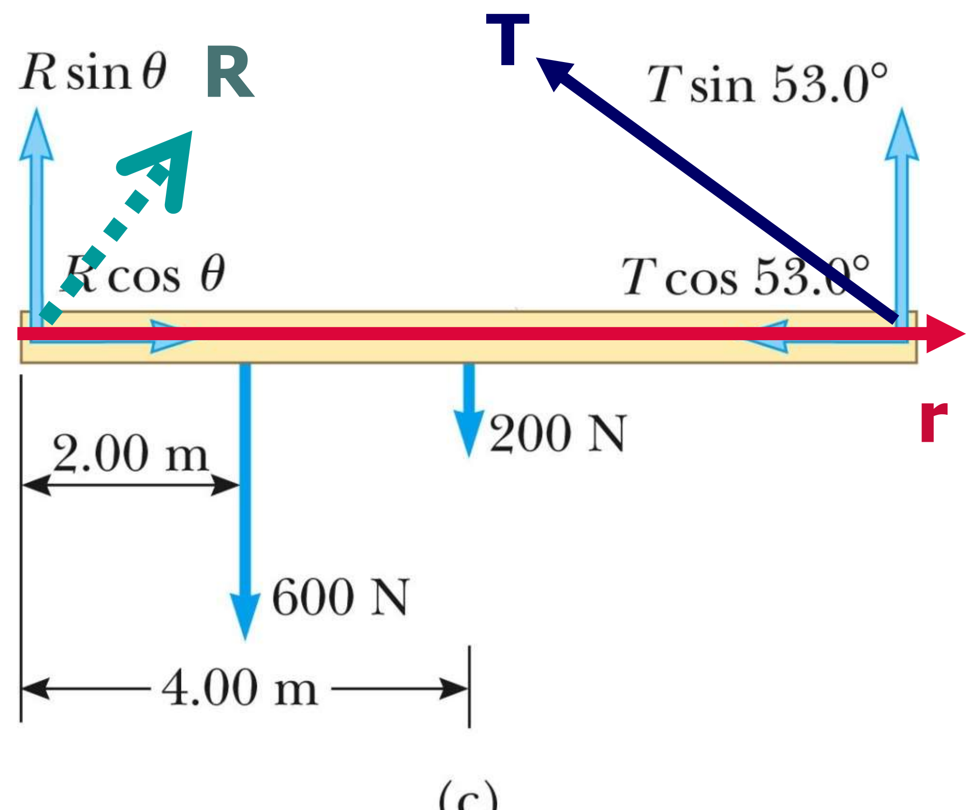
$$\sum F_y = Ry + T\sin 57 - 200 - 600 = 0$$

$$\frac{Ry}{Rx} = \tan\theta = \frac{800 - T\sin 57}{T\cos 57}$$

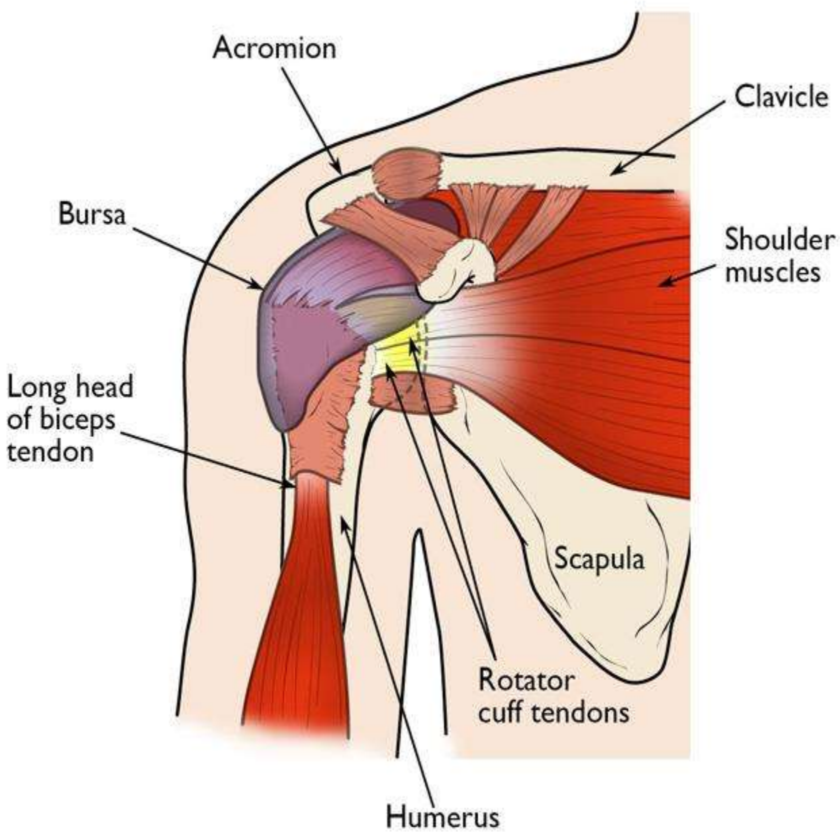
$$\theta = \tan^{-1}\left(\frac{663.4}{281.6}\right) = 71.7^\circ$$

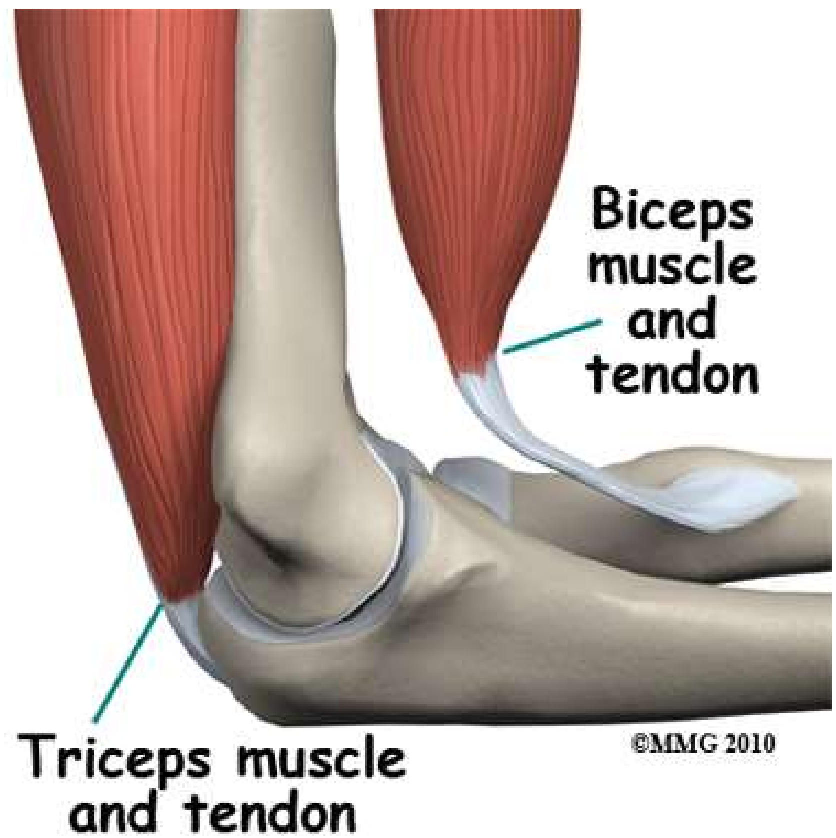
$$R = \frac{T\cos\varphi}{\cos\theta} = \frac{(313N)\cos 53^\circ}{\cos 71.7^\circ} = 581N$$

$$R = \sqrt{R_x^2 + R_y^2}$$



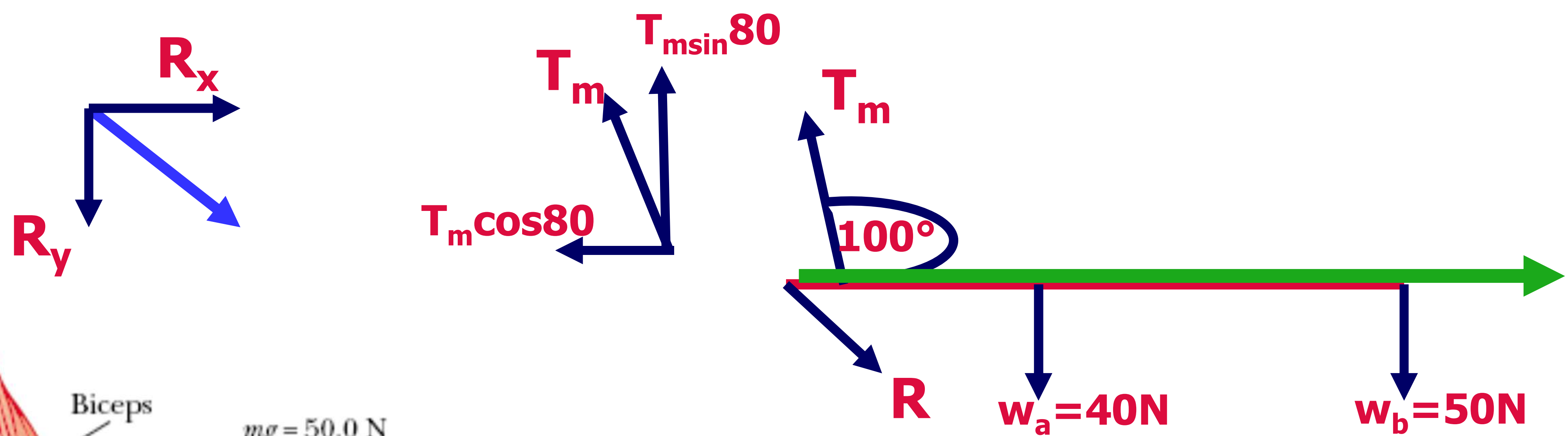
Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. Muscles apply torques against the torque exerted by the weight

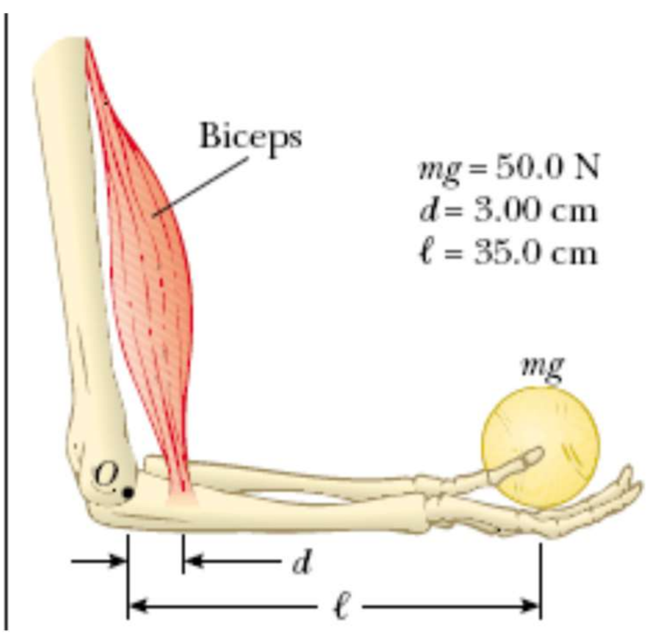




EXAMPLE

A person holds a 50 N sphere in his hand. The forearm is horizontal. The biceps muscle is attached 3 cm from the joint, and the sphere is 35 cm from the joint. Find the upward force exerted by the biceps on the forearm and the force exerted by the upper arm on the forearm and acting at the joint. Take the weight of the forearm is 40N acting at distance 15 cm from the joint.





$$\sum F_x = 0,\ R_x - T_m cos\theta = 0,\ R_x = T_m cos\theta \dots\dots (1)$$

$$\sum F_y = 0, T_m sin80 - R_y - w_a - w_b = 0\,,$$

$$R_y = T_m sin80 - 90 \dots\dots\dots\dots\dots\dots\dots\dots\dots .\,.\dots\dots\dots\dots (2)$$

$$\sum \vec{\tau} = 0,$$

$$\boldsymbol{\tau_R + \tau_{T_m} + \tau_{w_a} + \tau_{w_b} = 0}$$

0+T_m(3)sin100- 40(15) sin90-50 (35)sin90= 0

$$T_m = \frac{2350}{3sin100} = 795\ N,$$

Substitute in equ. 1 and 2

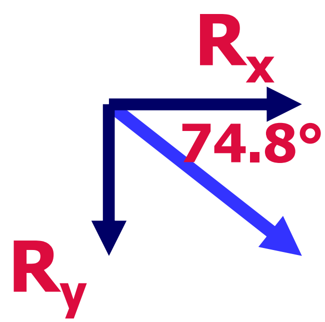
$$\boldsymbol{R_x = T_m cos80 = 138\ N}$$

$$\boldsymbol{R_y = T_m sin80 - 90 = 693\ N}$$

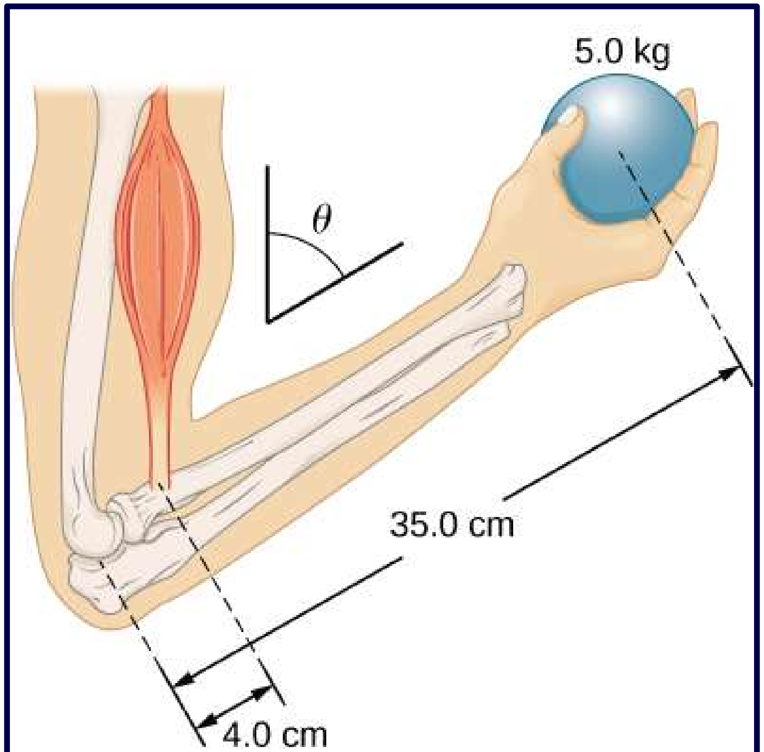
Then

$$\boldsymbol{R = \sqrt{R_x^2 + R_y^2} = 706\ N}$$

$$\boldsymbol{\theta = tan^{-1}\left(\frac{R_y}{R_x}\right) = tan^{-1}\left(\frac{168}{45.5}\right) = -78.7°}$$



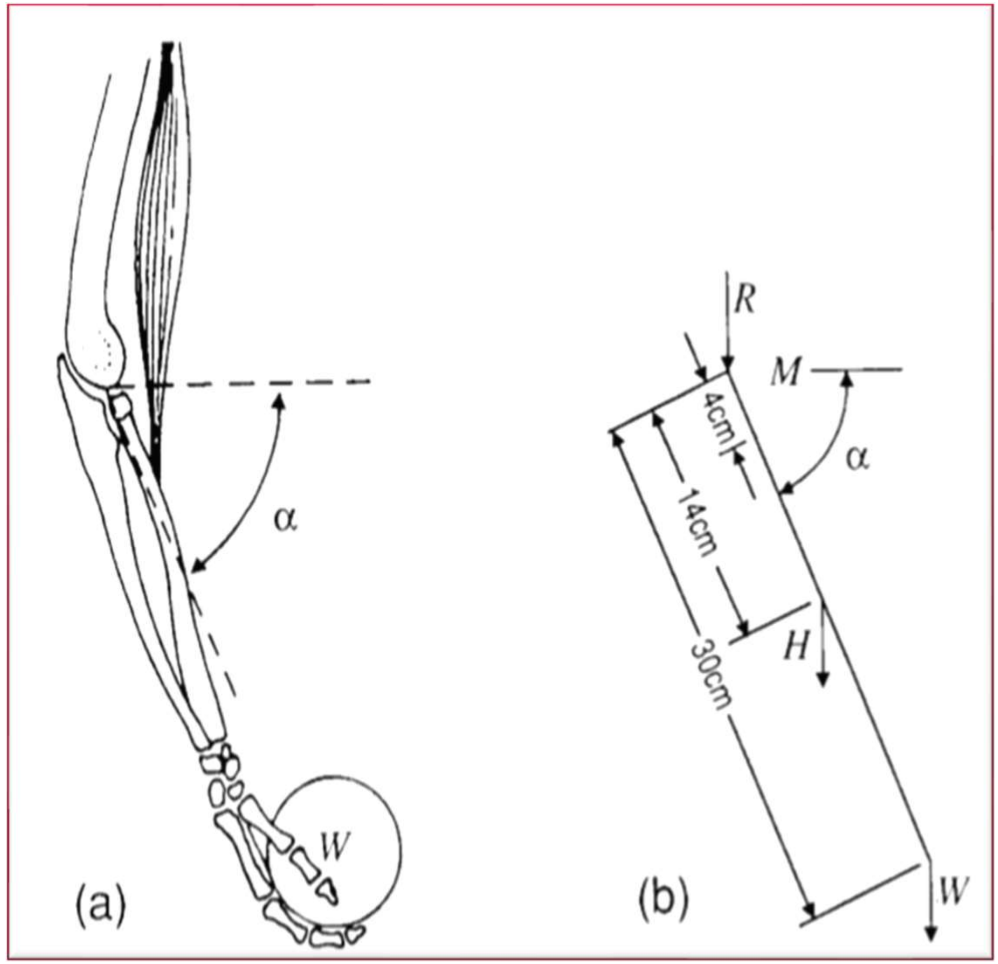
Find The Forcess T and R



In the shown free body diagram, Take the weight of the arm 3 kg affected

At distance 15 cm apart from the jointIf θ =45°, the angle that the arm

Makes with the hotizontal is 60°

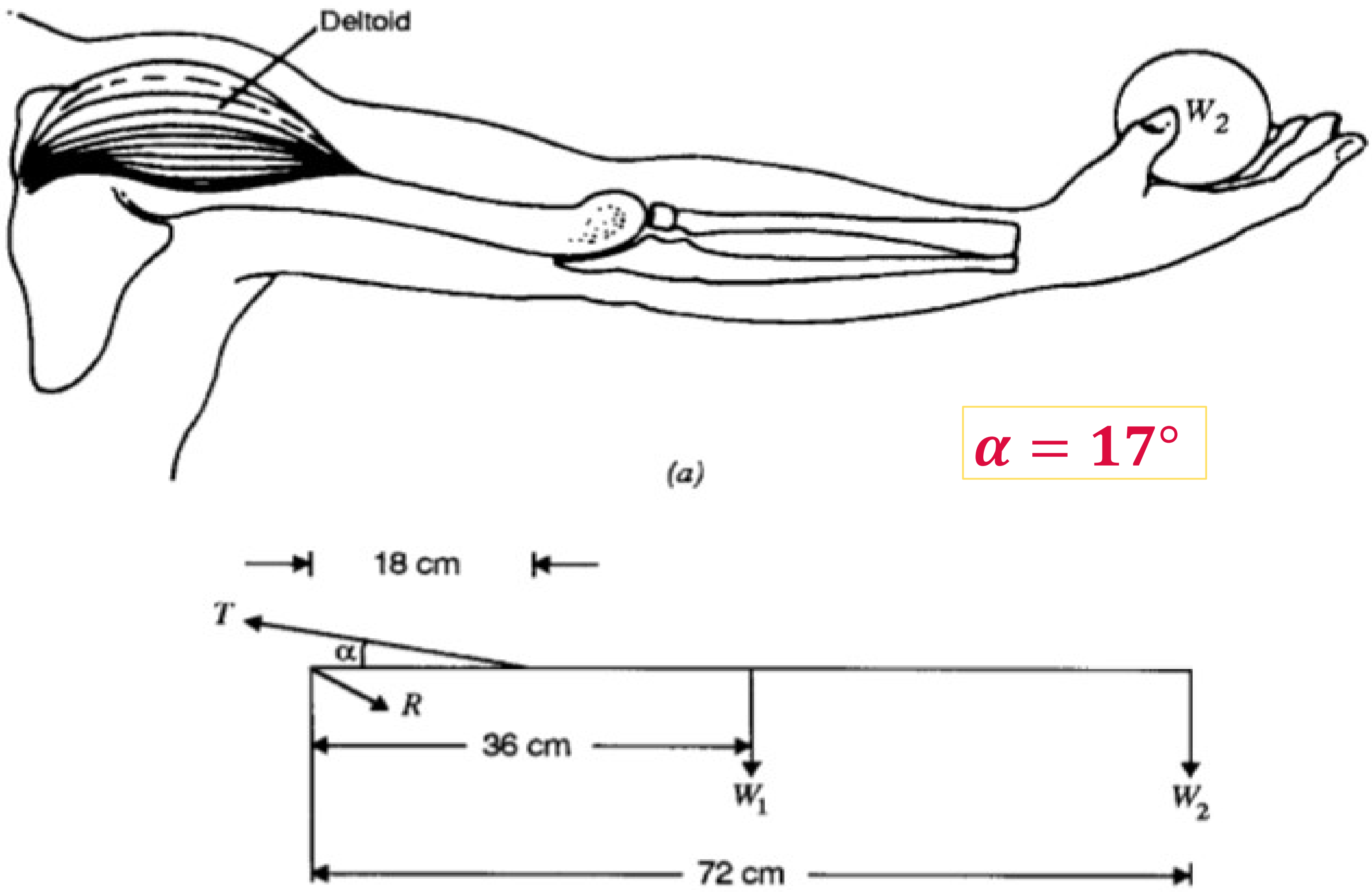


If $\alpha = 40° \; and \; the \; angle \; that \; the \; tension$

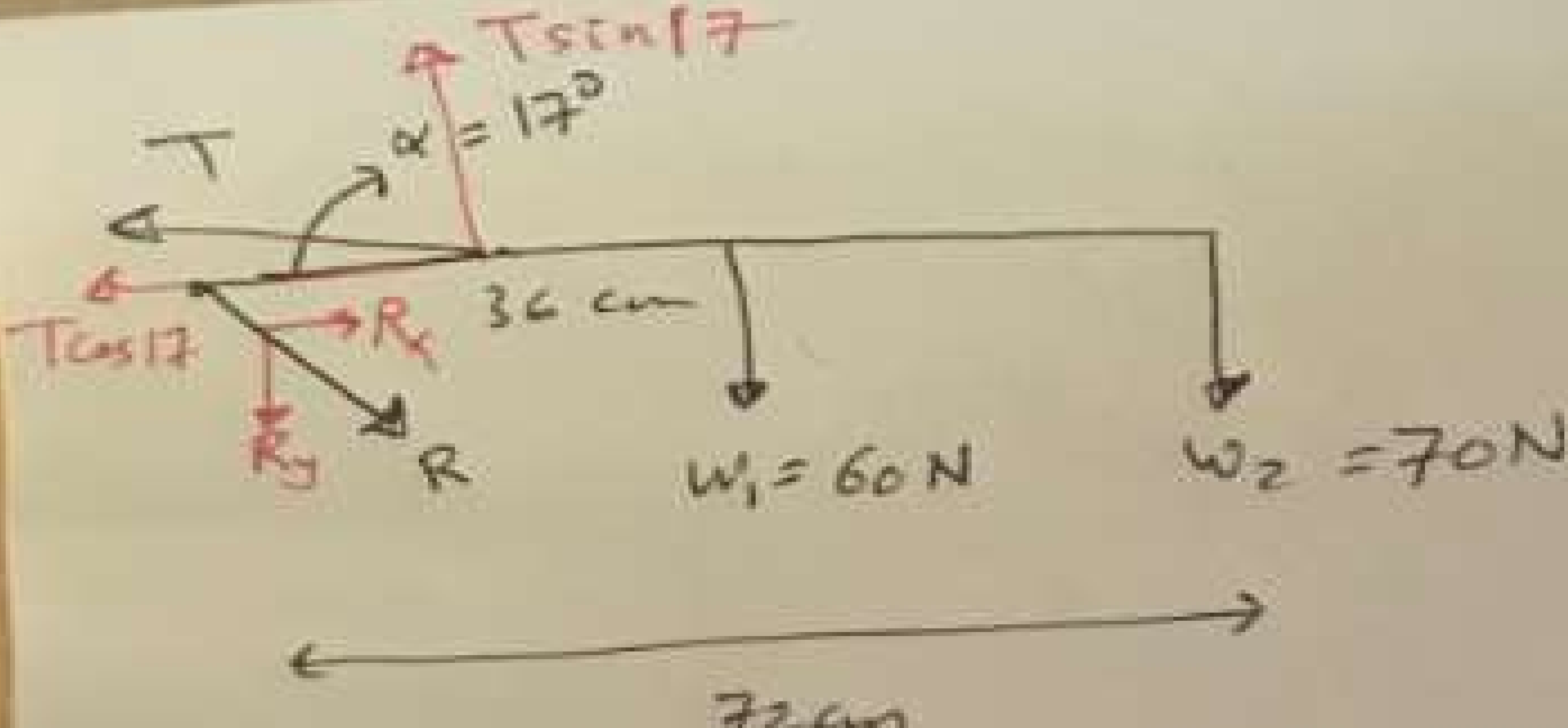
In the bicep muscle makes an angle 130° with the forearm

Deltoid Muscle

Find the tension in the deltoid muscle and the reaction force at the shoulder joint.



$\alpha = 17°$

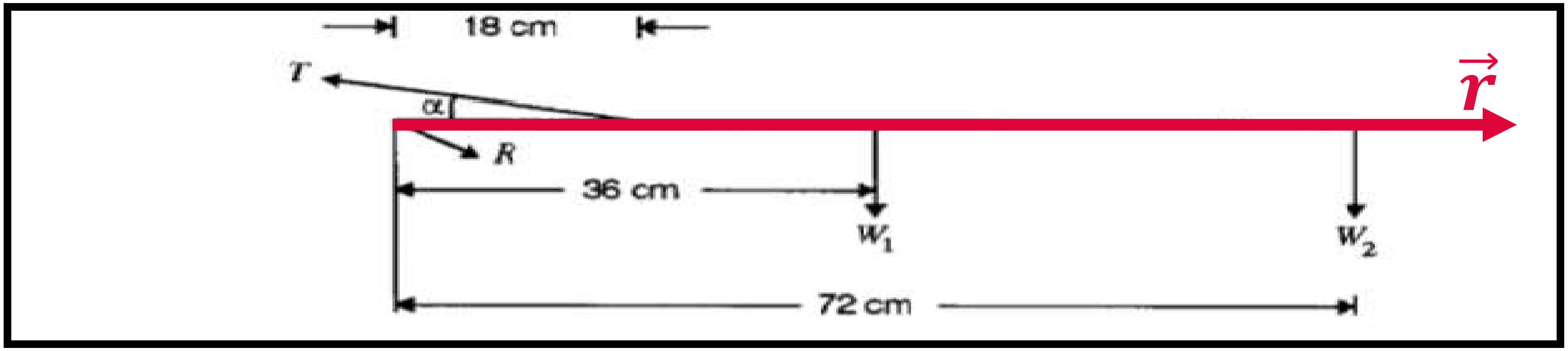


1) $\sum F_x = 0 \Rightarrow -T\cos 17 + R_x = 0$

$$R_x = T\cos 17 \quad \text{—— (1)}$$

2) $\sum F_y = 0 \Rightarrow T\sin 17 - R_y - W_1 - W_2 = 0$

$$R_y = T\sin 17 - 130 \quad \text{—— (2)}$$



3) $\sum \vec{\tau} = 0 \Rightarrow \tau_R + \tau_T + \tau_{w_1} + \tau_{w_2} = 0$

$\tau_R = 0$, $\tau_T = +18\,T\sin(180-17) = 18\,T\sin 17$

$\tau_{w_1} = -36(60)\sin 90 = -2160\,N$

$\tau_{w_2} = -72(70)\sin 90 = -5040\,N$

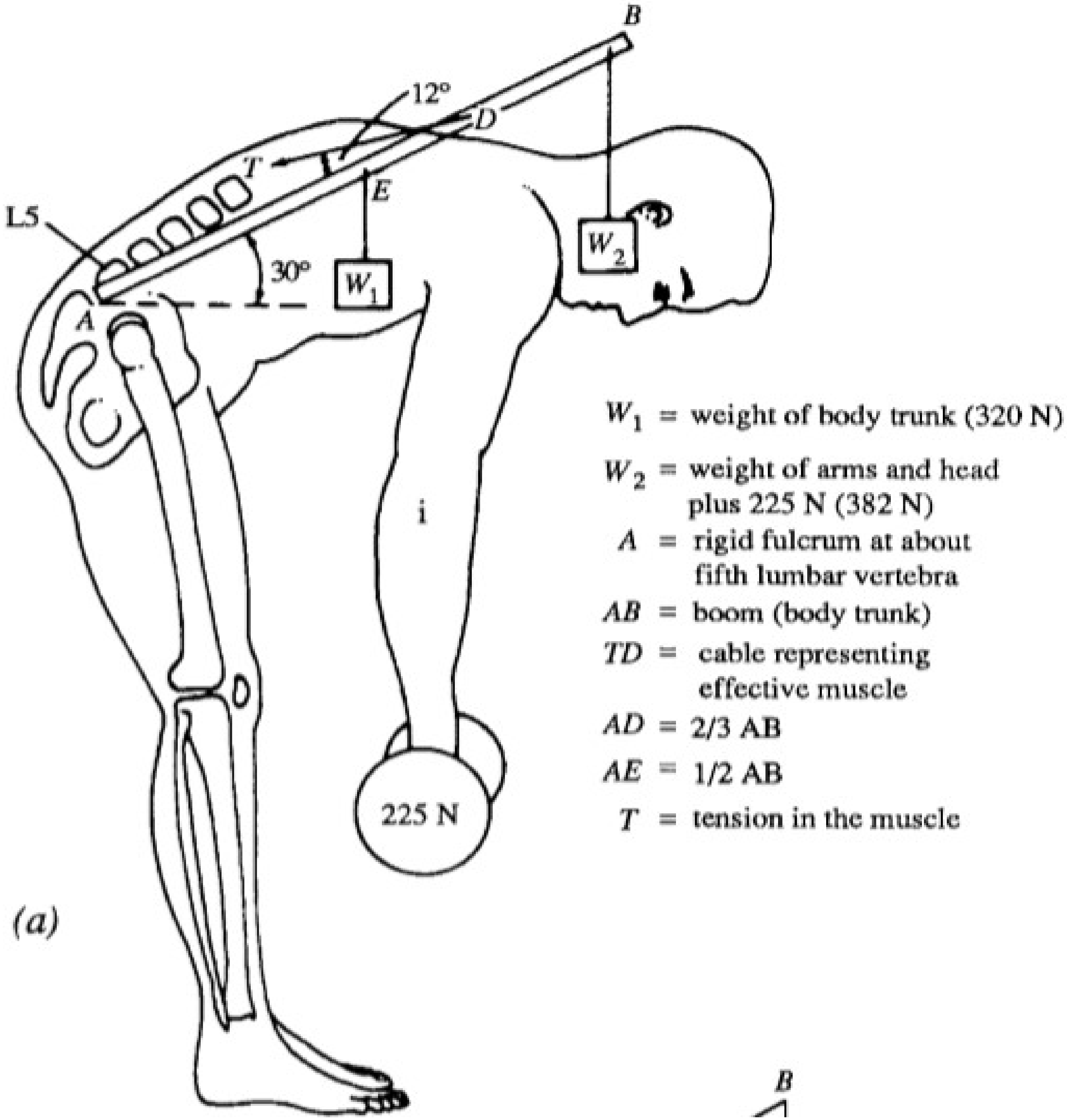
$\therefore 18\,T\sin 17 - 2160 - 5040 = 0$

$18\,T\sin 17 = 7200 \Rightarrow T = \frac{7200}{18\sin 17} = 1368\,N$

$\therefore R_x = T\cos 17 = 1368\cos 17 = 1308\,N$, $R_y = T\sin 17 - 130 \approx 270\,N$

$R = \sqrt{R_x^2 + R_y^2}$, $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$, اذاً R اتجاهه على

The Back and Spinal Column

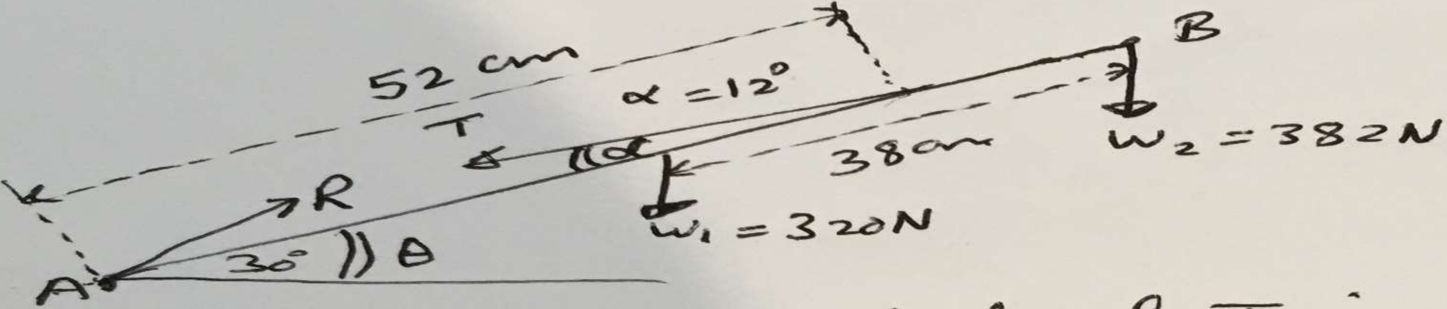


$\theta = 30°$

$AB = 78\ cm$

$AD = \frac{2}{3} AB = 52\ cm$, $AE = 38\ cm$

We can see in the FBD



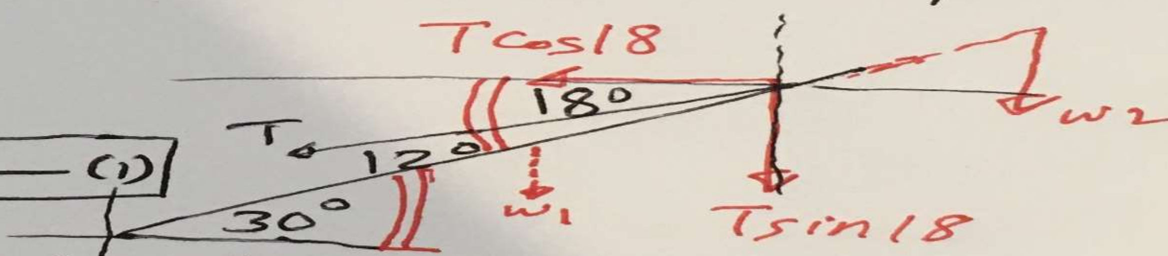
To find the magnitude of T in the back muscle and the joint reaction force ($\vec{R}$)

we analyze the T force into two components

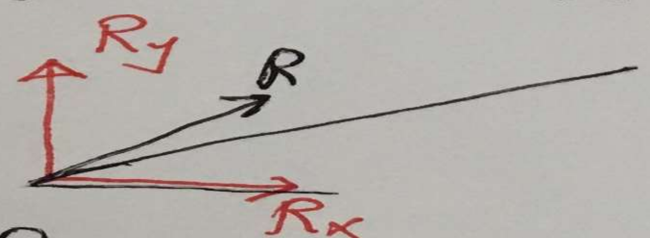
① $\sum F_x = 0$

$R_x - T\cos 18 = 0$

$$\boxed{R_x = T\cos 18 \quad \text{—— (1)}}$$



and analyze the force R



② $\sum F_y = 0$

$R_y - T\sin 18 - W_1 - W_2 = 0$

$$\boxed{R_y = T\sin 18 + 702 \quad \text{—— (2)}}$$

the $\Sigma \tau = 0$, take the torque around A

$$\tau_R + \tau_T + \tau_{W_1} + \tau_{W_2} = 0$$

$$\tau_R = 0 \quad , \quad \tau_T = +52\,T\sin(180-12) = \boxed{52\,T\sin 12}$$

$$\tau_{W_1} = -39\,W_1\sin(180-60) = -39(320)\sin 60 = -\boxed{10531\ N}$$

$$\tau_{W_2} = -78\,W_2\sin(180-60) = -78(382)\sin 60 = -\boxed{25840}$$

$$\therefore \quad 52\,T\sin 12 = 36335 \Rightarrow T = \frac{36335}{52\sin 12}$$

$$\boxed{T = 3361\ N}$$

from (1) $\Rightarrow R_x = T\cos 18 \approx \boxed{3196\ N}$

from (2) $\Rightarrow R_y = T\sin 18 + 702 \approx \boxed{1740\ N}$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(3196)^2 + (1740)^2}$$

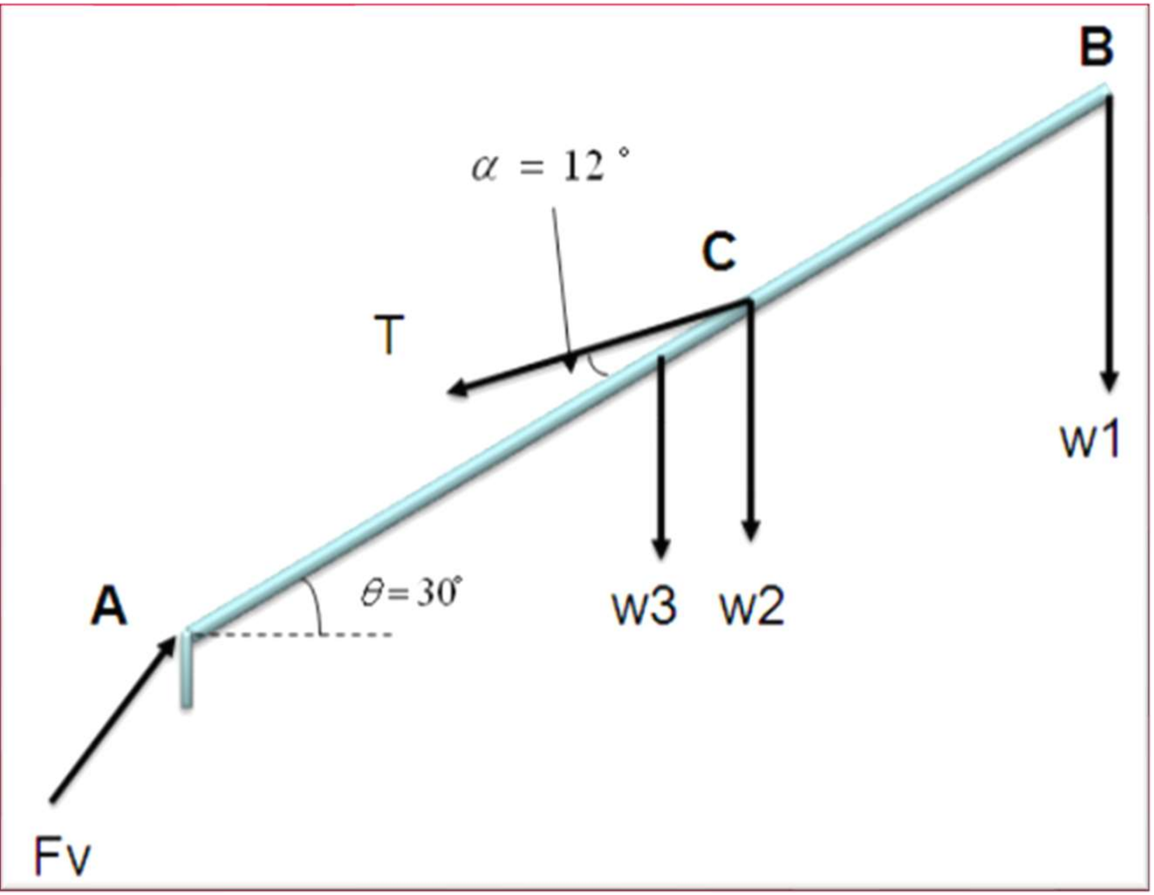
$$\boxed{R = 3639\ N}, \boxed{\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \approx 28.6^\circ}$$

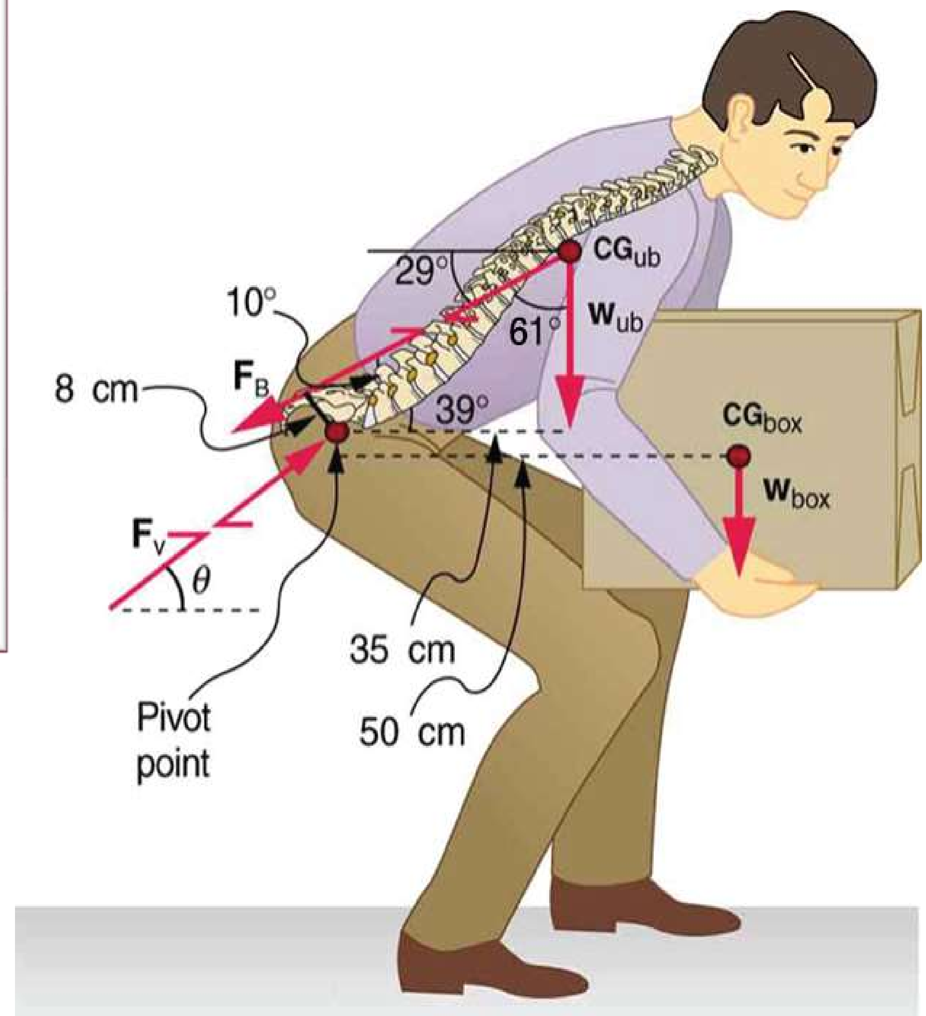
$$\frac{R}{W_1 + W_2} \approx \frac{3639}{702} \approx 52 \text{ times}$$

$$\frac{T}{W_1 + W_2} \approx 48 \text{ times}$$

... acting on the spinal Column

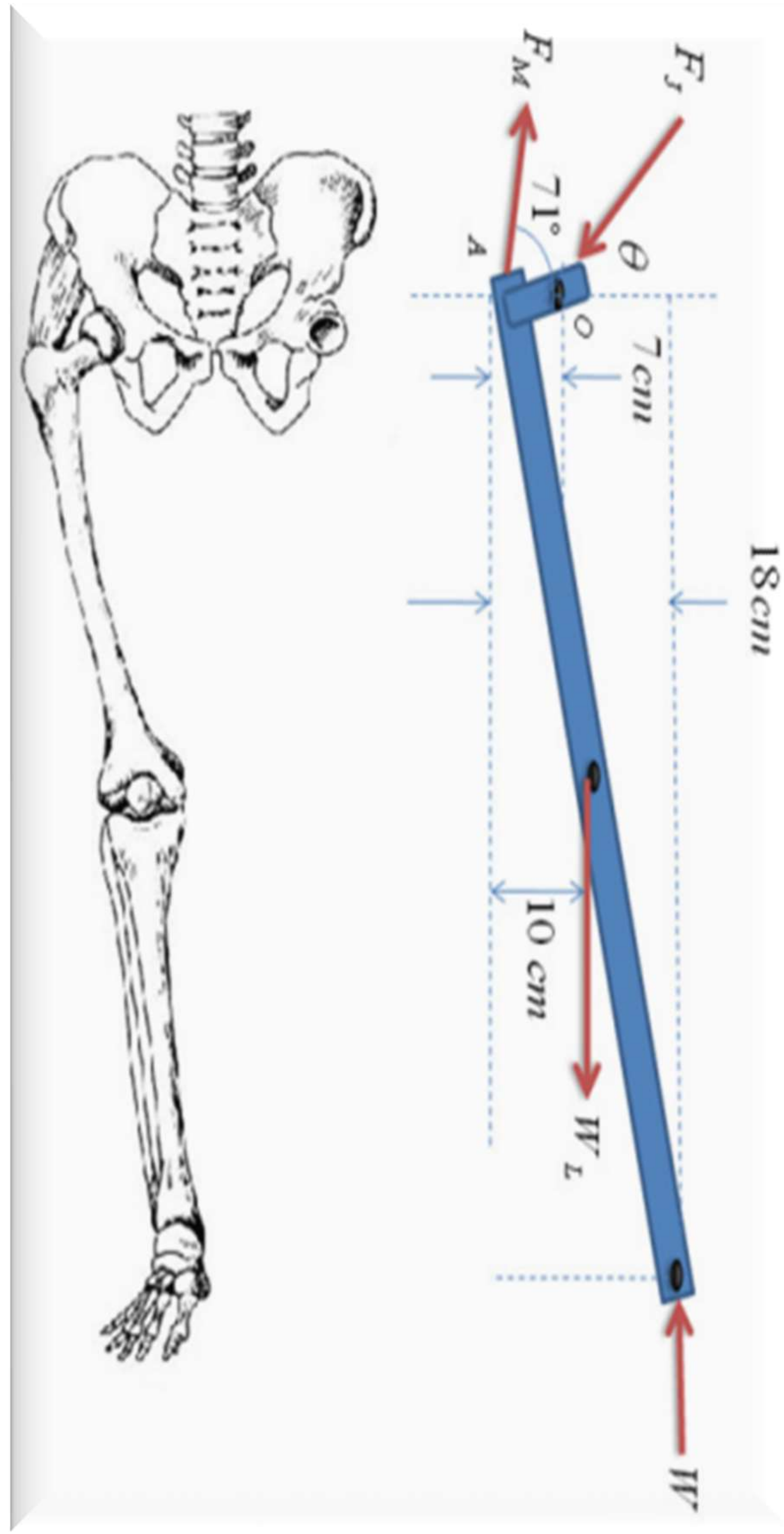
Back FBD

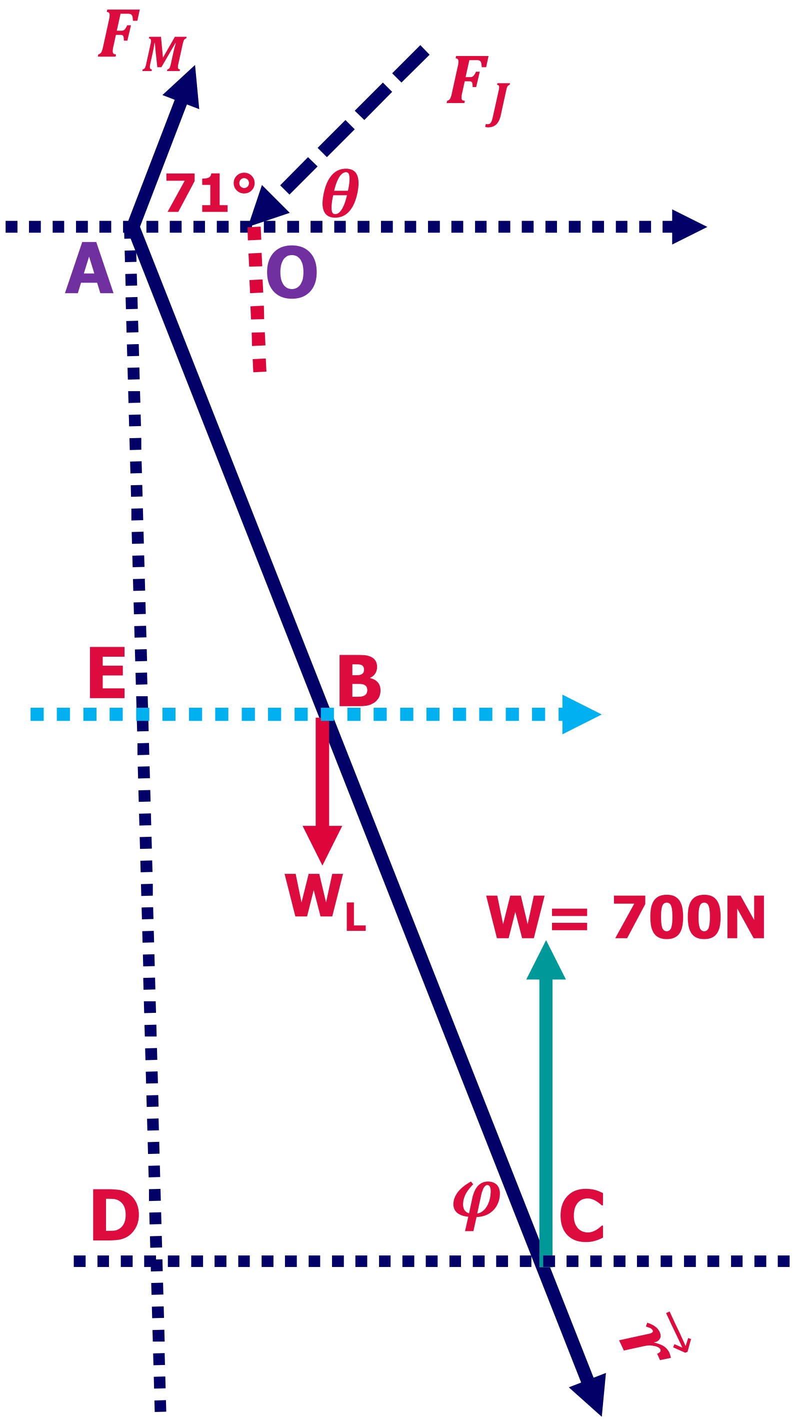

B
α = 12°
C
T
w1
A
θ = 30°
w3
w2
Fv

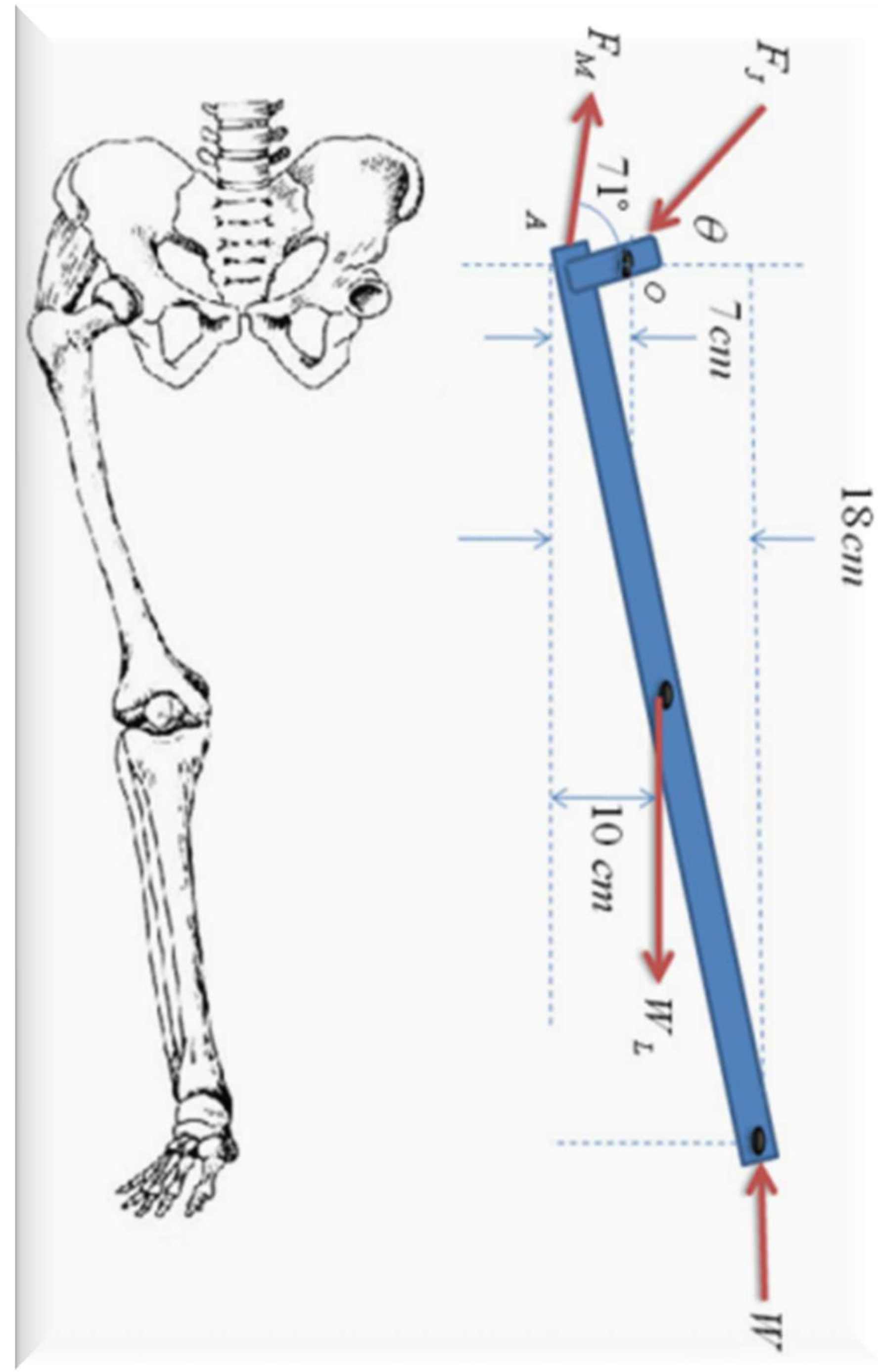

CGub
29°
10°
61°
Wub
8 cm
FB
39°
CGbox
Wbox
Fv
θ
35 cm
50 cm
Pivot point

The Hip Joint

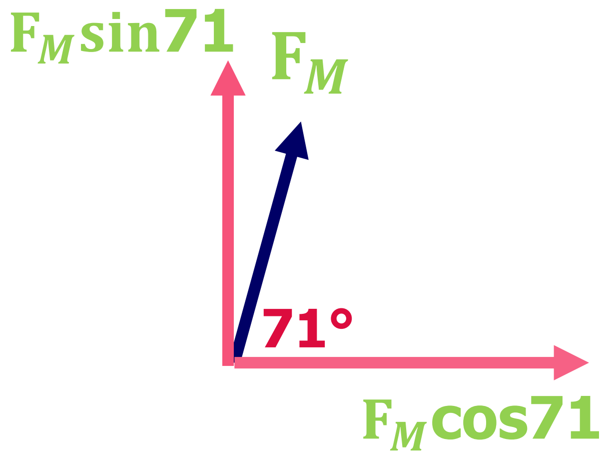
The hip joint and its simplified lever representation, with dimensions that are typical for a male body are shown in Figure 2.24. The hip is stabilized in its socket by a group of muscles, which is represented in Figure 2.24, (left) as a single resultant force F_M. When a person stands erect, the angle of this force is about 71° with respect to the horizon. W_L represents the combined weight of the leg, foot, and thigh. Typically, this weight is a fraction (0.185) of the total body weight W (i.e., $W_L = 0.185W$). The weight W_L is assumed to act vertically downward at the midpoint of the limb.





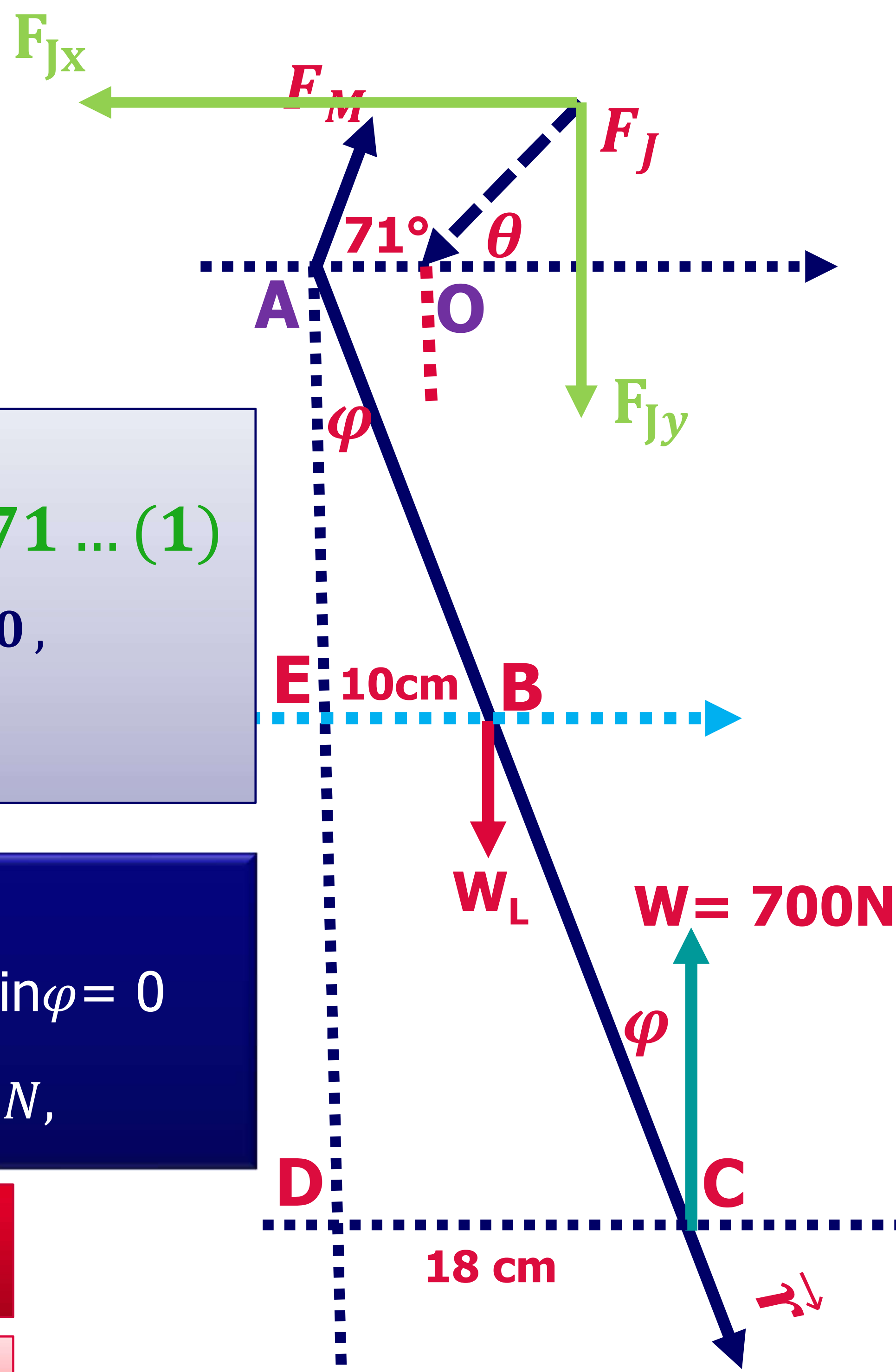


W_L=0.185W= 130N



$$\frac{EB}{AB} = \sin\varphi$$

$$\frac{DC}{AC} = \sin\varphi$$



$$\sum F_x = 0,$$

$$-F_{Jx} + F_M cos71 =, \quad F_{Jx} = F_M cos71 \dots (1)$$

$$\sum F_y = 0, \quad F_M sin71 - F_{Jy} - w_L + W = 0,$$

$$F_{Jy} = F_M sin71 + 570 \dots (2)$$

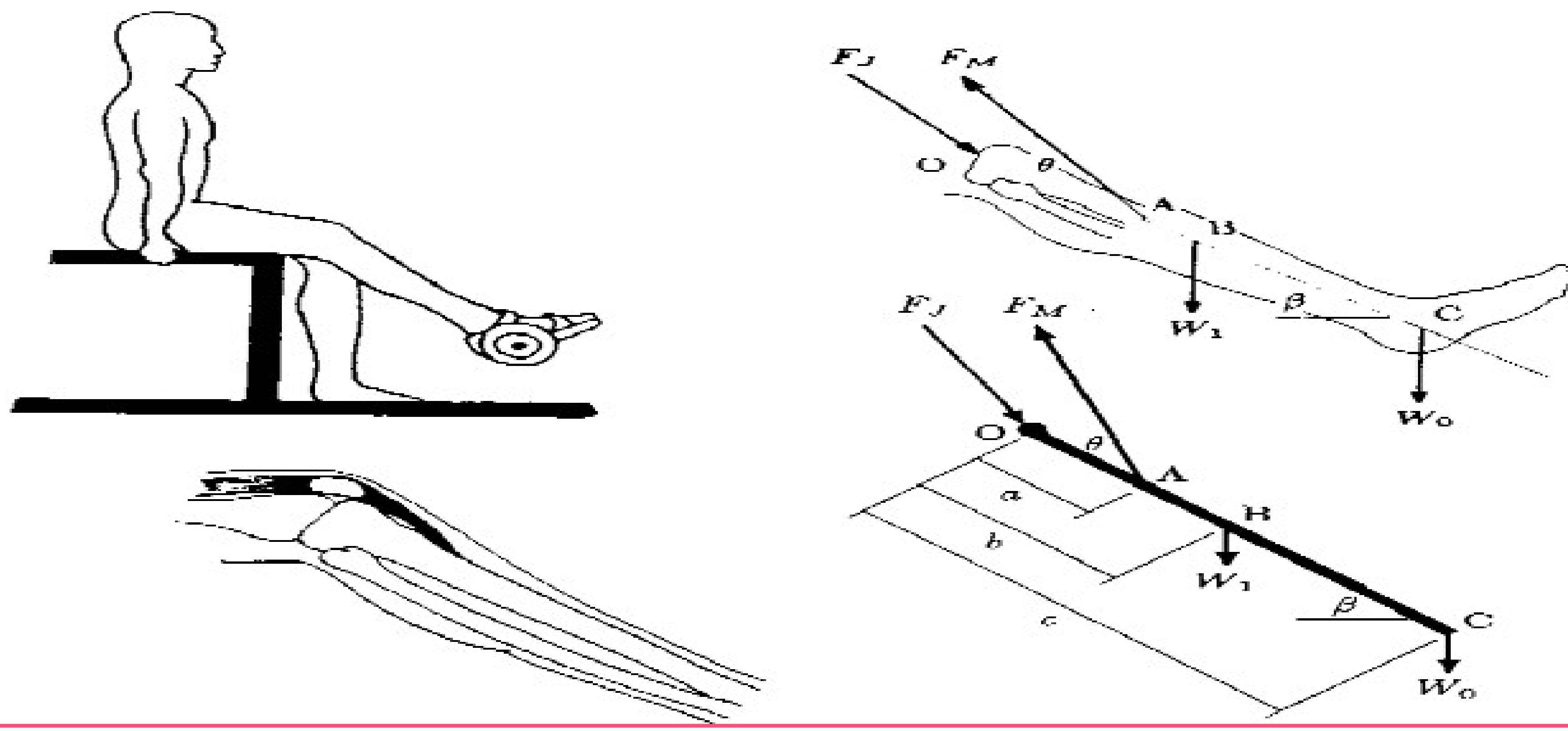
$$\sum \vec{\tau} = 0, \quad \tau_{F_M} + \tau_{F_J} + \tau_{w_L} + \tau_W = 0$$

$$0 - F_J(AO)\sin\theta - 130(AB)\sin\varphi + 700(AC)\sin\varphi = 0$$

$$F_{Jy} = \frac{700(18) - 130(10)}{7} = 1614\ N,$$

$$F_M = \frac{1614 - 570}{sin71} = 1104\ N$$

$$F_{Jx} = F_M cos71 = 1104 \cos 71 = 360\ N$$

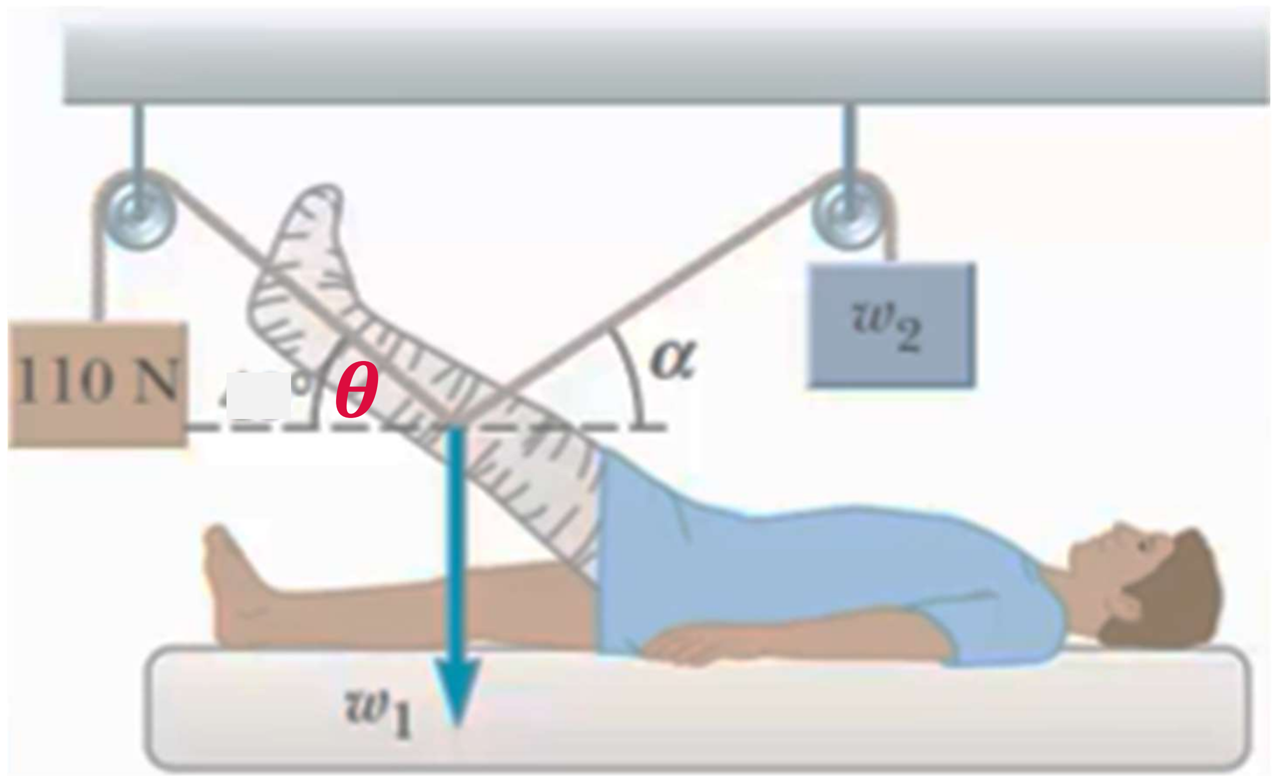


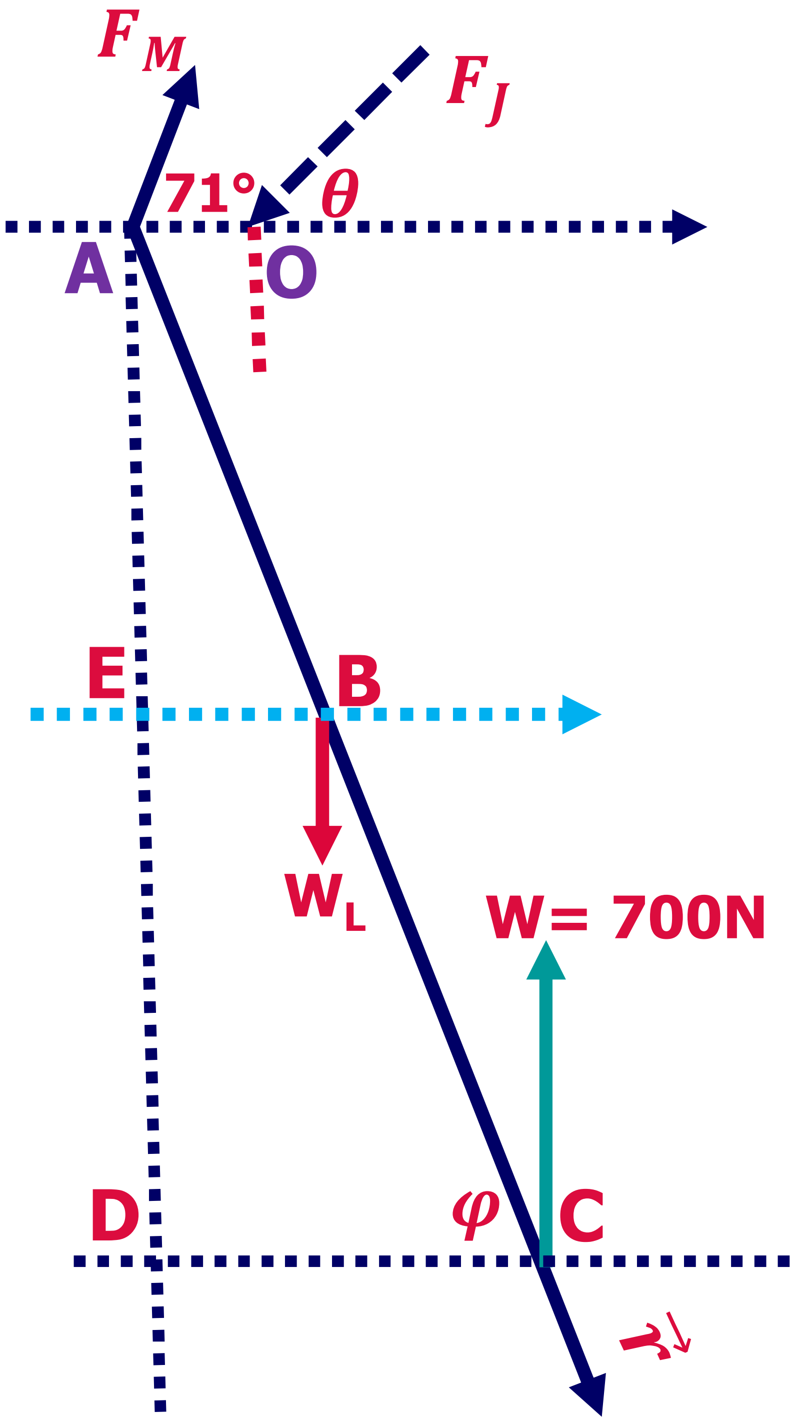
.The shown diagram illustrates the forces acting at knee and on tibia. The leg makes an angle of 30° with the horizontal. The weight of the leg W1=100N, and W0 = 75N. Find the bone-on-bone joint reaction force F_J and the force exerted by quadriceps muscle FM. Take $\theta = 32°$, $OA = 10cm$, $AB = 7$ cm and $BC = 15$ cm.

Example

The system of the shown leg and the cast is in equilibrium that no force is exerted on the hip joint by the leg and the cast , if w_1= 220N, and $\theta = 30°$, then the weight w_2 = ……………………

and the angle α =…………..





F_M
F_J
71°
θ
A
O
E
B
W_L
W= 700N
D
φ
C
$\vec{r}$

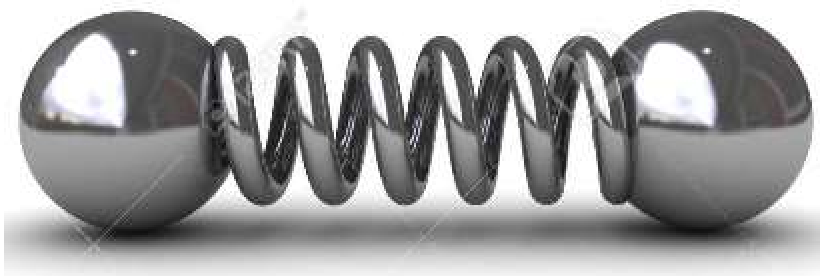
The shown free body diagram represent the hip joint for a standing person, where the forces at the muscle, the joints and the weights affecting the system are shown in the figure.

Take AO= 7 cm, AE= 20 cm, ED= 24 cm, W_L= 130 N, and $\varphi = 45°$, find the magnitude of F_M and F_J

Chapter 3

Elastic Properties of Material

Stress and Strain



States of Matter

- Solid
- Liquid
- Gas
- Plasmas

Solids: Stress and Strain

Stress = Measure of force felt by material

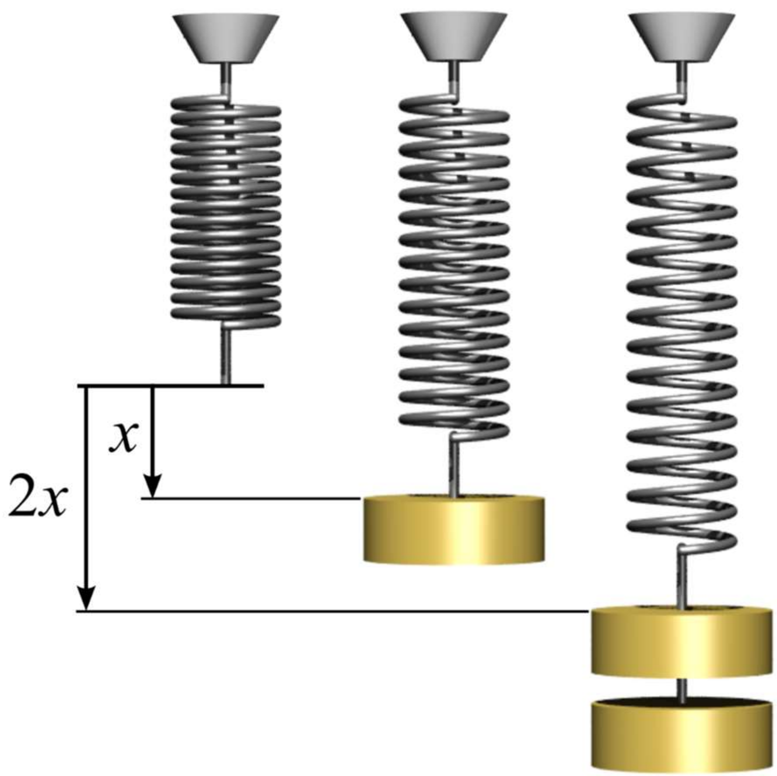
$$Stress\ (\sigma) = \frac{Force}{Area} = \frac{F}{A}$$

- **SI units are Pascals, 1 Pa = 1 N/m^2**
  (same as pressure)

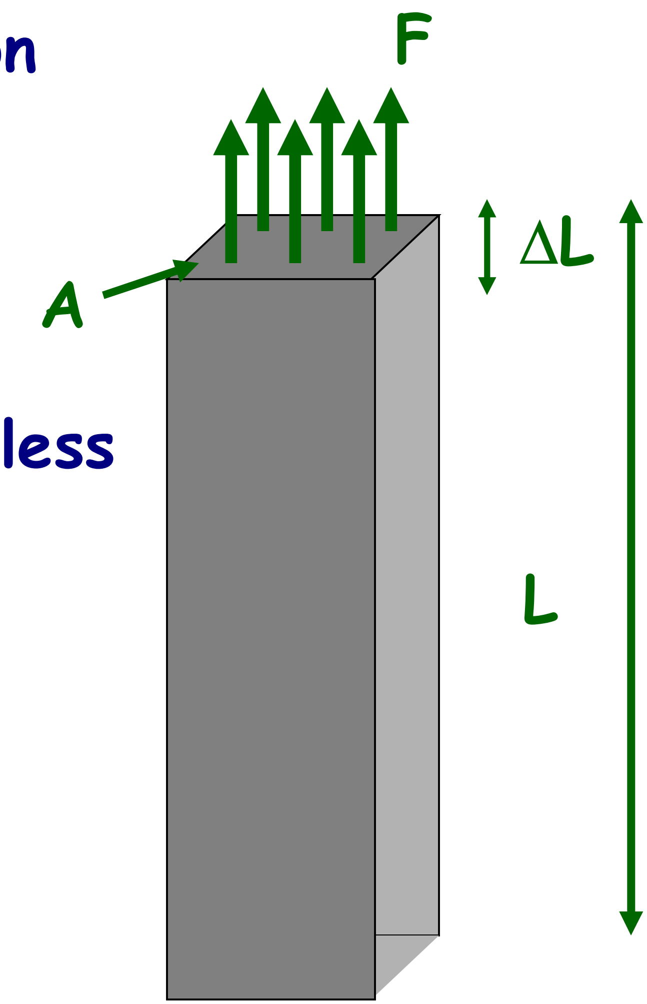
Solids: Stress and Strain

Strain = Measure of deformation

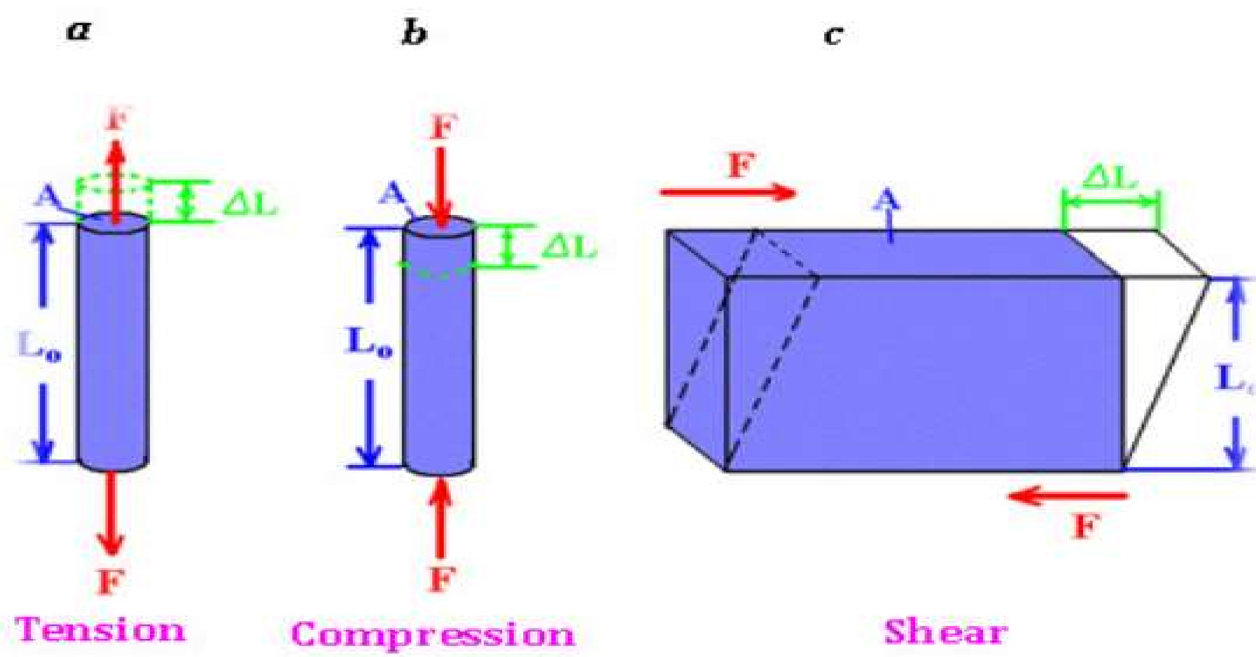
$$Strain = \frac{\Delta L}{L}$$



- **dimensionless**



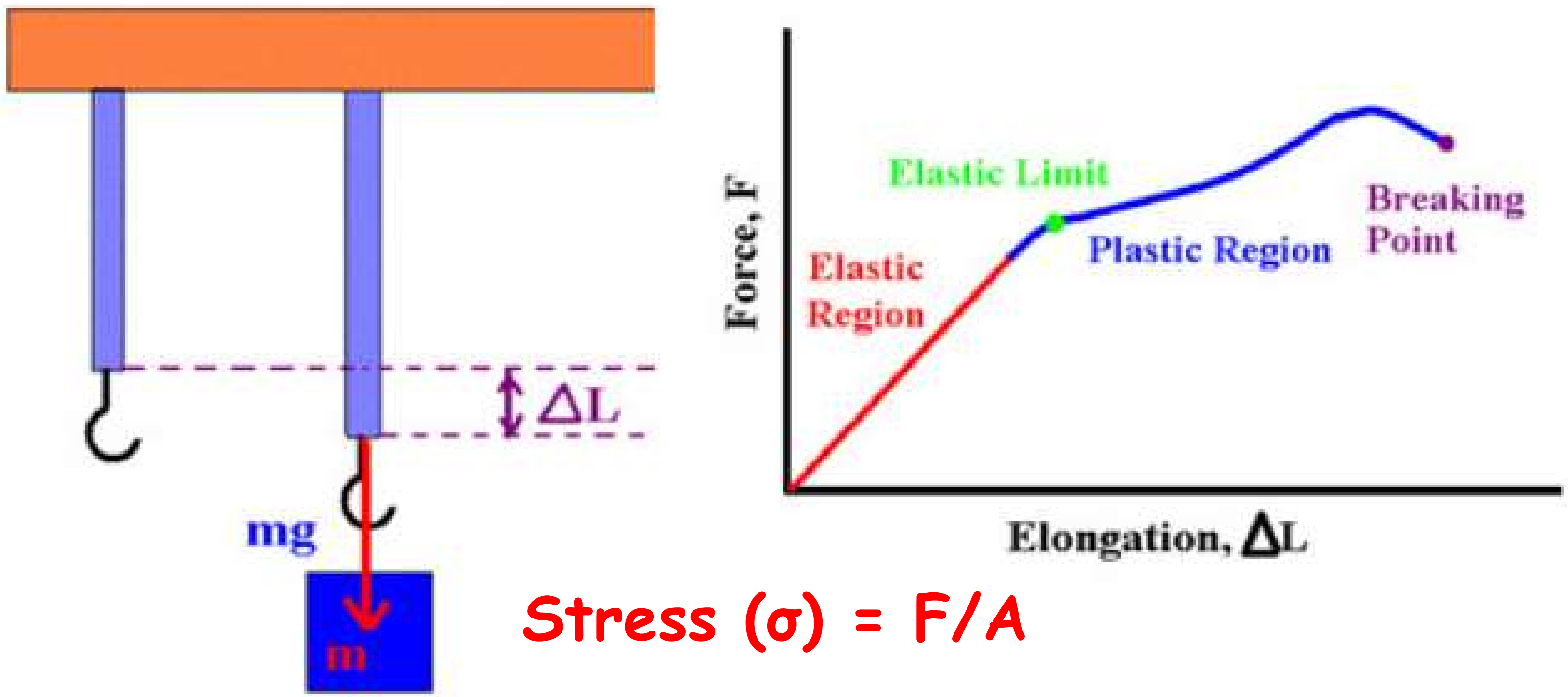
There are three kinds of stress commonly defined as:



extension as a fractional change in length. We call this the **tensile strain** which we define as **the extension per unit length**.

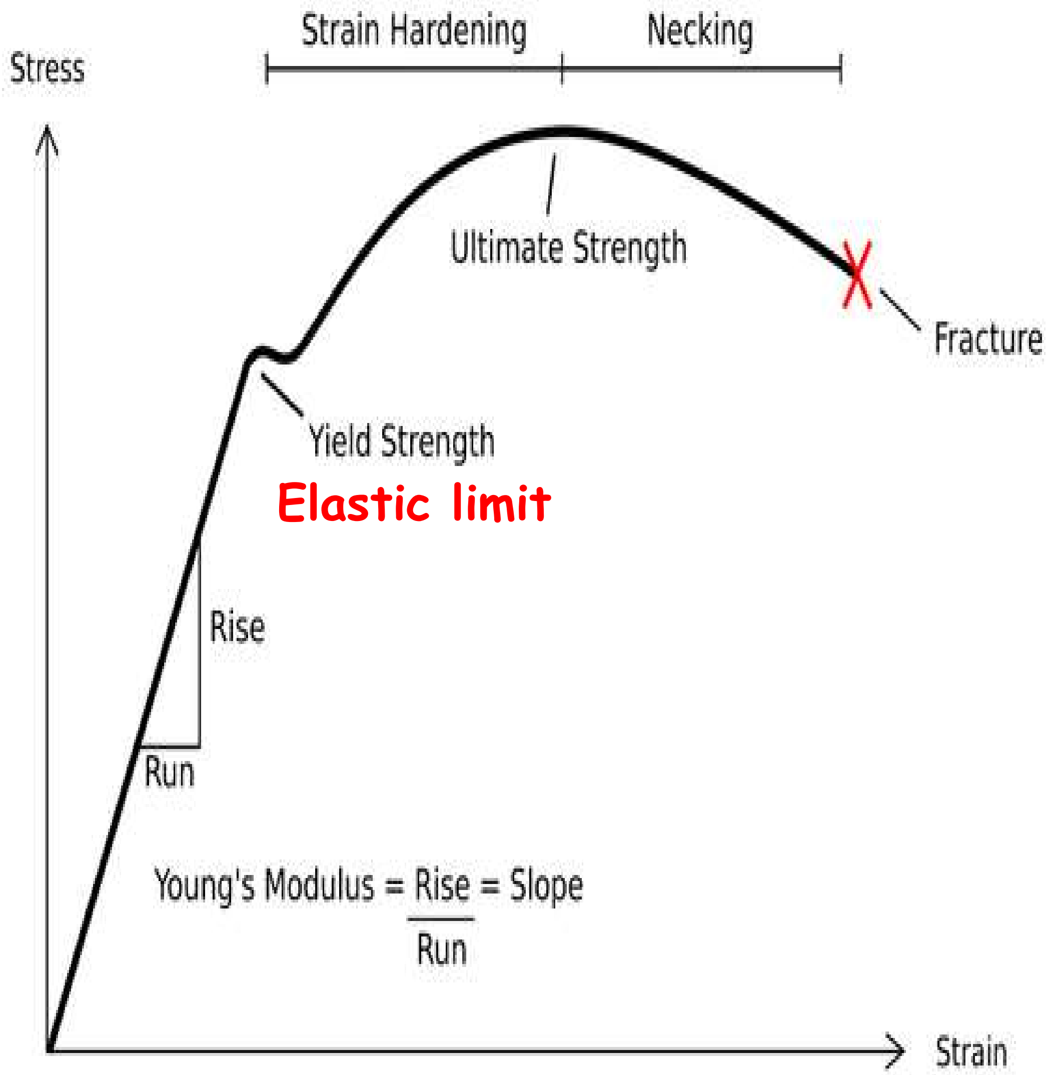
$$\varepsilon = \frac{\Delta L}{l} \tag{3.2}$$

Materials Beaviour under Stress



Stress (σ) = F/A

Elastic Region and Elastic Limit
Plastic Region Permanent Deformation
Ultimate strength
Fracture Point



Stress (σ) = F/A

Strain (ε) = ΔL / L

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L} = \frac{F\,L}{A\,\Delta L}$$

Linear Region (Elastic Region)

3.2 Young modulus

The form of Hooke's law represented in Eq. 3.3 is not the most useful form because the value of ***k*** is different not just for each material but also for different cross-sectional areas. So we need to introduce a new quantity known as Young modulus or elastic modulus, which depends only on the material type. Young modulus relates the elastic deformation of a solid to the associated stresses. It is defined ***as the ratio of stress over strain in the region in which Hooke's Law is obeyed for the material.*** This can be experimentally determined from the slope of a stress-strain curve created during tensile tests conducted on a sample of the material.

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L} = \frac{F\,L}{A\,\Delta L}$$

Units N/m², measure of the hardness (Stiffness) of the material

$$Y = \frac{\sigma}{\varepsilon}$$

$$Y = \frac{F/A}{\Delta l/l}$$

From which one can find

$$F = \frac{YA}{l}\Delta l$$

$F = k\ \Delta L$

K is the constant is called spring constant

K= YA/L measured in N/m

A 15 cm long animal tendon was found to stretch 3.7 mm by a force of 13.4 N. The tendon was approximately round with an average diameter of 8.5 mm. Calculate the elastic modulus of this tendon

$$l_0 = 15 \times 10^{-2}\ m$$

$$\Delta l = 3.7 \times 10^{-3}\ m\ ,\quad F = 13.4\ N$$

$$A = \pi r^2 = 3.14 \times \left(\frac{8.5}{2} \times 10^{-3}\right)^2$$

$$Y = \frac{F l_0}{A \Delta l} = \frac{\cancel{3.14} \times 15 \times 10^{-2}}{\cancel{3.14} \times (4.25 \times 10^{-3})^2 \times 3.7 \times 10^{-3}}$$

$$\boxed{Y = 2.2 \times 10^{8}\ N/m^2}$$

Values of Young´s Modulus for some Materials

Material	*Young's modulus ($N\ m^{-2}$)*
Steel	2.0×10^{11}
Brass	1.3×10^{11}
Cast iron	1.0×10^{10}
Aluminium	7.0×10^{10}
Marble	5.0×10^{10}
Concrete	2.0×10^{10}
Brick	1.4×10^{10}
Bone (human femur)	1.4×10^{10}
Timber (pine)	
parallel to the grain	1.0×10^{10}
across the grain	1.0×10^{9}
Nylon	5.0×10^{9}
Glass (crown)	7.1×10^{10}
Granite	4.5×10^{10}
Rubber	4.0×10^{6}

Example

(a) Find the minimum diameter of a steel wire 18 m long that elongates no more than 9 mm when a load of 380 kg is hung on its lower end. (b) If the elastic limit for this steel is $3 \times 10^8\ \mathrm{N/m^2}$, does permanent deformation occur with this load?

Steel wire, $l_o = 18m$, $\Delta l = 9 \times 10^{-3} m$

load $F = 3800 N$

Find the minimum diameter

from the table Young's Modulus for steel $= 2 \times 10^{11}\ N/m$

stress $\sigma = Y \varepsilon = Y \frac{\Delta l}{l_o}$

$$\frac{F}{A} = Y \frac{\Delta l}{l_o} \Rightarrow A = \frac{F l_o}{Y \Delta l}$$

$$\frac{F}{A} = Y\frac{\Delta l}{l_0} \Rightarrow A = \frac{F l_0}{Y \Delta l}$$

where $A = \pi r^2 = \frac{F l_0}{Y \Delta l}$

$$\therefore r^2 = \frac{F l_0}{\pi Y \Delta l} = \frac{3800 \cdot 18}{3.14 \times 2 \times 10^{11} \times 9 \times 10^{-3}}$$

$$r^2 = 12.1 \times 10^{-6} \Rightarrow r = 3.47 \times 10^{-3} \, m$$

eter $\Rightarrow$ $d = 6.95 \times 10^{-3} \, m \approx 7 \, mm$

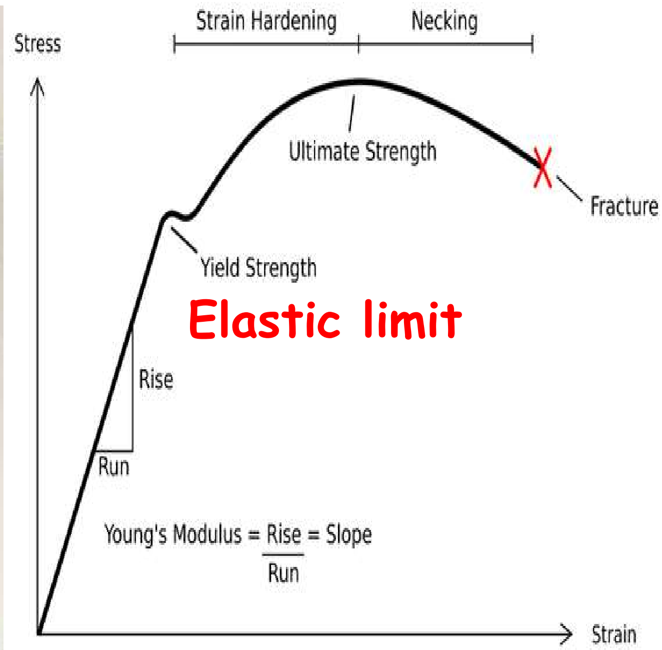
If the elastic limit for this steel is $3 \times 10^8\ \mathrm{N/m^2}$, does permanent deformation occur with this load?

$$\sigma = \text{Stress applied} = \frac{F}{A} = \frac{3800}{3.14 \times (3.47 \times 10^{-3})^2}$$

$$\sigma = 10^8\ \mathrm{N/m^2}$$

$$\text{elastic limit } \sigma_{el} = 3 \times 10^8\ \mathrm{N/m^2}$$

$\therefore$ $\sigma < \sigma_{el}$, so the deformation is temporary $\boxed{\text{[illegible]}}$



7. A pipe has an inner radius of $0.02\,m$ and an outer radius of $0.023\,m$. If subjected to a tension stress of $5 \times 10^7 \mathrm{Nm^{-2}}$, how large is the applied force?

$\sigma = 5 \times 10^7 \, N/m^2$

$F = \sigma A$

$A = \pi(r_{out}^2 - r_{in}^2)$

$r_{in} = 0.02$

$r_{out} = 0.023$

8. A man leg can be thought of as a shaft of bone $1.2\,m$ long. If the strain is 1.3×10^{-4} when the leg supports his weight, by how much is his leg shortened?

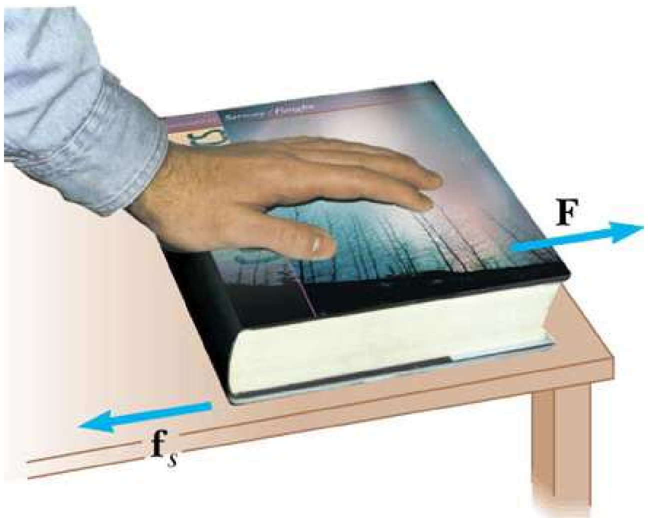
$l_0 = 1.2\,m$, strain $\varepsilon = \frac{\Delta l}{l_0} = 1.3 \times 10^{-4}$

Find Δl

$$\frac{\Delta l}{l_0} = \varepsilon_0 \Rightarrow \Delta l = l_0 \varepsilon$$

$$\Delta l = 1.2 \times 1.3 \times 10^{-4} = 1.56 \times 10^{-4}\,m = 156\,\mu m$$

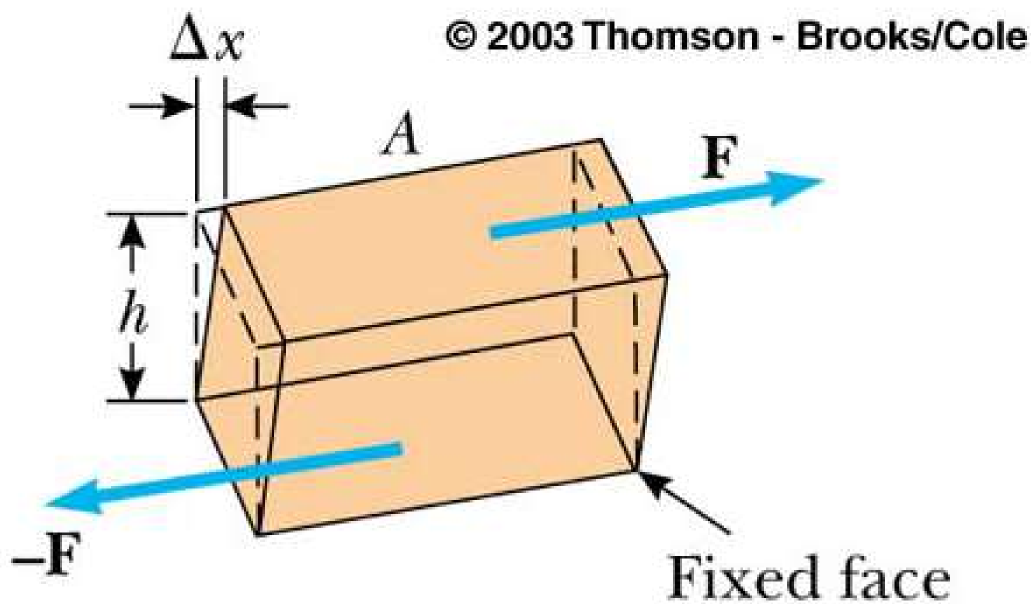
Shear Modulus



$$S = \frac{\left(F/A\right)}{\left(\Delta x/h\right)}$$

Sheer Stress

Sheer Strain



Stress in 3D

Bulk Modulus

$$B = -\frac{\Delta F/A}{\Delta V/V} = -\frac{\Delta P}{\left(\Delta V/V\right)}$$

Change in Pressure

Volume Strain

$$B = Y/3$$

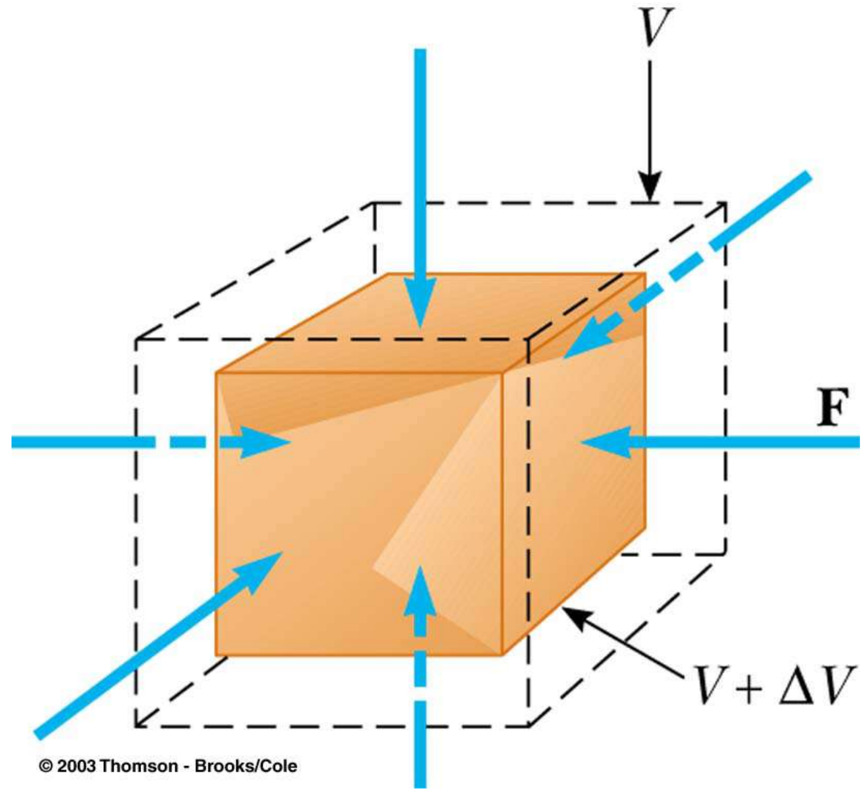
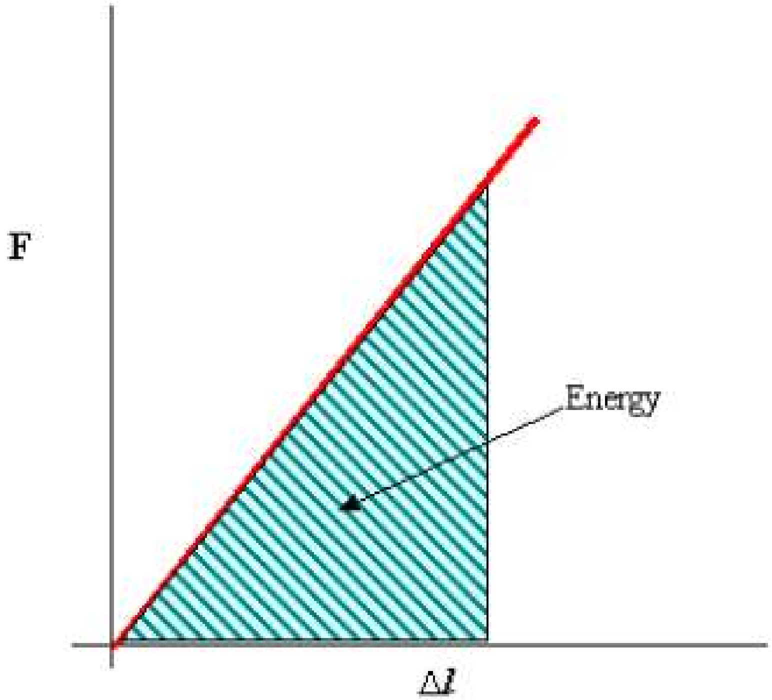


Table3.1: typical values for elastic Modulus

substance	young modulus(N/m^2)	Shear modulus(N/m^2)	bulk modulus(N/m^2)
Tungsten			
Steel			
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7×10^{10}	2.5×10^{10}	7×10^{10}
Glass	$6.5 - 7.8 \times 10^{10}$	$2.6 - 3.2 \times 10^{10}$	$5 - 5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	–	–	0.21×10^{10}

Elastic Strain Energy

When we stretch a wire, a work done on the wire. We are stretching the bonds between the atoms. If we release the wire, we can recover the energy stored in the wire due to stretch, which is called the **elastic strain energy**.



$$\mathcal{E} = \frac{1}{2} F \Delta l$$

Substituting for F from eq. 3.5 we get

$$F = \frac{YA\Delta l}{l}$$

$$\mathcal{E} = \frac{YA}{2l} \Delta l^2$$

Bone Fracture, Energy consideration

$$F_B = \sigma_B A = \frac{YA}{l}\Delta l$$

The compression Δl at the breaking point is, therefore,

$$\Delta l = \frac{\sigma_B l}{Y}$$

From Eq. 3.8, the energy stored in the compressed bone at the point of fracture is

$$\mathcal{E} = \frac{YA}{2l}\left(\frac{\sigma_B l}{Y}\right)^2$$

$$\boxed{\mathcal{E} = \frac{A l {\sigma_B}^2}{2Y}}$$

Example 3.6

Discuss the possibility of fracture of two leg bones that each have a length of about 90 cm and an average area of about 6 cm2 when a 70 kg person jump from a height of 60 cm.

The breaking stress of the bone σ_B is $10^9\ dyn/cm^2$, and Young's modulus for the bone is $14\ 10^{10}\ dyn/cm^2$. $(Hint: dyn = 10^{-5}N = 10\mu N, erg = 10^{-7}J,$ $dyn/cm^2 \equiv 10^{-1}\ N/m^2)$

The total energy absorbed by the bones of one leg at the point of compressive fracture is, from Eq. 3.11,

$$\mathcal{E} = \frac{1}{2}\frac{Al{\sigma_B}^2}{Y}$$

$$= \frac{1}{2}\frac{(6 \times 10^{-4}m^2)(0.9m)(10^8N/m^2)^2}{14 \times 10^9N/m^2} = 192.86J$$

Now we have to calculate the energy gained by jumping

$$\mathcal{E} = mgh = 70\ (9.8)(0.60) = 411.6\ J$$

Now, this energy is evenly distributed on both legs, thus each leg absorbs energy equals to $411.6J/2 = 205.8J$. This number exceeds the allowed energy to be stored at the bone break which will endanger the leg bones and it would cause bone fracture. If this person jumped on one leg this might lead to multiple fracture because of the huge difference between the allowed energy (less than $192.86J$) and the energy stored due to jumping ($411.6J$).

Examples

16. Assume that a 50 kg runner trips and falls on his extended hand. If the bones of one arm absorb all the kinetic energy (neglecting the energy of the fall), what is the minimum speed of the runner that will cause a fracture of the arm bone? Assume that the length of arm is 1 m and that the area of the bone is 4 cm^2.

$$m = 50\,kg \,,\ A = 4\,cm^2 = 4\times10^{-4}\,m^2 \,,\ l = 1\,m$$

$$E_k \equiv \text{Kinetic energy} = \frac{1}{2} m v^2 = \frac{1}{2}(50)v^2 = 25 v^2$$

$$\mathcal{E}_{fracture} = \frac{A l \sigma_B^2}{2Y} = \frac{4\times10^{-4}\times1\times(10^8)^2}{2\times14\times10^9} = 142.86\,J = 25 v^2$$

$$v^2 = \frac{142.86}{25} \Rightarrow v = 5.7\,m/s$$

A child slides across a floor in a pair of rubber-soled shoes. The frictional force acting on each foot is 20 N. The footprint area of each shoe's sole is 14 cm^2, and the thickness of each sole is 5 mm. Find the horizontal distance by which the upper and lower surfaces of each sole are offset. The shear modulus of the rubber is 3×10^6 N/m^2.

$$S = \frac{F/A}{\Delta x/h} \rightarrow\rightarrow \Delta x = \frac{Fh}{AS} = \frac{20\,(5\times10^{-3})}{(14\times10^{-4})(3\times10^{6})}$$

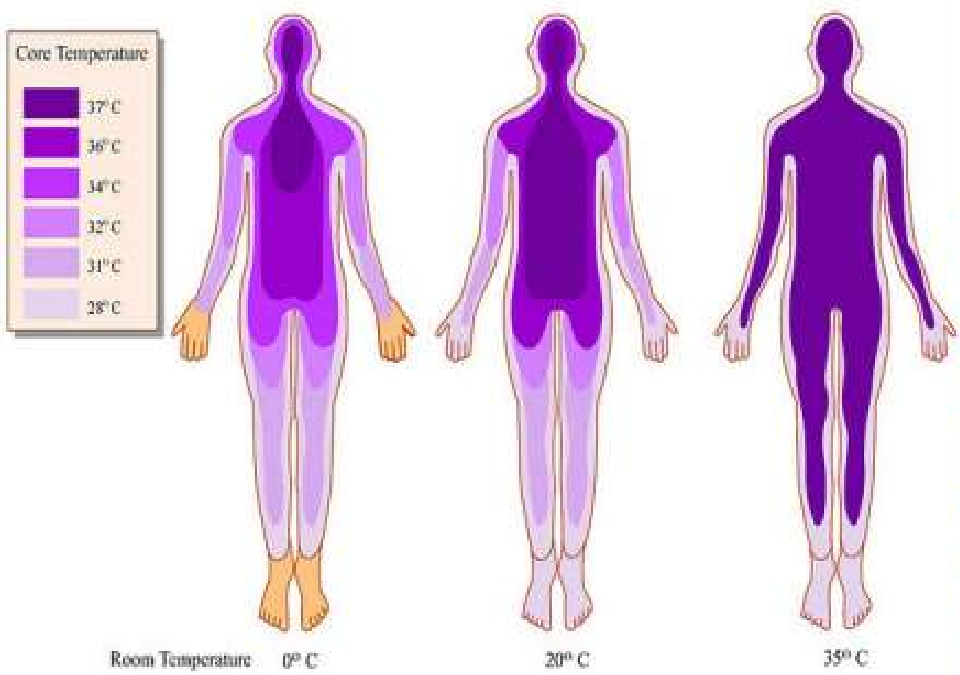
$$= \frac{10^{-3}}{42} = 2.4 \times 10^{-5} m$$

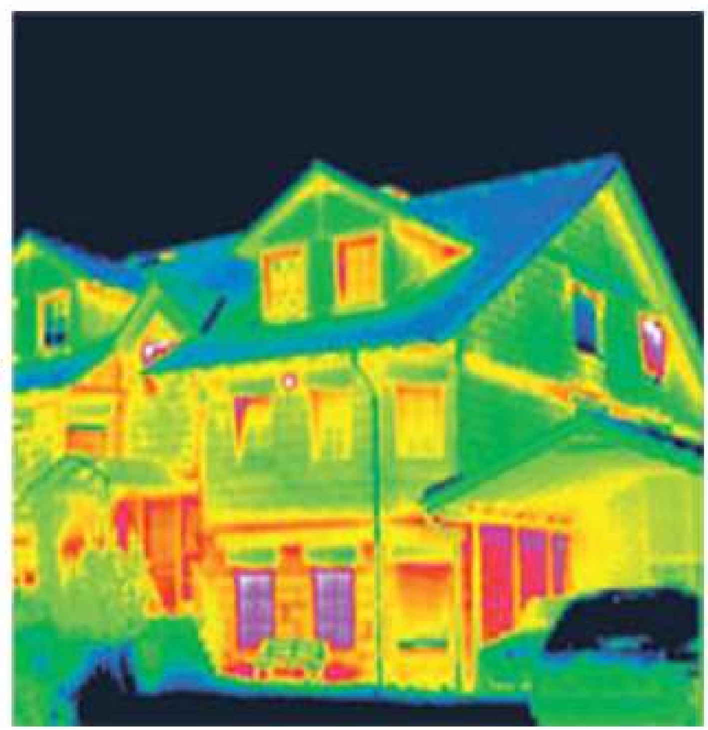
$$S = \frac{\left(F/A\right)}{\left(\Delta x/h\right)}$$

Sheer Stress

Sheer Strain

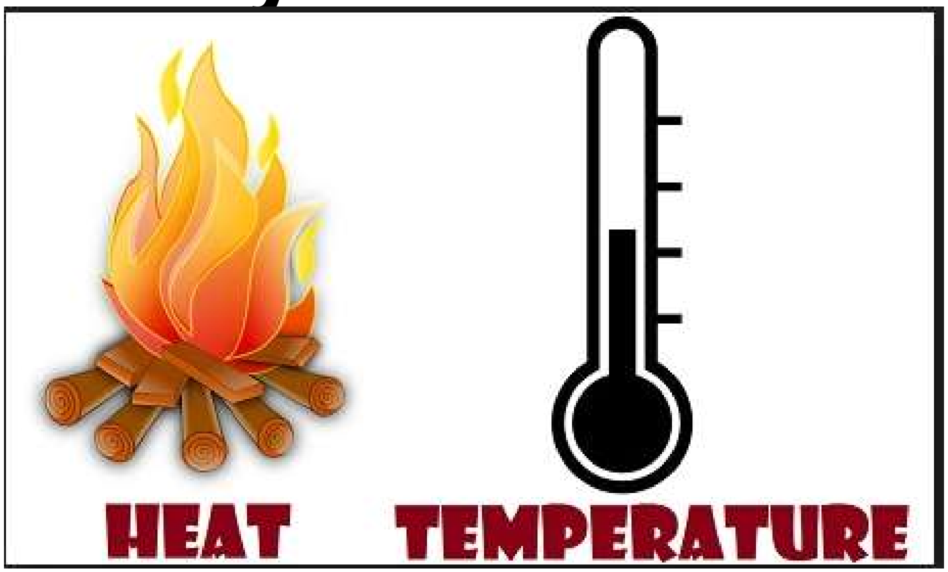
Thermal Properties of Matter





Definitions

- **Heat and Temperature (what is the difference)**
- **Thermal Contact – if energy can be exchanged between two objects, then they are in thermal contact**
- **Thermal Equilibrium – if two objects are in thermal contact and there is no exchange of energy, then they are in thermal equilibrium**



The Law of Equilibrium

- Zeroth Law of Thermodynamics
  - If objects A and B are separately in thermal equilibrium with object C, then A and B are in thermal equilibrium with each other

Definition of Temperature

- **If objects A and B are in thermal equilibrium, then they are at the same temperature.**

How to measure the Temperature

- ***Many physical properties of materials change sufficiently with temperature to be used as the bases for thermometers:***

⇨ The change in volume of a liquid

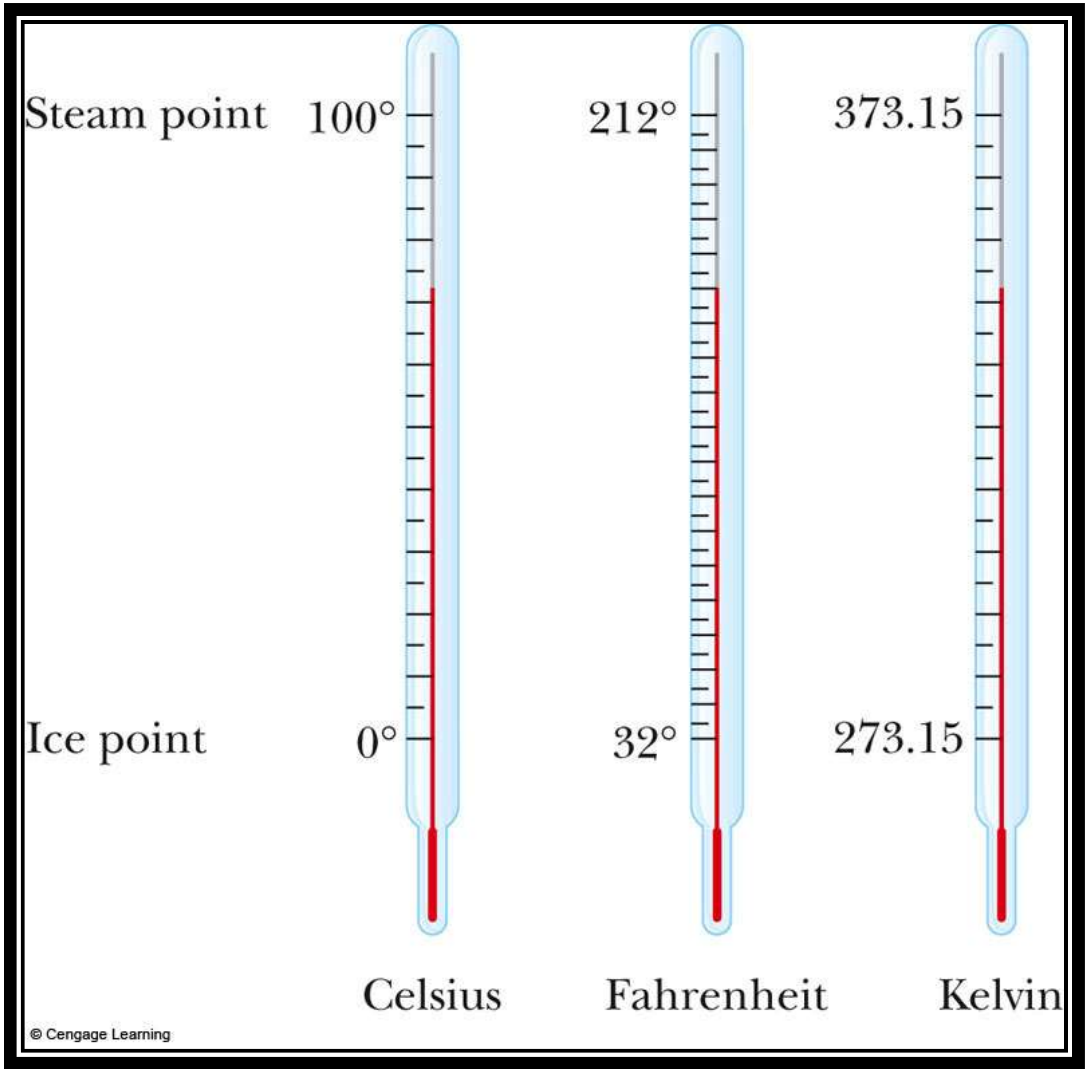
⇨ The change in length of a solid

⇨ The change in pressure of a gas held at constant volume

⇨ The change in electrical resistance of a conductor

⇨ The change in color of a very hot object.

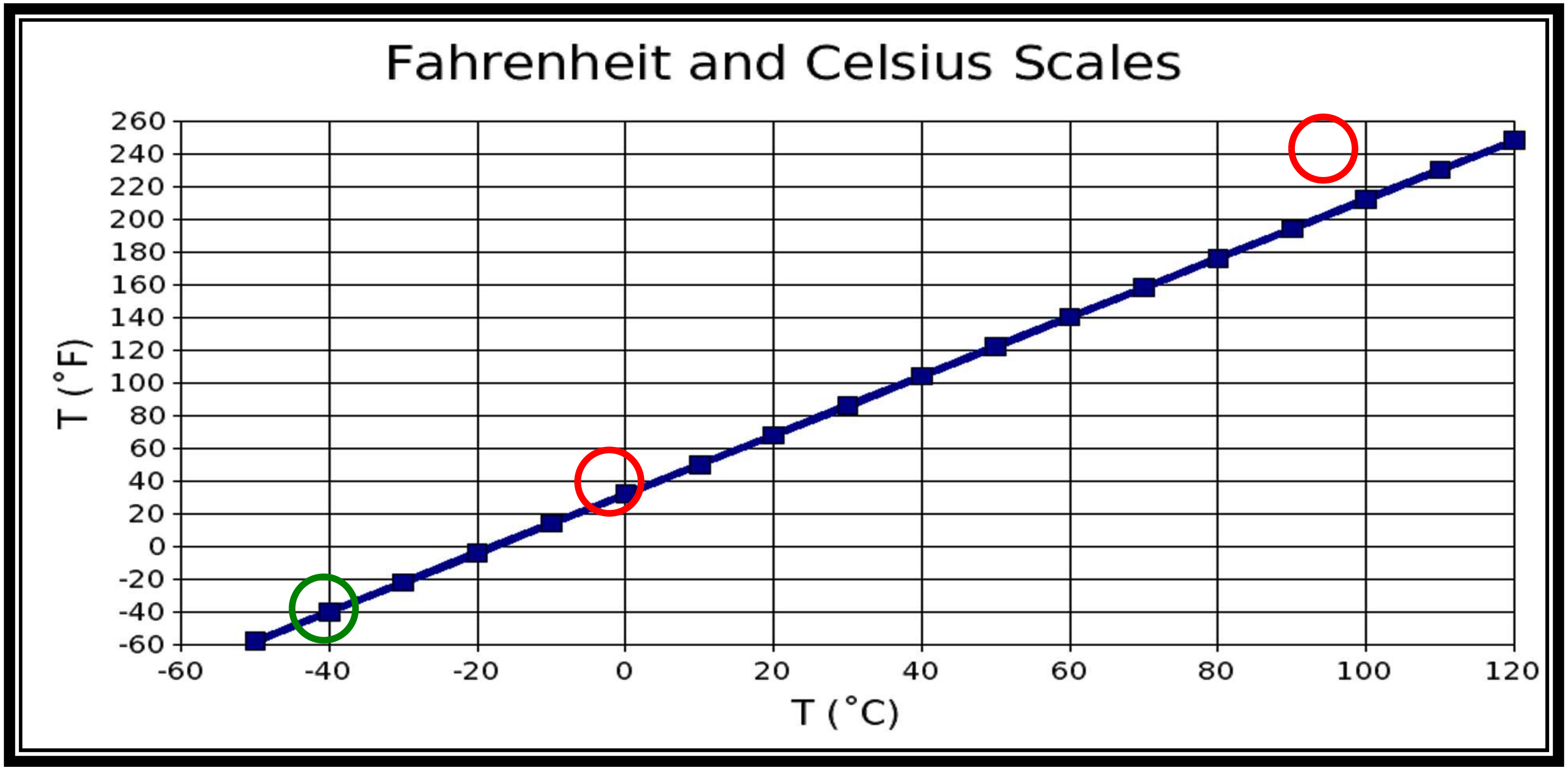
Temperature Scales



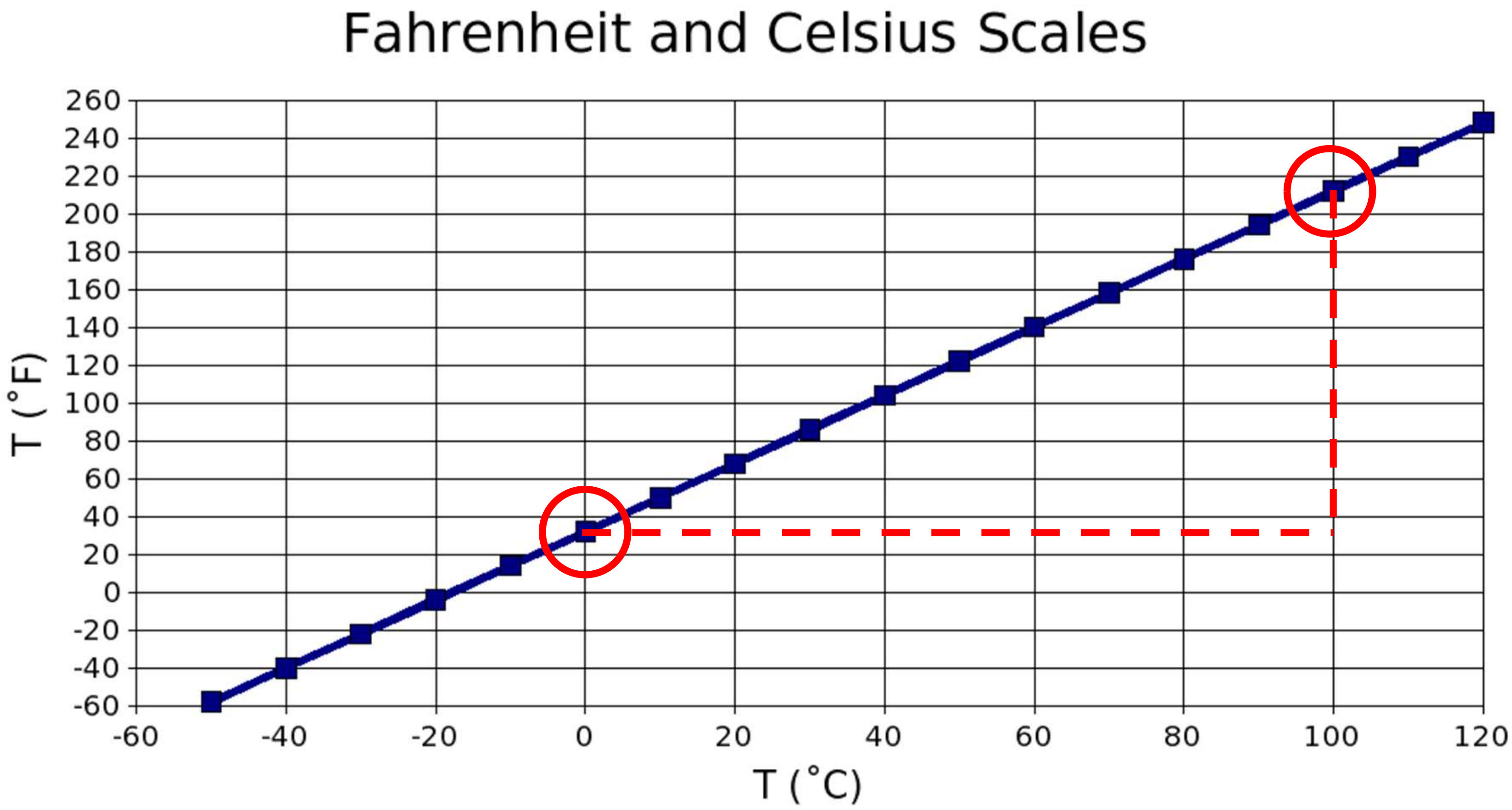
- Column of fluid changes height in response to warmth or coolness of surroundings
- Numbers assigned to the height establishes the *temperature scale*
- Each division in the scale is called a *degree*

Temperature Scales

- **Defined by:**
  - **Height of column when water freezes ($0°C = 32°F$)**
  - **Height of column when water boils ($100°C = 212°F$)**
- **Note: $-40°C = -40°F$**



Conversions



Slope: $$\frac{\Delta T_F}{\Delta T_C} = \frac{212°\text{F} - 32°\text{F}}{100°\text{C} - 0°\text{C}} = \frac{180°\text{F}}{100°\text{C}} = \frac{9}{5}$$

Intercept = 32°F

Conversions

	From °C to °F	From °F to °C
Temperature Reading	$T_F = \frac{9}{5} T_C + 32°$	$T_C = \frac{5}{9}\left(T_F - 32°\right)$
Temperature Change	$\Delta T_F = \frac{9}{5} \Delta T_C$	$\Delta T_C = \frac{5}{9} \Delta T_F$

Absolute Zero and Kelvin Scale

- Temperature is in units called kelvins (K)
- $T = 0$ K is called absolute zero
- Represents the temperature at which an ideal gas

$$T = T_C + 273.15\,\mathrm{K}$$

$$\Delta T = \Delta T_C$$

I. Characteristics of a Gas

A) Gases assume the shape and volume of a container.

B) Gases are the most compressible of all the states of matter.

C) Gases will mix evenly and completely when confined to the same container.

D) Gases have lower densities than liquids or solids.

C) The study of gases and its results formed the primary experimental evidence for our belief that matter is made up of particles. MATTER IS NOT CONTINUOUS.

III. Measurable Properties of a Gas

A) Mass - moles - symbol is n

B) Pressure - symbol is P

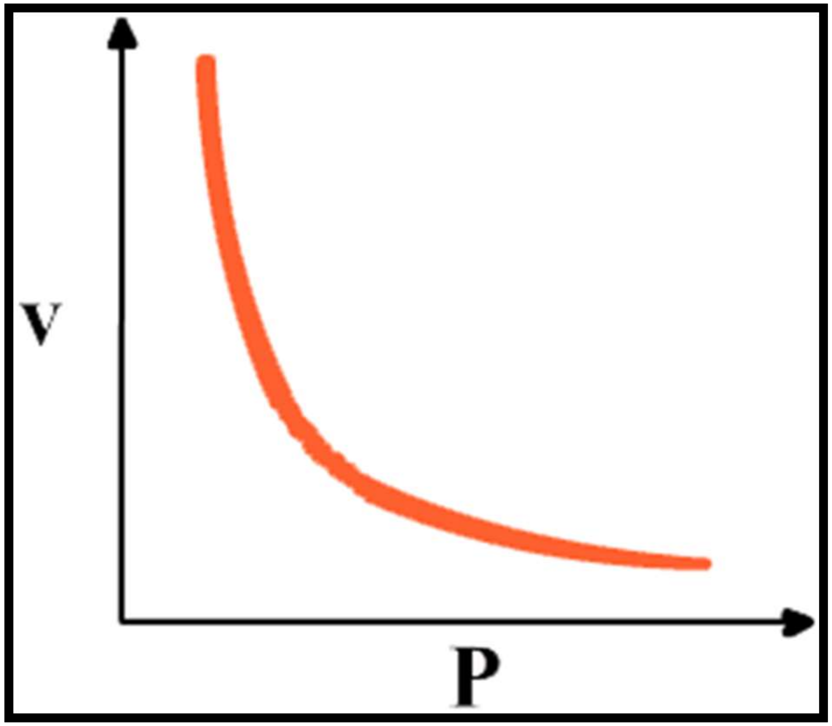
C) Volume - symbol is V

D) Temperature - must be in K - symbol is T

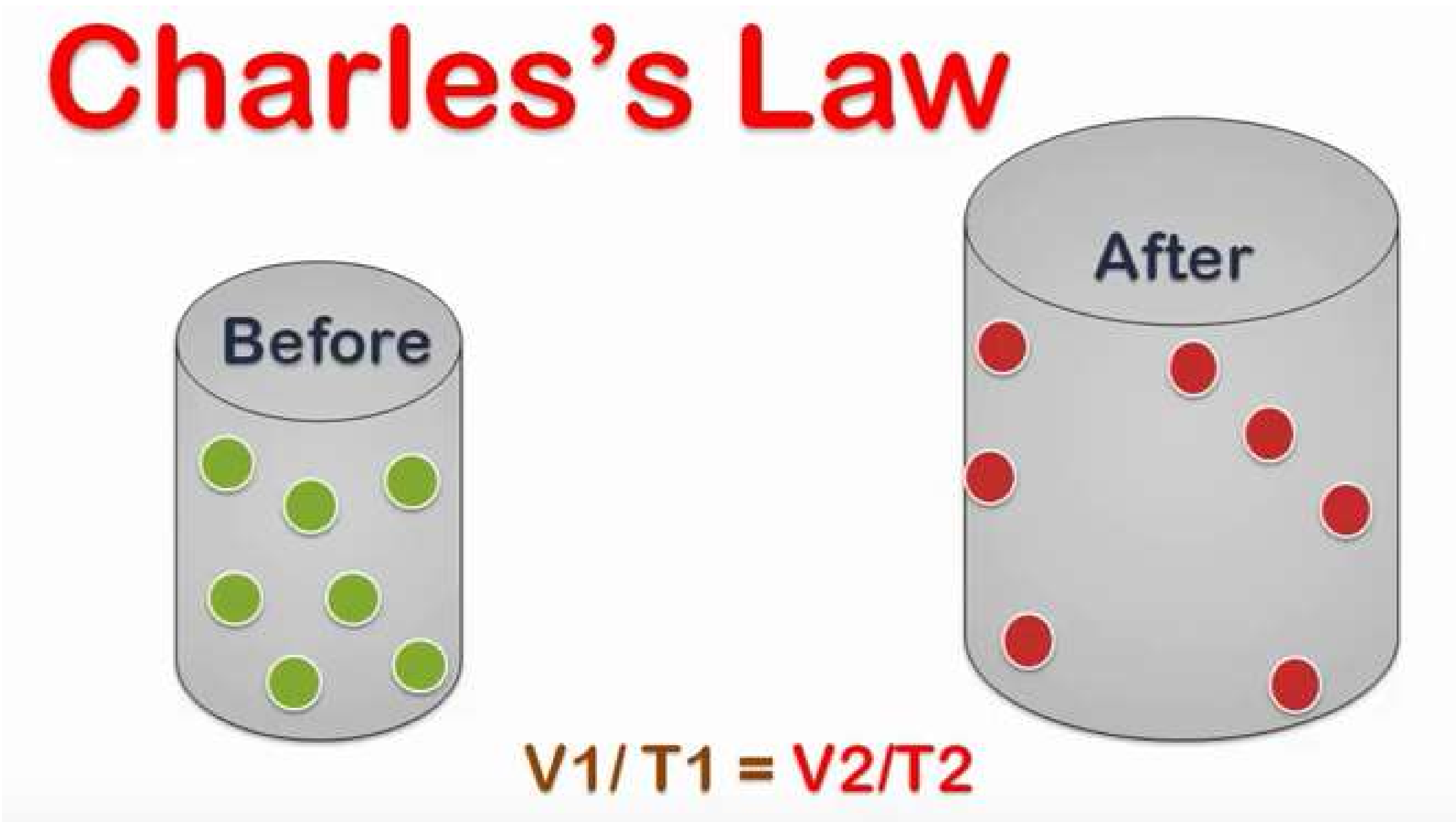
Boyle's Law: For a fixed amount of gas and constant temperature, **PV = constant.**

$$P_1V_1 = P_2V_2$$

T is constant



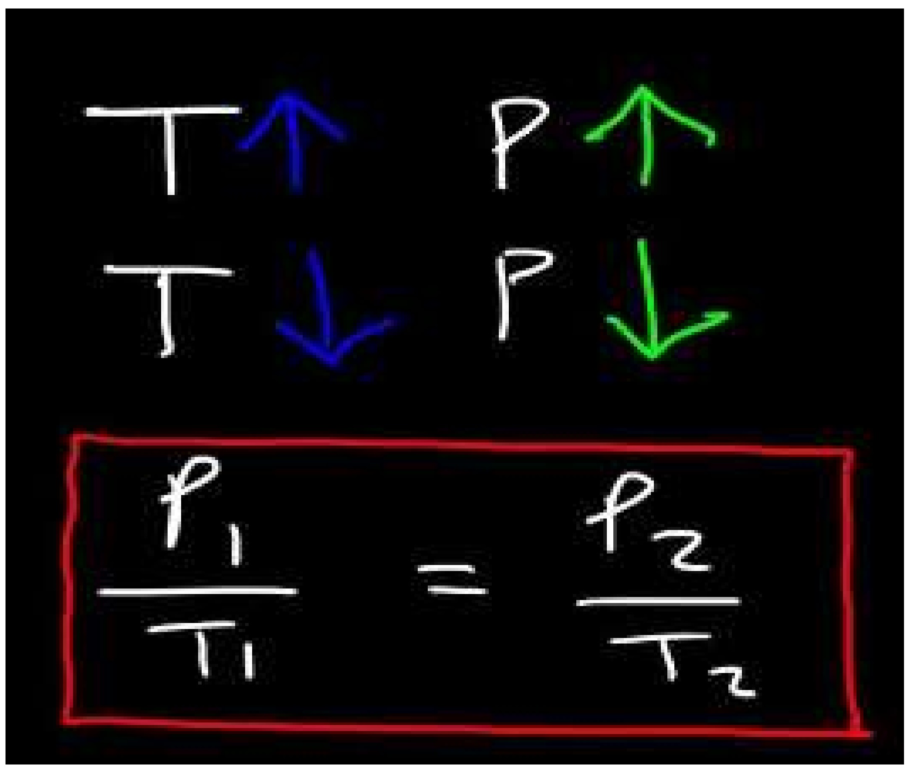
Charles's Law: at constant pressure the volume is linearly proportional to temperature. V/T = constant



Gay-Lussac’s Law

Old man Lussac studied the direct relationship between temperature and pressure of a gas at constant volume.

As the temperature increases the pressure of a gas exerts on its container increases.



The Ideal Gas Law

$PV = nRT$

P = Pressure (in kPa) V = Volume (in L)

T = Temperature (in K) n = moles

$$R = 8.31 \frac{kPa \cdot L}{K \cdot mol}$$

R is constant. If we are given three of P, V, n, or T, we can solve for the unknown value.

From Boyle's Law:

$P_iV_i = P_fV_f$ or $PV = constant$

From combined gas law:

$P_iV_i/T_i = P_fV_f/T_f$ or $PV/T = constant$

Ideal gas law the functional relationship between the pressure, volume, temperature and moles of a gas. $PV = nRT$; all gases are ideal at low pressure. $V = nRT$. Each of the individual laws is contained in this equation.

Sample problems

How many moles of H_2 is in a 3.1 L sample of H_2 measured at 300 kPa and 20°C?

$PV = nRT \quad P = 300\text{ kPa},\ V = 3.1\text{ L},\ T = 293\text{ K}$

$$(300\text{ kPa})(3.1\text{ L}) = n\,(8.31\text{ kPa}\bullet\text{L/K}\bullet\text{mol})(293\text{ K})$$

$$\frac{(300\text{ kPa})(3.1\text{ L})}{(8.31\text{ kPa}\bullet\text{L/K}\bullet\text{mol})(293\text{ K})} = n = 0.38\text{ mol}$$

Ideal Gas Law

The pressure, P, volume V, and temperature T of an ideal gas are related by a simple formula called the **ideal gas law**.

$$\boxed{PV = nRT}$$

$$R = 8.31\frac{J}{K \cdot mol}$$

$$R = 0.082\frac{L \cdot atm}{K \cdot mol}$$

$$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$$

2.50 g of $XeF4$ gas is placed into an evacuated 3.00 $liter$ container at 80°C. What is the pressure in the container?

Solution

$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$n = \frac{m}{Mw} = \frac{2.5}{207.28} = 0.012\, mole, R = 8.314\, J/mole.K,$$

$$T_K = T_C + 273.15 = 80 + 273 = 353\, K$$

$$V = 3 \times 10^{-3}\, m^3$$

So,

$$P = \frac{0.012 \times 8.314 \times 353}{3 \times 10^{-5}} = 1.17 \times 10^4\, Pa$$

A hydrogen gas thermometer is found to have a volume of $100.0\ cm^3$ when placed in an ice-water bath at $0°C$. When the same thermometer is immersed in boiling liquid chlorine, the volume of hydrogen at the same pressure is found to be $87.2\ cm^3$. What is the temperature of the boiling point of chlorine?

Since

$$P_1 = P_2 \quad \text{at } T_1 = 273\ K$$

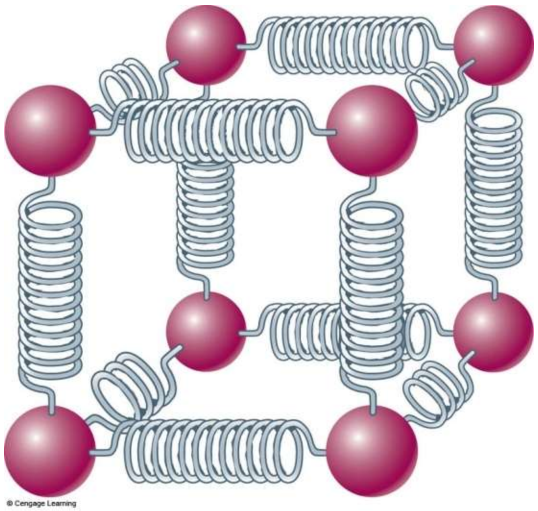
Thus,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \rightarrow T_2 = \frac{V_2}{V_1} T_1$$

$$T_2 = \frac{87.2}{100} \times 273 = 238.056\ K$$

Thermal Expansion

- **For solids and liquids:**
  - **Energy increase via heat input**
  - **Atoms vibrate with greater amplitude**
  - **Average separation increases**
- **Leads to macroscopic expansions**



(a)

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(b)

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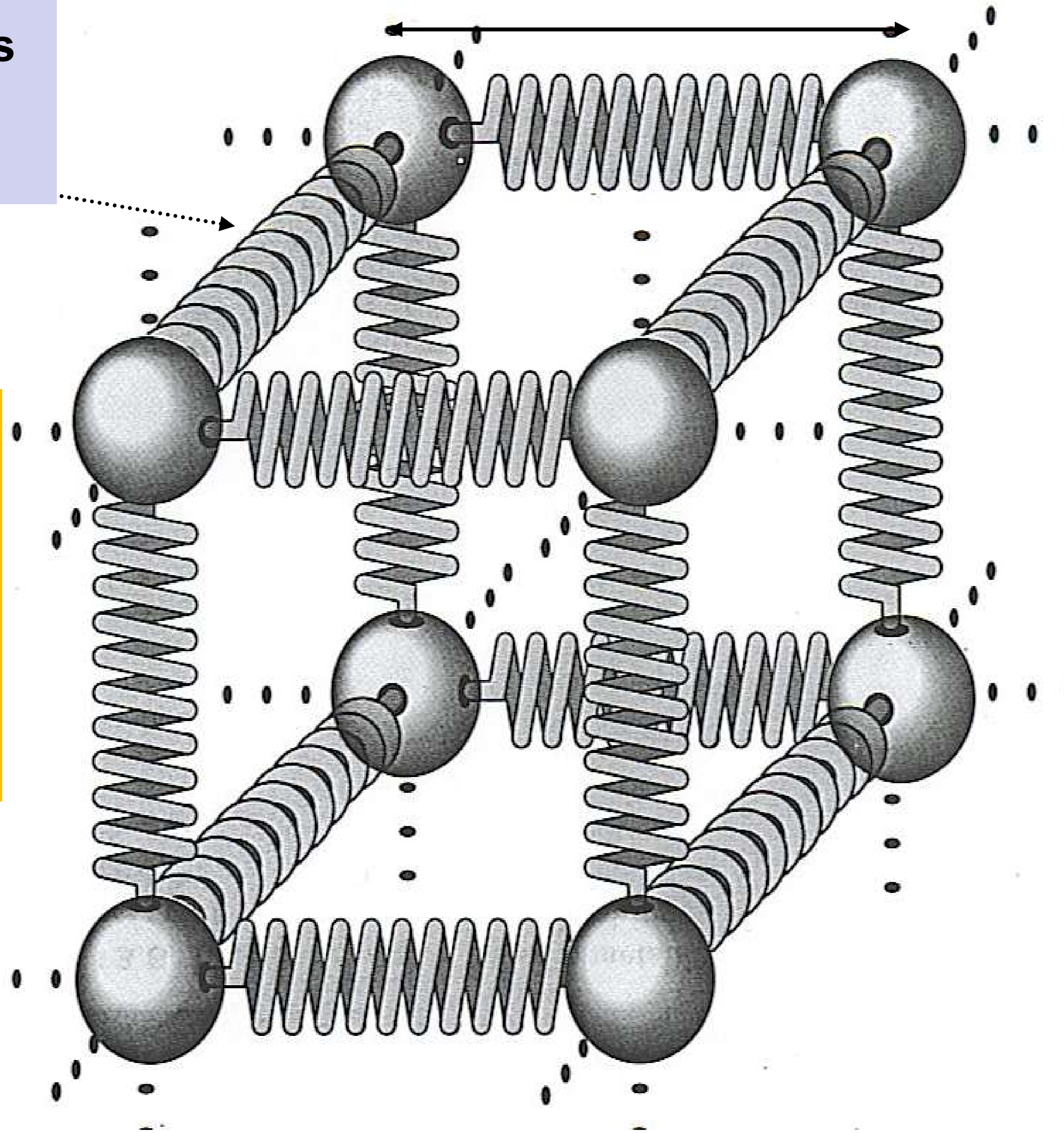
Most solids and liquids expand when heated. Why?

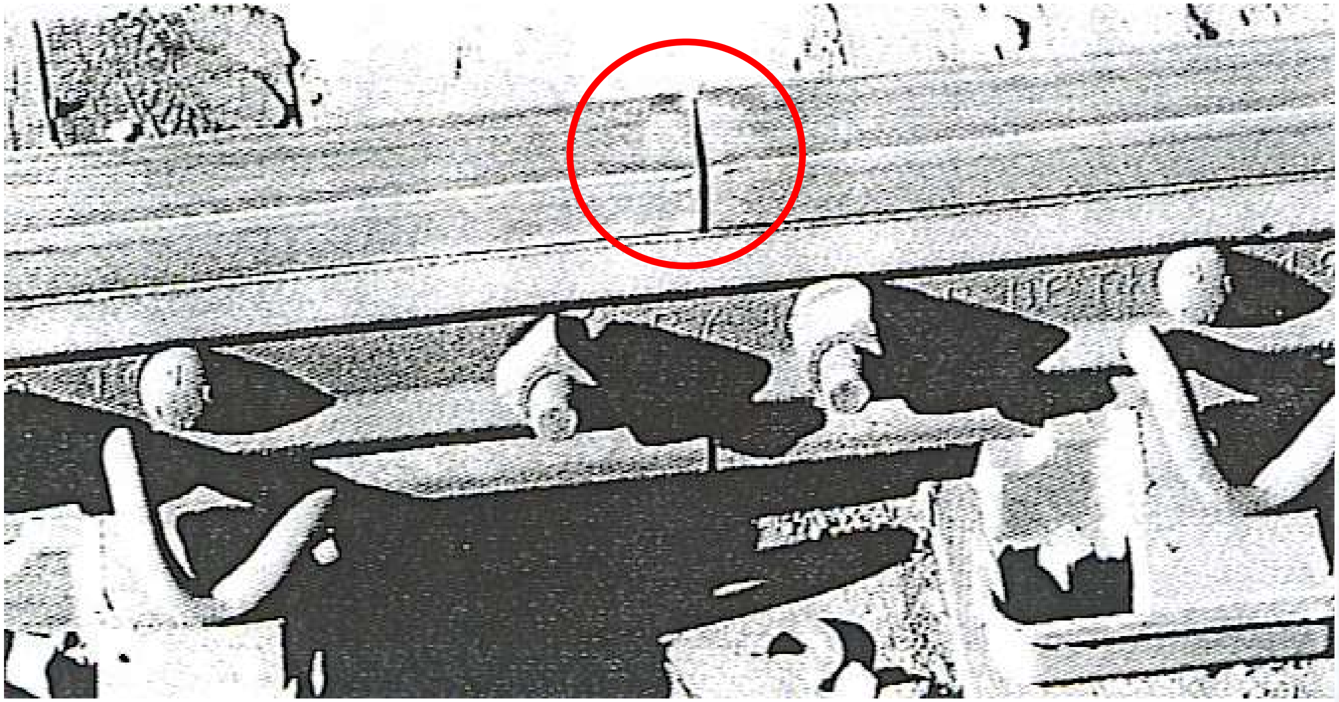
Inter-atomic forces

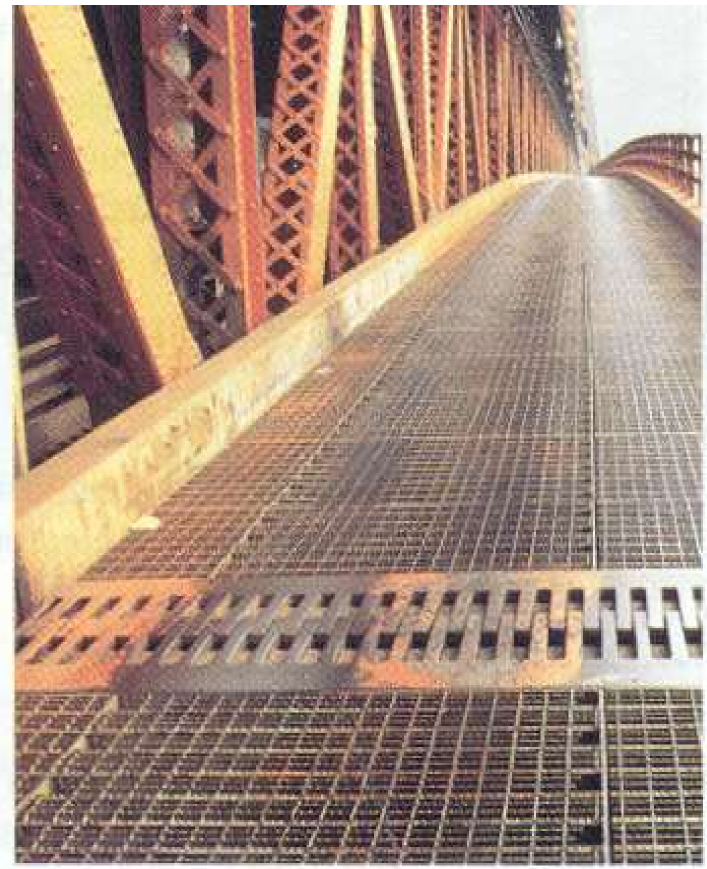
≡

"springs"



Internal Energy U is associated with the amplitude of the oscillation of the atoms



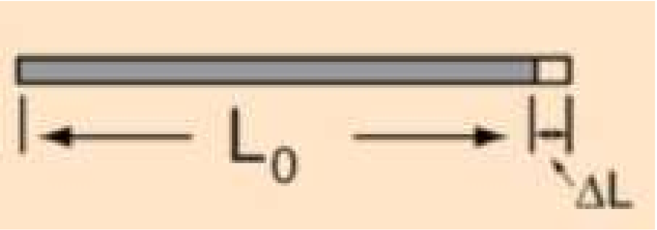


Linear Expansion of Solids

The change in one dimension of a solid (length, width or thickness) is called linear expansion. For small temperature changes, linear expansion is approximately proportional to the change in temperature; $\Delta T = T - T_0$. The fractional change in length is called the thermal strain, which is equal

$$\frac{L - L_0}{L_0} = \frac{\Delta L}{L_0},$$

Thermal strain



Where L_0 is the original length at the original temperature T_0. This is related to the change in temperature by

$$\frac{\Delta L}{L_0} = \alpha \Delta T \quad \text{or} \quad \Delta L = \alpha L_0 \Delta T, \tag{4.17}$$

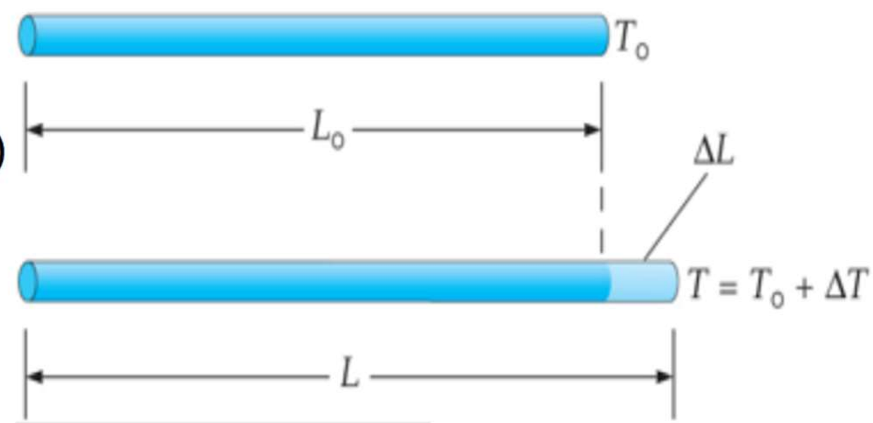
Where α is the thermal coefficient of linear expansion, which has units of K^{-1}. We can also write down an expression for the final length L after a temperature change:

$$
\begin{aligned}
\Delta L &= \alpha L_0 \Delta T \\
L - L_0 &= \alpha L_0 \Delta T \\
L &= L_0 + \alpha L_0 \Delta T \\
L &= L_0 (1 + \alpha \Delta T)
\end{aligned}
$$

Linear Expansion

$$\frac{\Delta L}{L_o} = \alpha \Delta T \qquad L = L_o(1 + \alpha \Delta T)$$

α – thermal coefficient of linear expansion



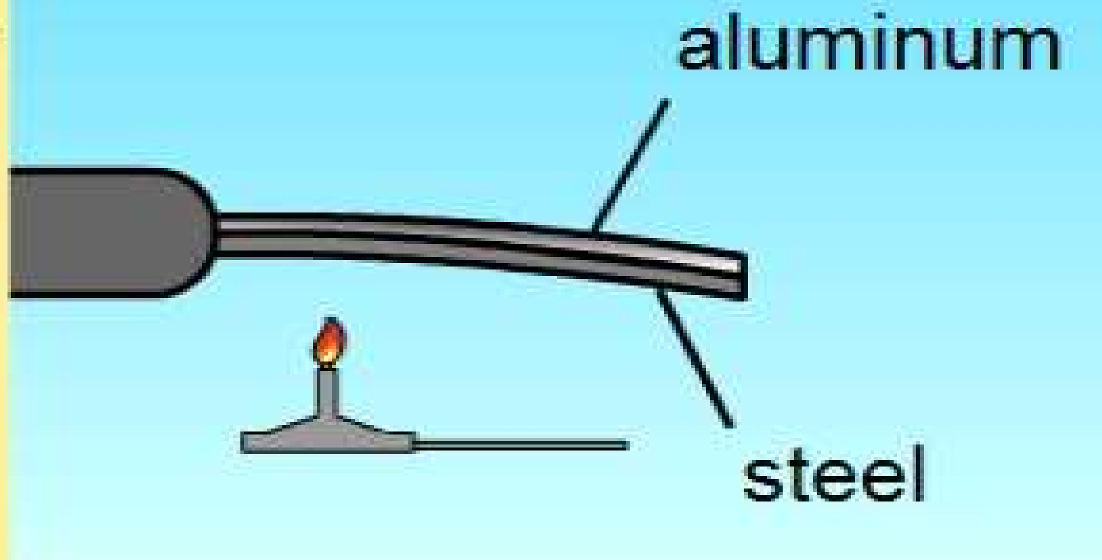
Linear Thermal Expansion Coefficient

The coefficient of expansion may vary slightly for different temperature ranges. Since this variation is negligible for most applications, we (usually) consider α to be a constant and independent of temperature.

Different substances expand at different rates, which because they have different thermal expansion coefficient.

Coefficient of Volume Expansion, at 20 C

Material	Coefficeint of Linear Expansion, α (C)$^{-1}$
Aluminum	25×10^{-6}
Steel	12×10^{-6}
Glass	9×10^{-6}
Concrete	$\approx 12 \times 10^{-6}$



Ceramics
low expansion coefficients
$\alpha \sim 10^{-6}\ K^{-1}$
Polymers **high expansion coefficients**
$\alpha \sim 10^{-4}\ K^{-1}$
Metals
$\alpha \sim 10^{-5}\ K^{-1}$

Linear Thermal Expansion

$$\Delta L = \alpha L_0 \Delta T$$

$$\Delta L = L - L_0$$

$$\Delta T = T - T_0$$

A concrete slab has a length of 12 m at $-5\ ^{\circ}C$ on a winter's day. What is the change in length from winter to summer, when the temperature is $35\ ^{\circ}C$? The linear expansion coefficient of concrete is $1 \times 10^{-5}\,^{\circ}C^{-1}$.

Solution

$$\Delta T = (T_f - T_0) = (35 - (-5)) = 40\ ^{\circ}C$$

$$\Delta l = \alpha l_0 \Delta T = 1 \times 10^{-5}\,^{\circ}C^{-1} \times 12m \times 40\ ^{\circ}C = 4.80\ mm$$

A steel rod 4.00 cm in **diameter** is heated so that its temperature increases by $70.0C^{\circ}$. It is then fastened between two rigid supports. The rod is allowed to cool to its original temperature. Assuming that Young's modulus for the steel is $20.6 \times 10^{10}\ N/m^2$ and that its average coefficient of linear expansion is $11 \times 10^{-6}\ ^{\circ}C^{-1}$, calculate the tension in the rod.

$$r = 2\ cm = 2 \times 10^{-2} m, Y_{steel} = 20.6 \times 10^{10}\ \frac{N}{m^2}, and\ \alpha = 11 \times\ 10^{-6}\ ^{\circ}C^{-1}$$

$$\sigma = Y_{steel}\ \varepsilon$$

Where

$$\sigma = \frac{F}{A}, and\ \varepsilon = \frac{\Delta l}{l_0}$$

Thus,

$$F = Y_{steel}\ \times A \times \frac{\Delta l}{l_0}$$

$$F = Y_{steel} \times A \times \frac{\Delta l}{l_0}$$

The change in length due to increase in temperature is

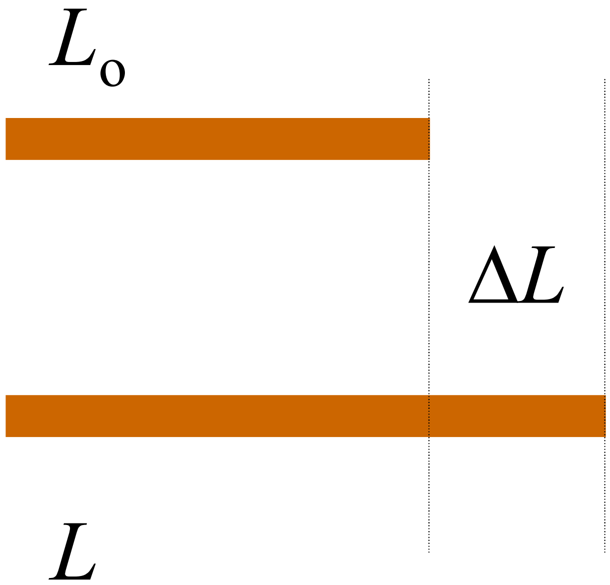
$$\Delta l = \alpha l_0 \Delta T \rightarrow \frac{\Delta l}{l_0} = \alpha \Delta T$$

$$F = Y_{steel} \times A \times \left(\frac{\Delta l}{l_0}\right) = Y_{steel} \times A \times (\alpha \Delta T)$$

$$F = 20.6 \times 10^{10} \frac{N}{m^2} \times \pi(2 \times 10^{-2} m)^2 \times (11 \times 10^{-6}\ {}^{\circ}C^{-1} \times 70.0 C^{\circ})$$

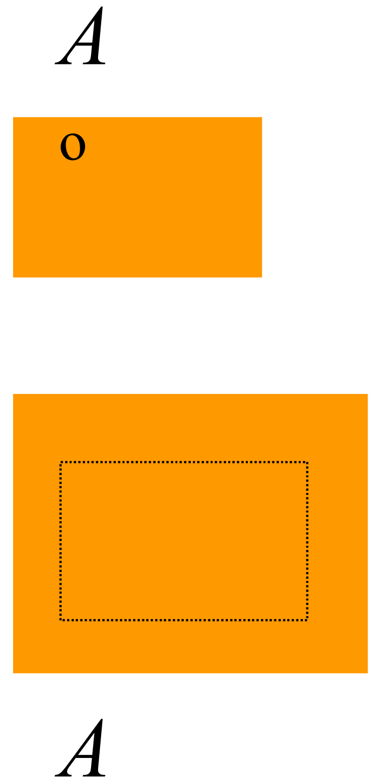
$$F = 1.99 \times 10^5\ N \cong 2 \times 10^5\ N$$

Linear



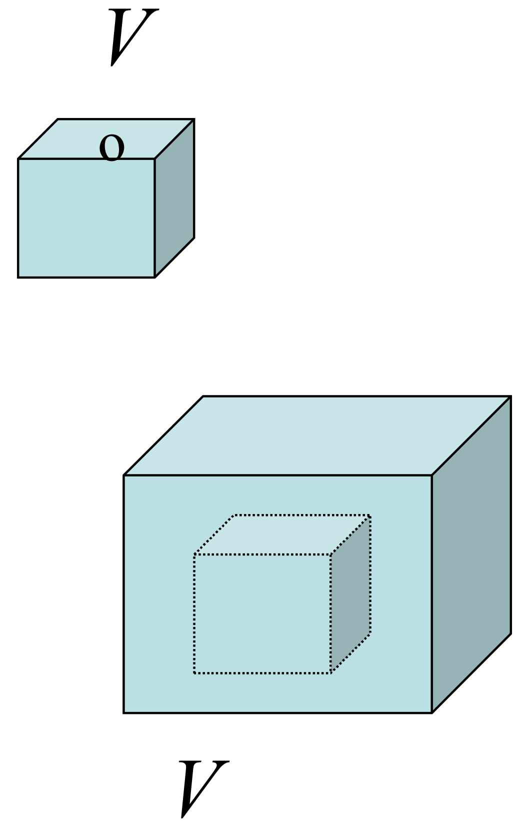
$$\Delta L = \alpha L_o \Delta T$$

Area



$$\Delta A = 2\alpha A_o \Delta T$$

Volume



$$\Delta V = 3\alpha V_o \Delta T = \beta V_o \Delta T$$

Volume expansion – solid cube

$$V_o = L_o^3$$

$$V = L^3 = (L_o + \alpha\, L_o\, \Delta T)^3 = L_o^3(1 + \alpha\, \Delta T)^3$$

$$V = L_o^3\,(1 + 3\,\alpha\,\Delta T + 3\,\alpha^2\,\Delta T^2 + \alpha^3\,\Delta T^3)$$

$$V = L_o^3\,(1 + 3\,\alpha\,\Delta T) \qquad \text{(ignoring higher order terms)}$$

$$V - V_o = \Delta V = 3\,\alpha\,L_o^3\Delta T = \beta\,V_o\,\Delta T$$

α **coefficient of linear expansion**

β **coefficient of volume expansion**

15. A thermometer has a mercury-filled glass bulb with a volume of $2 \times 10^{-7}\ m^3$ attached to a thin glass capillary tube with an inner radius of $5 \times 10^{-5}\ m$. If the temperature increases by 100°C, how far will the mercury rise in the tube? (volume thermal expansion coefficient of mercury= $1.82 \times 10^{-4}\ K^{-1}$).

$V_0 = \pi r^2 h_0$ = $2 \times 10^{-7}\ m^3$, where $r = 5 \times 10^{-5} m$, *is the inner radius of the capillary,*
and h_0 *is the initial height of the mercury*, so
$h_0 = \frac{V_0}{\pi r^2} = \frac{2\times10^{-7}}{3,14\times(5\times10^{-5})^2}$= $25.4\ m$

$\Delta T = 100\ ^0C$

The new volume of the mercury after the expansion
$V = V_0[1 + \beta\Delta T]$, where $\beta = 1.82 \times 10^{-4} K^{-1}$ *and* $V\ = \pi r^2 h$, *and h is the*
New height of the mercury, so
$\pi r^2 h$= $\pi r^2 h_0\ [1 + \beta\Delta T]$ ⋙⋙ h= $h_0\ [1 + \beta\Delta T]$
$h = 25.4\ [1 + 1.82 \times 10^{-4}(100)]$= $25.4 \times 1.0182 = 25.86\ m$

Expansion Coefficients

$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

Linear Expansion – 1D

$$\boxed{\Delta A = \gamma A_0 \Delta T}$$

Area Expansion – 2D

$$\boxed{\Delta V = \beta V_0 \Delta T}$$

Volume Expansion – 3D

If α is the same in all directions then

$$\gamma = 2\alpha \qquad \beta = 3\alpha$$

Heat as a form of Energy

What is HEAT?

- **Form of energy and measured in JOULES, Ergs, Calorie or BTU (Thermal Units)**
- **Particles move about more and take up more room if heated – this is why things expand if heated**
- **It is also why substances change from: solids liquids gases when heated**

Heating and Cooling

- If an object has become hotter, it means that it has gained heat energy.

$$\Delta T > 0 \text{ (positive)}$$

- If an object cools down, it means it has lost energy

$$\Delta T < 0 \text{ (Negative)}$$

- Heat energy always moves from:

HOT object → COOLER object

Cup of water at 20 °C in a room at 30°C - gains heat energy and heats up – its temperature rises

Cup of water at 20 °C in a room at 10°C loses heat energy and cools down – its temperature will fall.

Heat Capacity

Heat capacity is the amount of energy required to raise the temperature of a substance by 1K or 1ºC.

Different substances will have different thermal capacities, and different masses of the same kind of substance will also have different thermal capacities.

Heat Supplied (Q) = Heat Capacity (C) x Change in temperature (ΔT)

$$\text{Heat Capacity (C)} = \frac{Q\ (Joul)}{\Delta T\ (K)}$$

Specific Heat Capacity

- Specific Heat Capacity can be thought of as a measure of how much heat energy is needed to warm the substance up.
- You will possibly have noticed that it is easier to warm up a saucepan full of oil than it is to warm up one full of water.

- **Specific Heat Capacity (C) of a substance is the amount of heat required to raise the temperature of 1g of the substance by 1°C (or by 1 K).**
- **The units of specific heat capacity are J $^{o}C^{-1}$ g^{-1} or J K^{-1} $g^{-1.}$ Sometimes the mass is expressed in kg so the units could also be J $^{o}C^{-1}$ g^{-1} or J K^{-1} kg^{-1}**

The amount of heat energy (q) gained or lost by a substance = mass of substance (m) X specific heat capacity (C) X change in temperature (ΔT)

$$Q = m \times c \times \Delta T$$

- **Approximate values in J / kg ° K of the Specific Heat Capacities of some substances are:**

Air	1000	Lead	125
Aluminium	900	Mercury	14
Asbestos	840	Nylon	1700
Brass	400	Paraffin	2100
Brick	750	Platinum	135
Concrete	3300	Polythene	2200
Cork	2000	Polystyrene	1300
Glass	600	Rubber	1600
Gold	130	Silver	235
Ice	2100	Steel	450
Iron	500	Water	4200

How much energy would be needed to heat 450 grams of copper metal from a temperature of 25.0ºC to a temperature of 75ºC? The specific heat of copper at 25ºC is 0.385 J/g ºC.)

The change in temperature (*ΔT)* is:

75ºC - 25ºC = 50ºC

Given mass, two temperatures, and a specific heat capacity, you have enough values to plug into the specific heat equation

q = m x C x ΔT .

and plugging in your values you get

Q = (450 g) x (0.385 J/g ºC) x (50ºC)

= 8700 J

$$Q = m\,c\,\Delta T$$

Q = energy transfered (joules)
m = mass of water (grams)
c = specific heat capacity
ΔT = temperature change (K or ºC)

The sum of heat gain and heat lost equal zero

assume that m_x is the mass of unknown substance, whose specific heat c_x is to be determined, and then it is heated to a certain temperature T_x. Likewise take an amount of water m_w which its specific heat is known c_w and its temperature T_w is known. If both substances are mixed and left until their temperature remain constant (final temperature T_f), so we can apply the conservation of energy as,

$$Q_{gain}(w) + Q_{lost}(x) = 0 \Longrightarrow m_w c_w (T_f - T_w) + m_x c_x (T_f - T_x) = 0$$

Solving this equation for c_x, we find

$$c_x = \frac{m_w c_w (T_f - T_w)}{m_x (T_x - T_f)} \tag{4.25}$$

A quantity of hot water at 91 ℃ and another cold one at 12 ℃. How much kilogram of each one is needed to make an 800 liter of water bath at temperature of 35 ℃.

Th(hot)= 91C, Tc(cold)= 12; Tm (mixture)= 35C

M(hot)+ m(cold) = 800 kg............................(1)

Q(gain by cold water)+ Q (lost by hot water) = 0
m(cold)x c(water) x (Tm- Tc)+ m(h)x c(water)x (Tm-Th)= 0
23 m(cold) – 79 m(hot) = 0..........................(2)

Heat Transfer

- Heat is transferred through a material by being passed from one particle to the next
- Particles at the warm end move faster and this then causes the next particles to move faster and so on.
- In this way heat in an object travels from:

the HOT end → the cold end

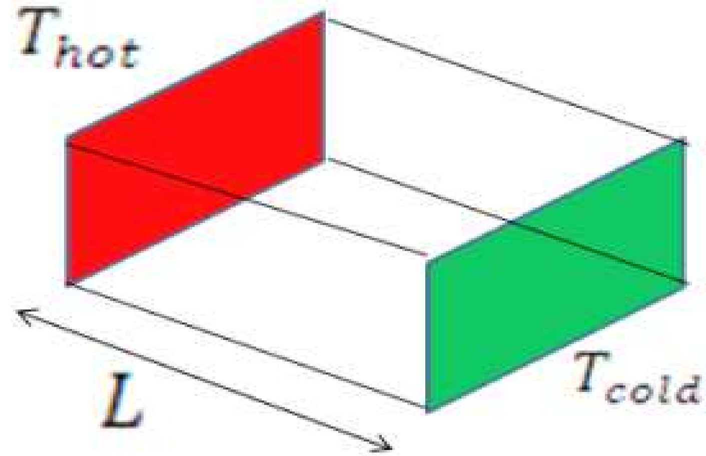
Conduction

- Occurs by the particles hitting each other and so energy is transferred.
- Can happen in solids, liquids and gases,
- Happens **best in solids**-particles very close together
- Conduction does not occur very quickly in liquids or gases

Conduction

- Materials that conduct heat quickly are called conductors
- All metals are good conductors of heat
- Copper is a very good conductor of heat
- Pans for cooking are usually made with a copper or aluminium bottom and plastic handles

Heat Rate $H = \Delta q / \Delta t$



For heat transfer between two plane surfaces, such as heat loss through the wall of a house, the rate of conduction heat transfer (H) is the energy transferred per unit of time:

$$H = \frac{Q}{t} = \kappa A \frac{(T_{hot} - T_{cold})}{L}, \quad measured\ in\, Watt \tag{4.26}$$

Where L is the separation distance (thickness) between the hot and the cold sides, A is the contact area, $(T_{hot} - T_{cold})$ is the temperature gradient, and κ is a constant called the thermal conductivity coefficient of the material measured in W/mK. This constant is a characteristic of the material. Table 4.5 shows some values of κ for different materials. Those with high κ like metals are good heat conductor and with small κ like gases and nonmetals are good insulator.

Table 4.5: The thermal conductivity coefficient for different materials.

Substance	$(W\,m^{-1}K^{-1})$	Substance	$(W\,m^{-1}K^{-1})$
Silver	427	Ice	2
Copper	397	Water	0.6
Aluminum	238	Wood	0.08
Gold	314	Air	0.023
Concrete	0.8	Hydrogen	0.1
Glass	0.8	Helium	0.138

Heat Rate $H = \Delta q / \Delta t$

$$H = \kappa \frac{A \Delta T}{L}$$

An aluminum pot contains water that is kept steadily boiling (100 ºC). The bottom surface of the pot, which is 12 mm thick and $1.5\text{x}10^4$ mm^2 in area, is maintained at a temperature of 102 ℃ by an electric heating unit. Find the rate at which heat is transferred through the bottom surface. Compare this with a copper based pot. The thermal conductivities for aluminum and copper are shown in the table.

The thickness of ghe pot is L = 12mm= 0.012 m
and the temperature differece ΔT= 102-100= 2°C
The surface area A= $1.5\text{x}10^4$ mm^2 = $1.5\text{x}10^{-2}$ m^2

$$H_{al} = \kappa_{Al} \frac{A\Delta T}{L} = 238 \frac{1.5\times10^{-2}(2)}{0.012} = 595 \text{ Watt}$$

$$H_{Cu} = \kappa_{Cu} \frac{A\Delta T}{L} = 397 \frac{1.5\times10^{-2}(2)}{0.012} = 992 \text{ Watt}$$

A box with a total surface area of 1.20 m2 and a wall thickness of 4.00 cm is made of an insulating material. A 10.0-W electric heater inside the box maintains the inside temperature at 15.0°C above the outside temperature. Find the thermal conductivity k of the insulating material.

Surface area A= 1.2 m^2 and the wall thickness L = 4cm = 0.04 m

The heat rate is the power of the electric heater H= 10 Watt

The temperature difference between inside and outside is ΔT= 15°C

$$H= \kappa \frac{A\Delta T}{L} \rightarrow\rightarrow \kappa = \frac{H\ L}{A\Delta T} = \frac{10\ (0.04)}{1.2\ (15)} = 0.022\ {}^{W}/_{mK}$$

Heat Transfer by Radiation

- Transfer of heat directly form the source to the object by a wave, travelling as rays.
- Heat radiation is also known as

  INFRA-RED Radiation

- All objects that are hotter than their surroundings give out heat as infra-red radiation
- Heat transfer by radiation does not need particles to occur and is the only way energy can be transferred across empty space

The color and texture of different surfaces determines how well they absorb the radiation.

(1) Black objects absorb more radiation than white objects.

(2) Matt and rough surfaces absorb more than shiny and smooth surfaces.

Radiation

The relationship governing radiation from hot objects is called the Stefan-Boltzmann Law:

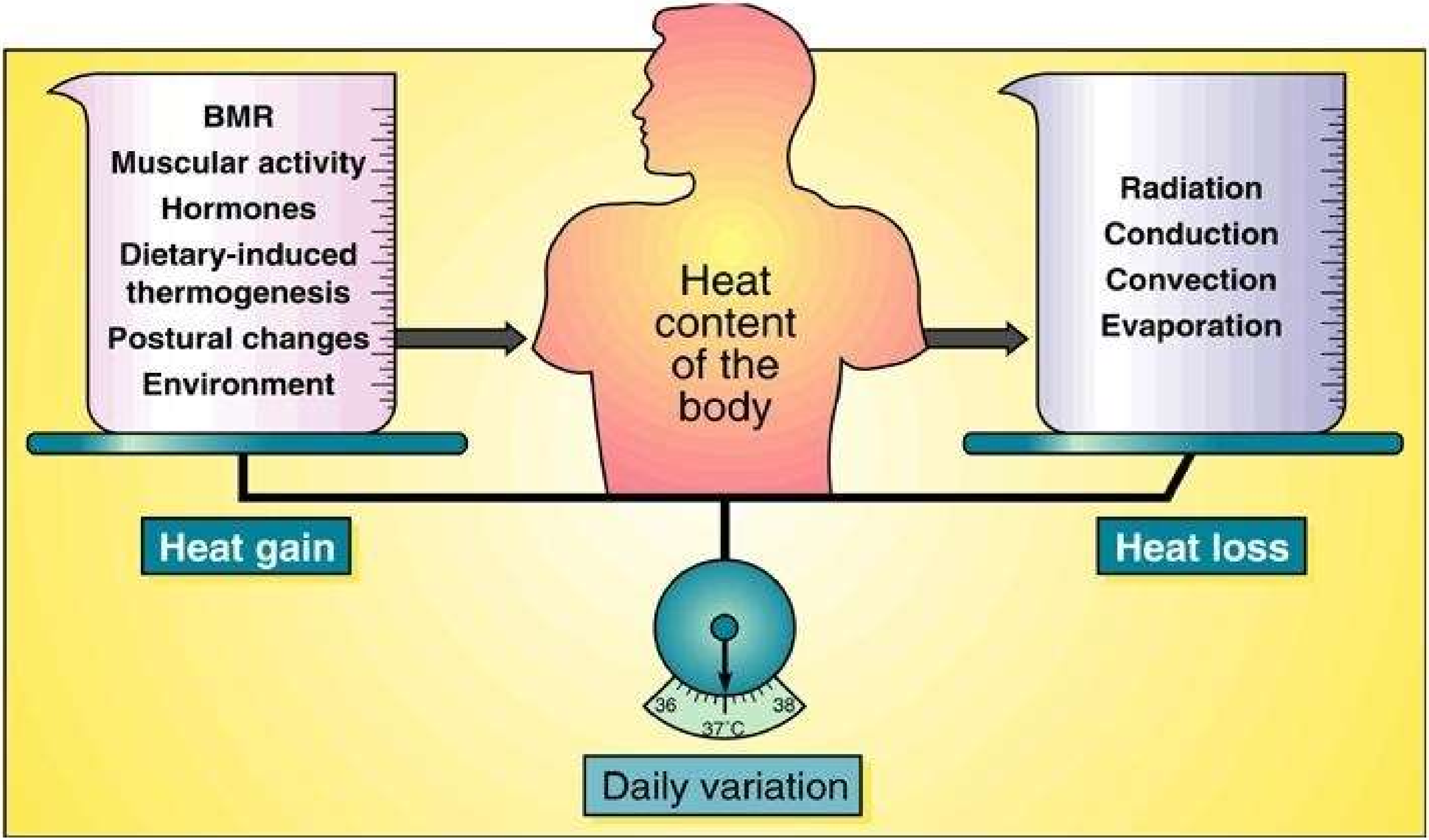
$$P = e\,\sigma\,A\,(T^4 - T_S^4) \tag{4.28}$$

Where P is the net radiated power measured in Watt, e is the emissivity (=1 for ideal radiator), A is the radiation area in m^2, T is the temperature of the radiator in Kelvin, Ts is the temperature of the surroundings in Kelvin, and σ = $5.67\text{x}10^{-8}$ Watt/m^2 K^4 is a constant called Stefan-Boltzmann constant.

<u>**Example 4.12**</u>

A student tries to decide what to wear is staying in a room that is at 20℃. If the skin temperature is 37℃, how much heat is lost from the body in 10 minutes? Assume that the emissivity of the body is 0.9 and the surface area of the student is 1.5 m^2.

Thermoregulation in Heat



- **Heat Loss by Conduction**
  - Direct transfer of heat through a liquid, solid, or gas from one molecule to another.
  - A small amount of body heat moves by conduction directly through deep tissues to cooler surface. Heat loss involves the warming of air molecules and cooler surfaces in contact with the skin.
  - The rate of conductive heat loss depends on thermal gradient.

- **Heat Loss by Evaporation (~ 55%)**
  - Heat transferred as water is vaporized from respiratory passages and skin surfaces.
  - For each liter of water vaporized, 580 kcal transferred to the environment.
  - When sweat comes in contact with the skin, a cooling effect occurs as sweat evaporates.
  - The cooled skin serves to cool the blood.

A 2 m length of copper pipe of 10 cm diameter containing hot water at 80C. If the surroundings are at 20C, at what rate does the pipe lose thermal energy due to radiation? (take e = 1)

A cylinder of radius r= 0.05 m and length l= 2m

The hot side has temperature = 273+ 80= 353 K

The cold side has temperature= 273+20= 293K

P = $eA\sigma\left(T_h^4 - T_c^4\right) = 1\,(2\pi r l)5.67 \times 10^{-8}\left[353^4 - 293^4\right]$

= 2(3.14) 0.05(2) $5.67 \times 10^{-8}\left[353^4 - 293^4\right]$

= 290.46 W

A girl having a surface area of 1.2 m^2 wears a down jackets 3 cm thick with thermal conduction coefficient of 0.02 $Wm^{-1}K^{-1}$. If her skin temperature is 34 °C and she can safely lose 85 W by conduction, what is the lowest temperature for which the clothing is adequate?

$$H = \kappa \frac{A\Delta T}{L}$$

FLUID MECHANICS
Biofluids

- **Fluid**

A continuous, amorphous substance whose molecules move freely past one another and that has the ability to flow and assume the shape of its container; a liquid or gas.

- **Fluids share the properties of not resisting deformation and the ability to flow.**

  These properties are typically a function of their inability to support a shear stress in static equilibrium.

While in a solid, stress is a function of strain, in a fluid stress is a function of rate of strain.

Objectives

1. comprehend the concepts necessary to analyse fluids in motion.
2. identify differences between steady/unsteady, uniform/non-uniform and compressible/incompressible flow.
3. construct streamlines and stream tubes.
4. appreciate the Continuity principle through Conservation of Mass and Control Volumes.
5. derive the Bernoulli (energy) equation.
6. familiarise with the momentum equation for a fluid flow.

Characteristics of Fluids

Force & Pressure

Pressure is a normal force exerted by a fluid per unit area

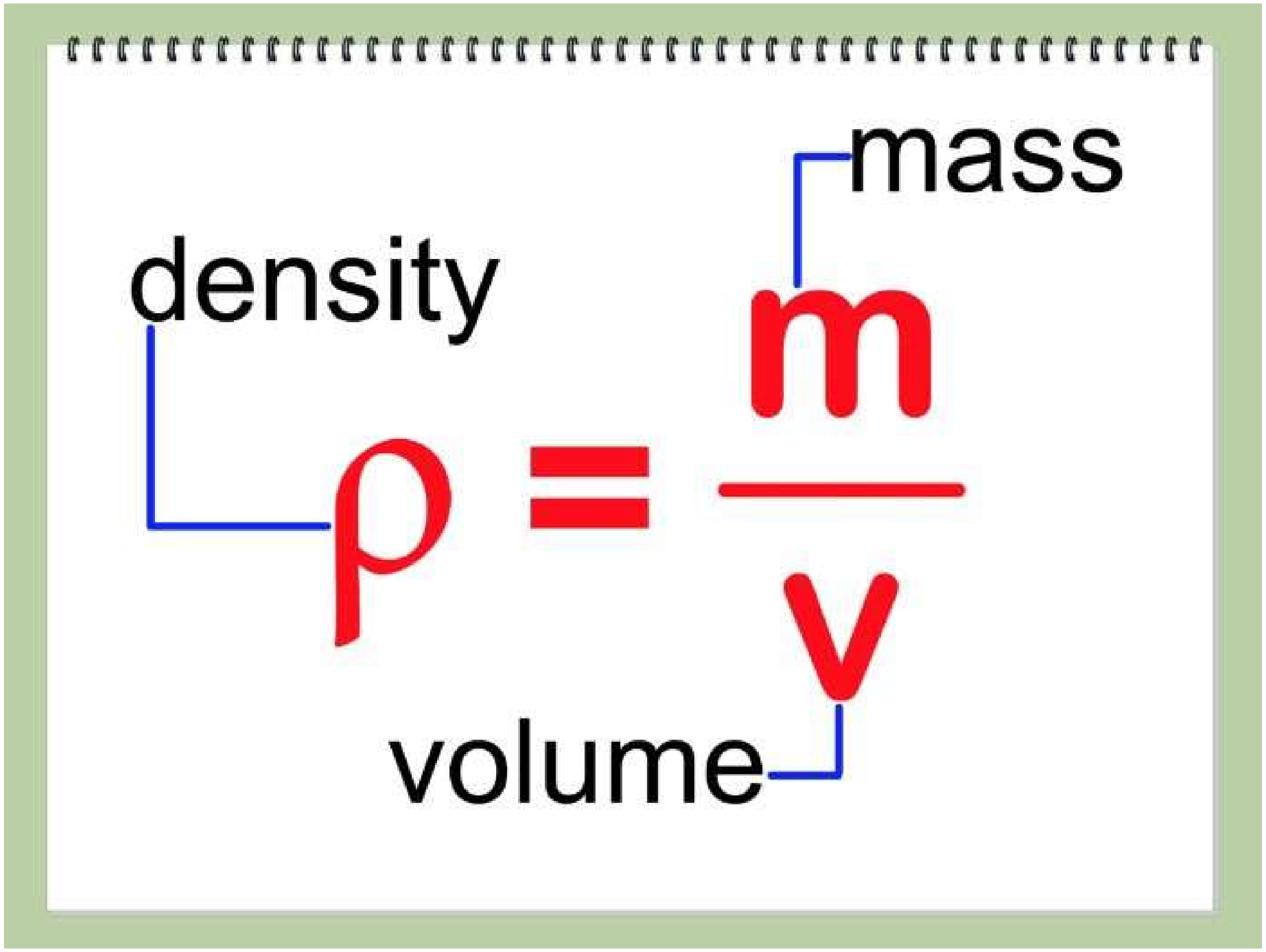
$$= (p_1 - p_2)A = \Delta p A \tag{5.1}$$

The standard unit for pressure is the Pascal, which is a Newton per square meter (N/m^2). Pressure in a fluid can be seen to be a measure of energy per unit volume by means of the definition of work, that is, this energy is related to other forms of fluid energy by the Bernoulli equation.

$$p = \frac{Force}{Area} = \frac{F}{A} = \frac{F.\Delta x}{A.\Delta x} = \frac{W}{\Delta V} = \frac{Energy}{volume}$$

Where ΔV is the unit volume, which related to the unit mass Δm, of the fluid by the density ρ, as $\Delta m = \rho \Delta V$.

Mass and Density



Kinetic energy density: The kinetic energy of a moving fluid has a great importance when dealing with fluids at motion: more useful in applications like the Bernoulli equation when it is expressed as kinetic energy per unit volume. For a unit mass Δm of a fluid moving with average speed $\bar{v}$, its kinetic energy is given by

$$K.E = \frac{1}{2}\Delta m\overline{v^2} = \frac{1}{2}\rho\Delta V\overline{v^2}$$

So that the kinetic energy density is written as

$$K.E.density = \frac{kinetic\ energy}{volume} = \frac{\frac{1}{2}\rho\Delta V\overline{v^2}}{\Delta V} = \frac{1}{2}\rho\overline{v^2} \qquad (5.2)$$

Potential energy density: Similarly, like the kinetic energy density, it is the potential energy per unit volume, for a certain mass of fluid at distance h above the ground; then the potential energy density is

$$P.E.density = \frac{potential\ energy}{volume} = \frac{\Delta m\, g\, h}{\Delta V} = \frac{\rho\Delta V gh}{\Delta V} = \rho gh \qquad (5.3)$$

Uniform Flow, Steady Flow

***uniform flow*:** flow velocity is the same magnitude and direction at every point in the fluid.

***non-uniform*:** If at a given instant, the velocity is not the same at every point the flow. (In practice, by this definition, every fluid that flows near a solid boundary will be non-uniform - as the fluid at the boundary must take the speed of the boundary, usually zero. However if the size and shape of the of the cross-section of the stream of fluid is constant the flow is considered *uniform*.)

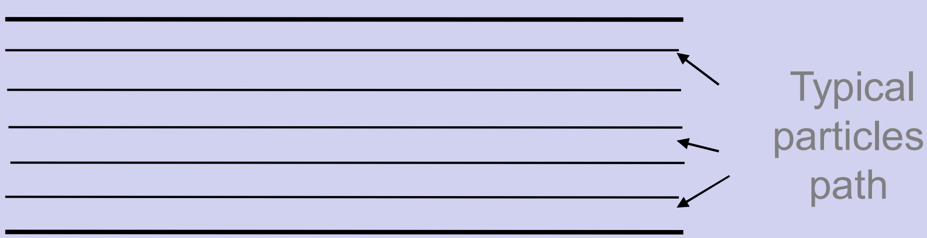
***steady*:** A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but DO NOT change with time.

***unsteady*:** If at any point in the fluid, the conditions change with time, the flow is described as *unsteady*. (In practice there is always slight variations in velocity and pressure, but if the average values are constant, the flow is considered *steady*.)

Laminar and Turbulent Flow

Laminar flow

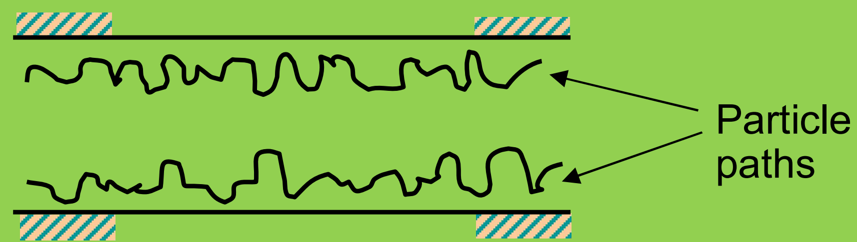
- all the particles proceed along smooth parallel paths (Stream lines) and all particles on any path will follow it without deviation.
- Hence all particles have a low velocity only in the direction of flow.



Laminar flow

Turbulent Flow

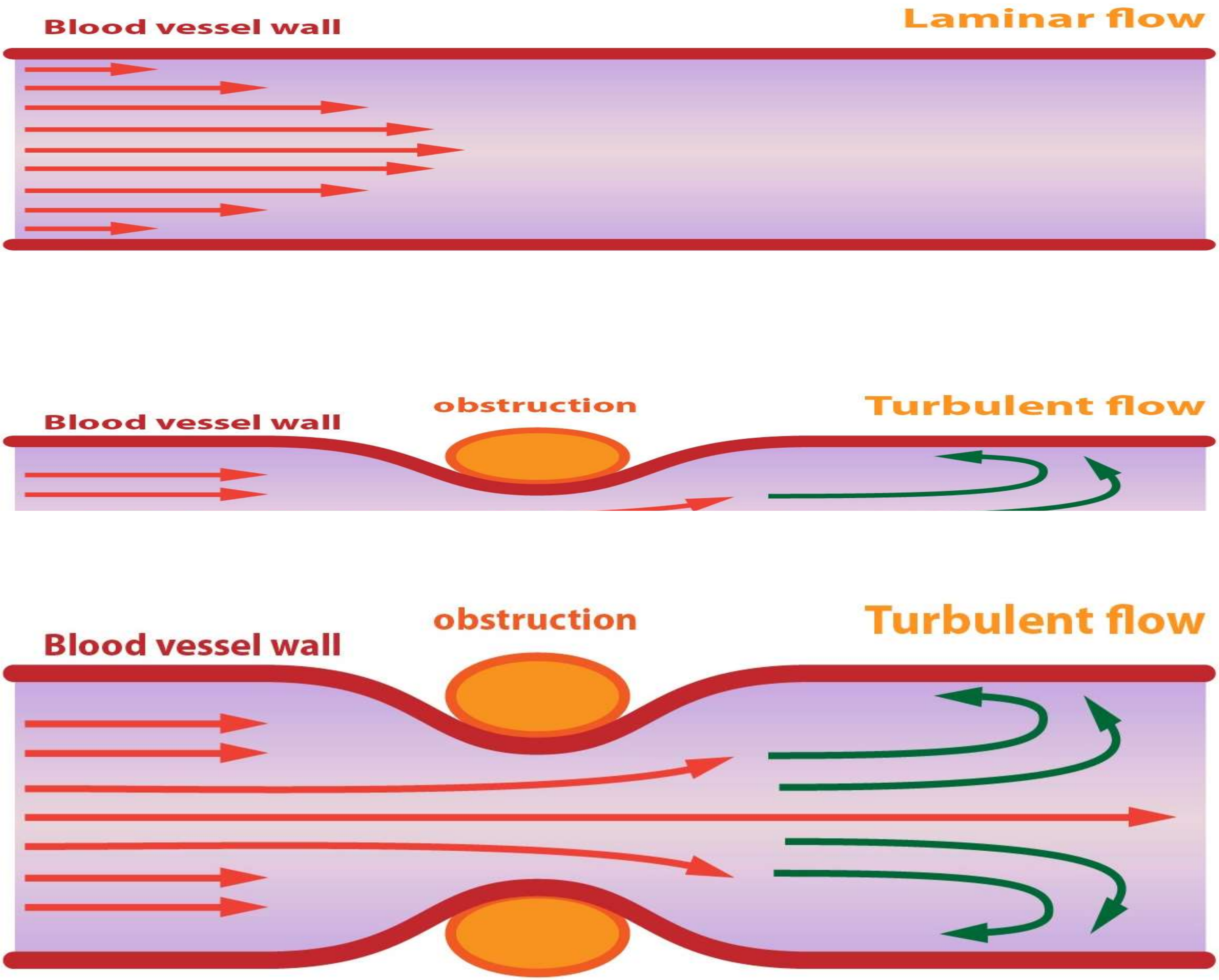
- the particles move very fast (high speed) in an irregular manner through the flow field. Intersection of stream lines
- Each particle has superimposed on its mean velocity fluctuating velocity components both transverse to and in the direction of the net flow.



Turbulent flow

Transition Flow

- exists between laminar and turbulent flow.
- In this region, the flow is very unpredictable and often changeable back and forth between laminar and turbulent states.
- Modern experimentation has demonstrated that this type of flow may comprise short 'burst' of turbulence embedded in a laminar flow.


Blood vessel wall
Laminar flow
obstruction
Blood vessel wall
Turbulent flow
obstruction
Turbulent flow
Blood vessel wall

Compressible or Incompressible

- All fluids are compressible - even water - their density will change as pressure changes.
- Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density.
- As you will appreciate, *liquids are quite difficult to compress* - so under most steady conditions they are treated as incompressible.

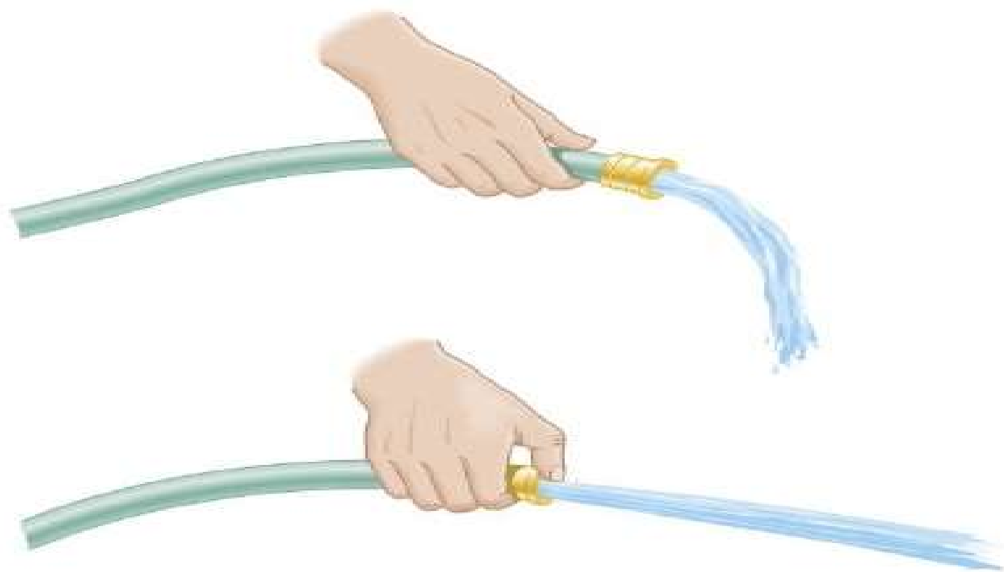
Viscous and Non viscous

Mass flow rate or Volume flow rate

$$\text{mass flow rate} = m = \frac{\text{mass of fluid}}{\text{time taken to collect the fluid}}$$

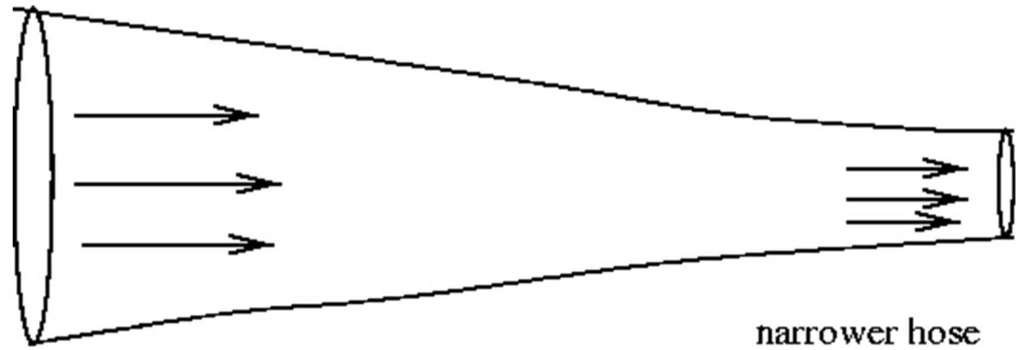
$$\text{time} = \frac{\text{mass}}{\text{mass flow rate}}$$

The mass of fluid per second that flows through a tube is called the ***mass flow rate***.



Volume flow rate - Discharge

Flow Rate (Discharge) = Q = $\dfrac{\text{volume of fluid} \quad \Delta V}{\text{Time} \quad \Delta t}$



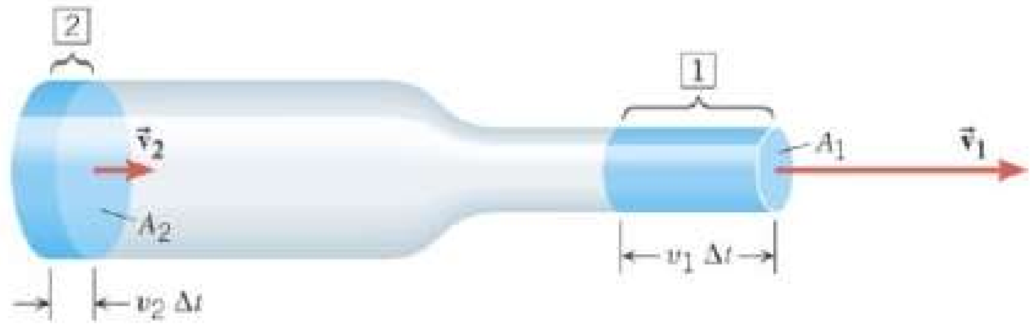
The flow rate is measured in the units of volume per unit time, m^3/s.

"The water all has to go somewhere"

The rate a fluid enters a pipe must equal the rate the fluid leaves the pipe.

$$Q_{in} = Q_{out} \qquad Q = \frac{\Delta V}{\Delta t} = constant$$

Contiuity Equation



$$Q = \frac{\Delta V}{\Delta t} = \frac{A\,\Delta x}{\Delta t} = A\bar{v} = constant \tag{5.6}$$

Where $\bar{v} = \frac{\Delta x}{\Delta t}$, is the average speed of flow at any point. Take $A = \pi r^2$, so the continuity equation can be rewritten as

$$A_1\,\overline{v_1} = A_2\overline{v_2}$$

Or,

$$r_1^2\overline{v_1} = r_2^2\overline{v_2} \tag{5.7}$$

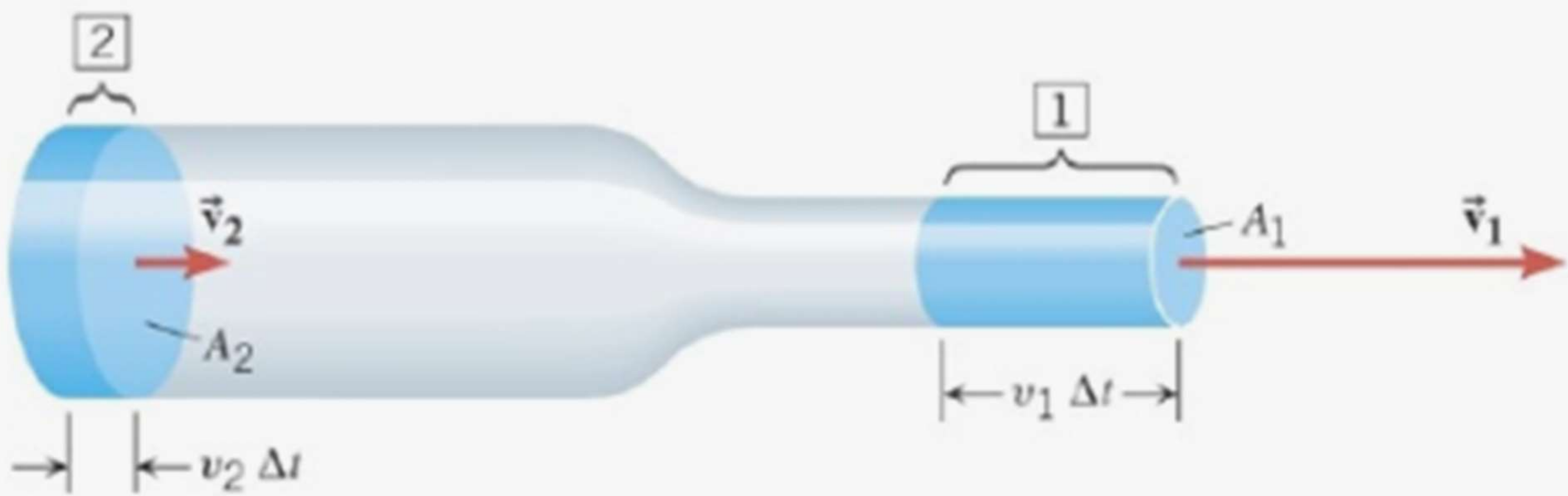
This means that the speed of flow increases obviously by a small decrease of the diameter of the tube.

EQUATION OF CONTINUITY

The mass flow rate has the same value at every position along a tube that has a single entry and a single exit for fluid flow.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Incompressible fluid: $A_1 v_1 = A_2 v_2$



Example 3.2

- If we know the mass flow is 1.7 kg/s, how long will it take to fill a container with 8 kg of fluid?

$$time = \frac{mass}{mass\ flow\ rate}$$

$$= \frac{8}{1.7} = 4.7\text{s}$$

Example 3.3

- If the density of the fluid in the above example is 850 kg/m^3 what is the volume per unit time (the discharge)?

$$Q = \frac{\textit{mass fluid rate}}{\textit{density}} = \frac{m/\Delta t}{\rho}$$

$$= \frac{0.857}{850}$$

$$= 0.00108\ m^3/s\ (m^3 s^{-1})$$

$$= 1.008 \times 10^{-3}\ m^3/s$$

but $1\ \textit{litre} = 1.0 \times 10^{-3} m^3$,

so $Q = 1.008\ l/s$

Example

A garden hose has an unobstructed opening with a cross sectional area of $2.85 \times 10^{-4} m^2$. It fills a bucket with a volume of $8.00 \times 10^{-3} m^3$ in 30 seconds.

Find the speed of the water that leaves the hose through (a) the unobstructed opening and (b) an obstructed opening with half as much area.

(a)

$$Q = Av$$

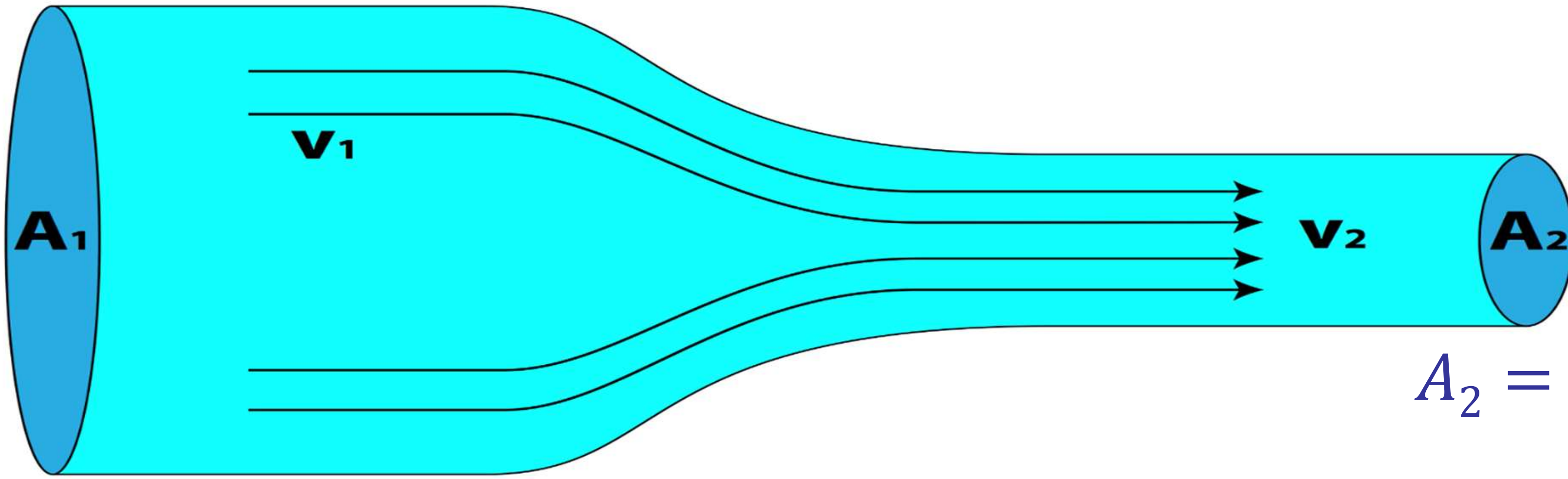
$$v = \frac{Q}{A} = \frac{(8.00 \times 10^{-3}\,\text{m}^3)/(30.0\,\text{s})}{2.85 \times 10^{-4}\,\text{m}^2} = 0.936\,\text{m/s}$$

(b)

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = (2)(0.936\,\text{m/s}) = 1.87\,\text{m/s}$$

A river is 40m wide, 2.2m deep and flows at 4.5 m/s. It passes through a 3.7-m wide gorge, where the flow speed increases to 6.0 m/s. How deep is the gorge?

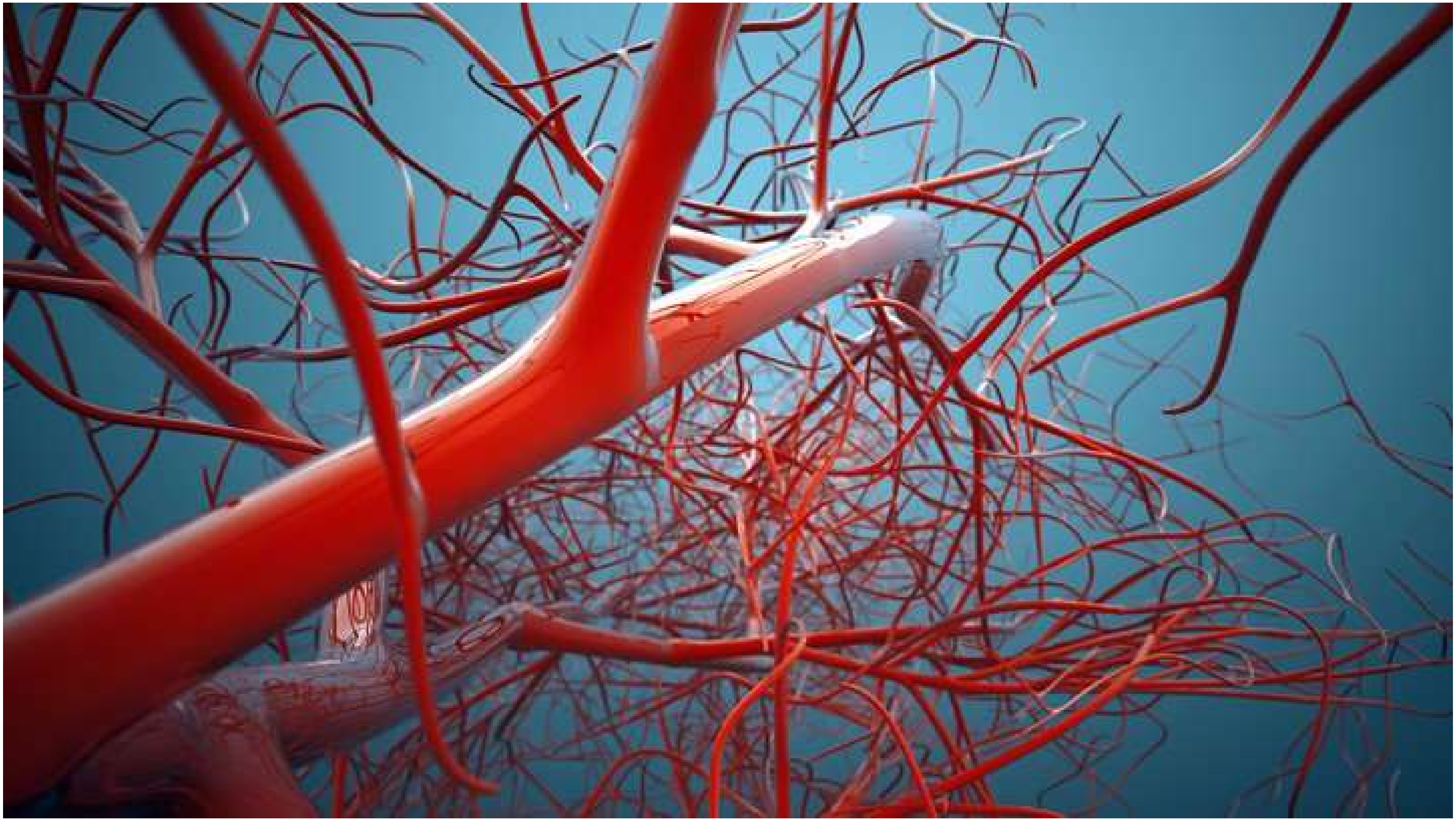


$$A_2 = w_2 d_2$$

$$A_1 = w_1 d_1$$

$$Continuity\ equation: A_1 v_1 = A_2 v_2 \rightarrow w_1 d_1 v_1 = w_2 d_2 v_2$$

$$d_2 = \frac{w_1 d_1 v_1}{w_2 v_2} = \frac{40 \times 2.2 \times 4.5}{3.7 \times 6.0} = 18\ m$$



The Blood flows in capillaries with lower speed than in the main blood vessel

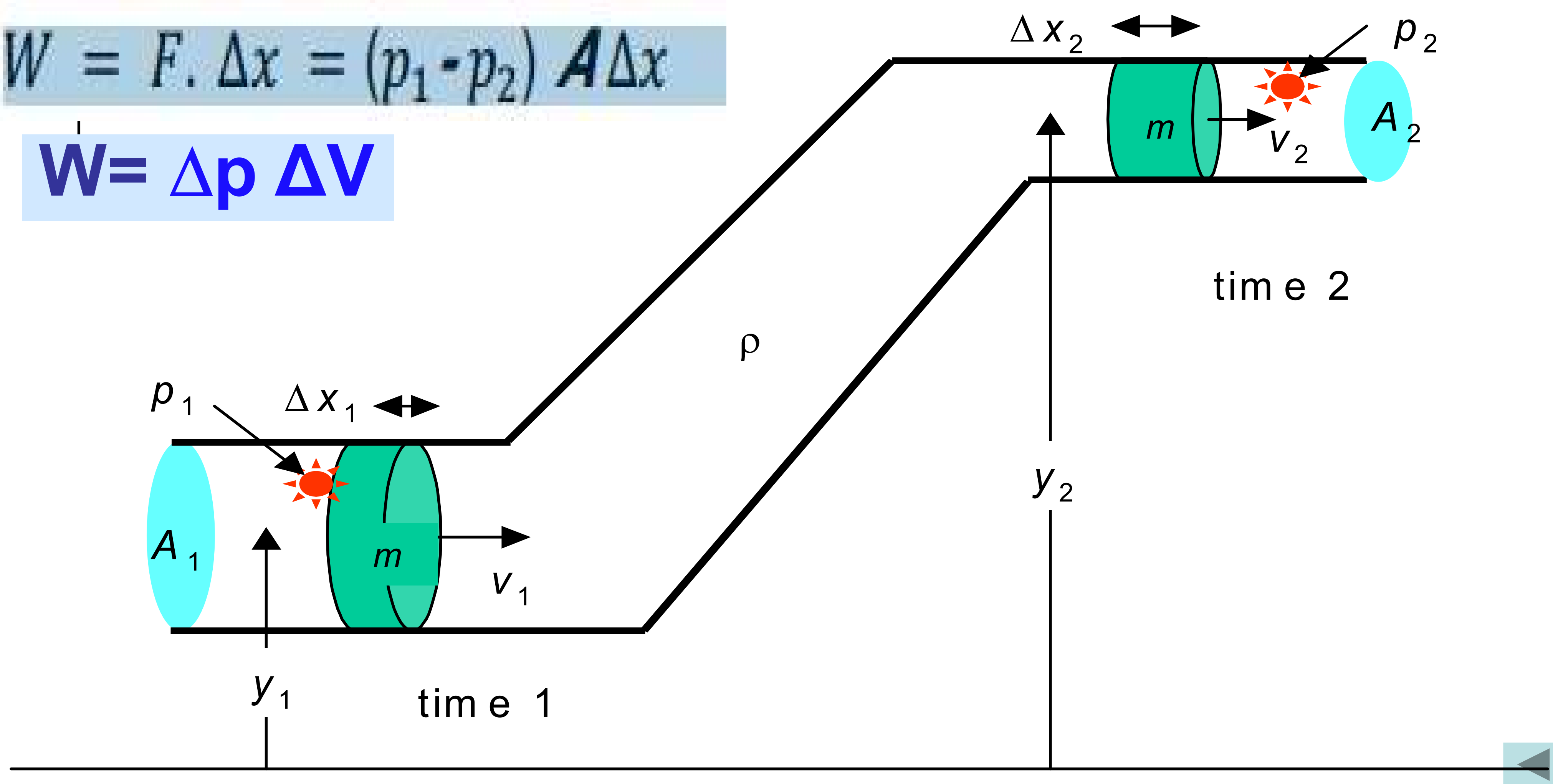
$$A_v < nA_c$$

Conservation of Energy: Bernoulli's Eqn.

The work done on a fluid when it flows from place to another one is equal to the change of its mechanichal energy

$$W = F.\Delta x = (p_1 - p_2)\, A\Delta x$$

$$W = \Delta p\, \Delta V$$



Change in Mechanical Energy

$$\Delta K = \tfrac{1}{2}\,\Delta m\, v_2^2 - \tfrac{1}{2}\,\Delta m\, v_1^2 = \tfrac{1}{2}\,\rho\,\Delta V\, v_2^2 - \tfrac{1}{2}\,\rho\,\Delta V\, v_1^2$$

$$\Delta U = \Delta m\, g\, y_2 - \Delta m\, g\, y_1 = \rho\,\Delta V\, g\, y_2 - \rho\,\Delta V\, g\, y_1$$

Net Work

$$p_1\,\Delta V - p_2\,\Delta V = \tfrac{1}{2}\,\rho\,\Delta V\, v_2^2 - \tfrac{1}{2}\,\rho\,\Delta V\, v_1^2 + \rho\,\Delta V\, g\, y_2 - \rho\,\Delta V\, g\, y_1$$

Rearranging

$$p_1 + \tfrac{1}{2}\,\rho\, v_1^2 + \rho\, g\, y_1 = p_2 + \tfrac{1}{2}\,\rho\, v_2^2 + \rho\, g\, y_2$$

Applies only to an ideal fluid (zero viscosity)

$$p + \tfrac{1}{2}\,\rho\, v^2 + \rho\, g\, y = \text{constant}$$

for any point along a flow tube or streamline

Bernoulli's Equation

for any point along a flow tube or streamline

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Dimensions

p [Pa] = [N.m^{-2}] = [N.m.m^{-3}] = [J.m^{-3}]

$\frac{1}{2}\rho v^2$ [kg.m^{-3}.m^2.s^{-2}] = [kg.m^{-1}.s^{-2}] = [N.m.m^{-3}] = [J.m^{-3}]

ρ g h [kg.m^{-3} m.s^{-2}. m] = [kg.m.s^{-2}.m.m^{-3}] = [N.m.m^{-3}] = [J.m^{-3}]

Each term has the dimensions of energy / volume or energy density.

$\frac{1}{2}\rho v$ 2 KE of bulk motion of fluid

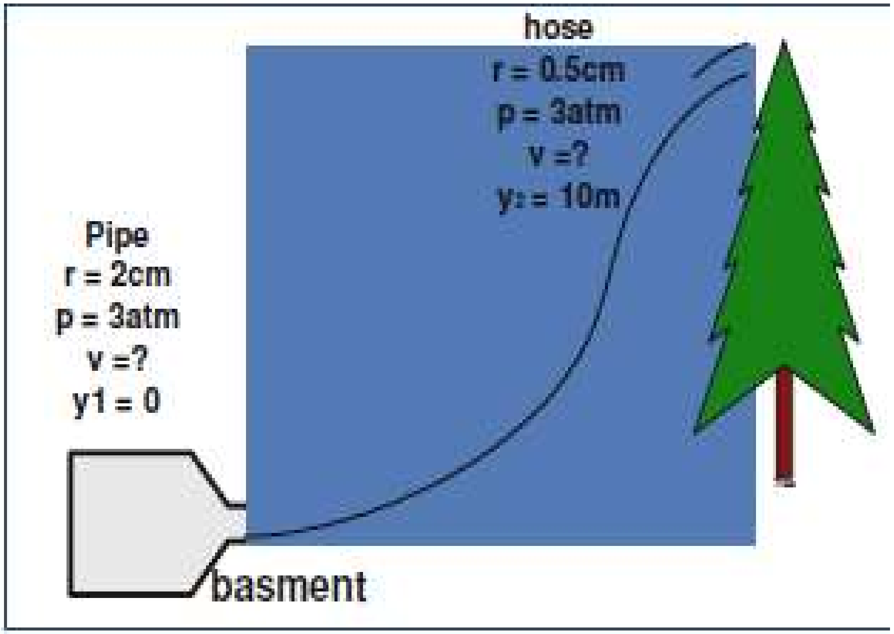
$\rho g h$ GPE for location of fluid

p pressure energy density arising from internal forces within moving fluid (similar to energy stored in a spring)

Example 5.2

Water enters the basement through a pipe 2 cm in radius at an absolute pressure of 3 atm. A hose with a 0.5 cm radius is used to water plants 10 m above the basement. Find the speed of water as it leaves the hose?

We have two points of interest. Point 1 in the pipe at the basement, where $p_1 = 3atm, y_1 = 0m, v_1 = ?$ Point 2 is in the hose just at the moment the water leaving the hose for planting the tree, where $p_2 = 1atm, y_2 = 10m, v_2 = ?$ By applying Bernoulli's equation, we have



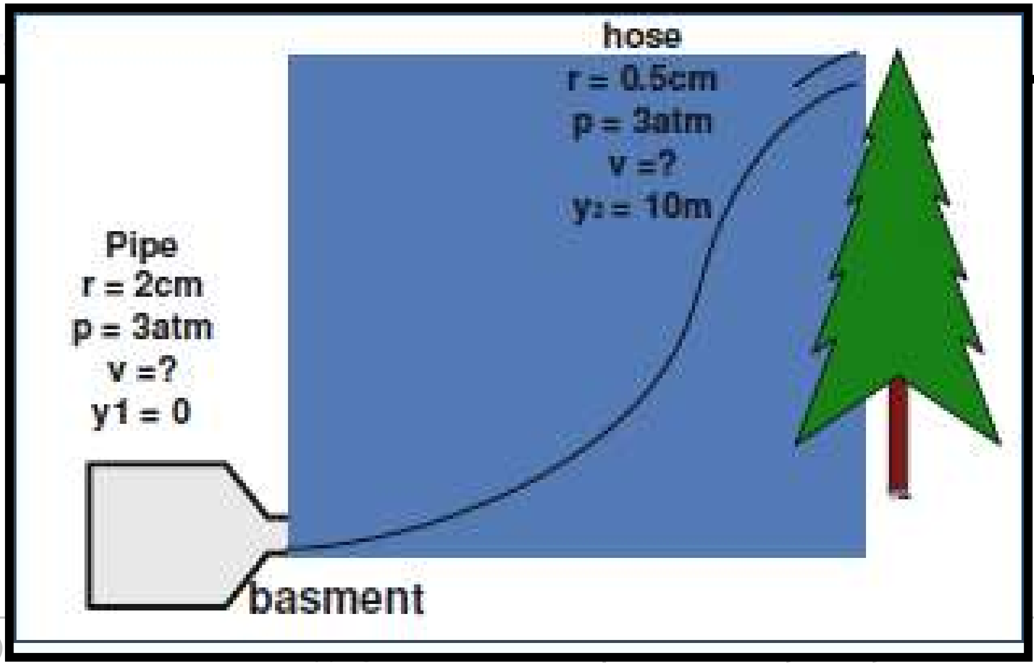
$$p_1 - p_2 = \frac{1}{2}\rho\left(\overline{v_2^2} - \overline{v_1^2}\right) + \rho g (y_2 - y_1)$$

We have two unknowns v_1 and v_2 , so we can reduce them to only one unknown by using the continuity equation, where $v_1 = \frac{r_2^2}{r_1^2} v_2$

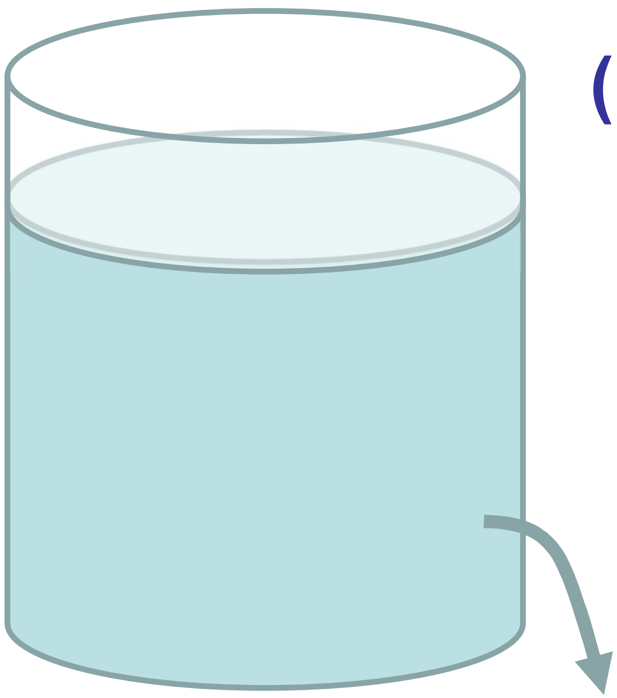
So $v_1 = \frac{1}{16} v_2$, and $\overline{v_2^2} - \overline{v_1^2} = \frac{255}{256} v_2^2$, then we substitute in the last equation to get:

$$(3-1) \times 1.013 \times 10^5 = \frac{1}{2}(1000) \frac{255}{256} v_2^2 + 1000(10)(10-0)$$

Solve for v_2 , we get $v_2 = 14.35\, ms^{-1}$.



Water is flowing from a hole of 1 cm radius at the bottom of a closed cylindrical container of 2m diameter. If the height of the water in the container is 2 m and the pressure over the surface of water is 3 atm., calculate how much time it took until the container became empty?



(1) Point on surface of liquid

(2) Point just outside hole

Applications of Bernoulli Equation

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

❑ Static Consequences

$v = 0$

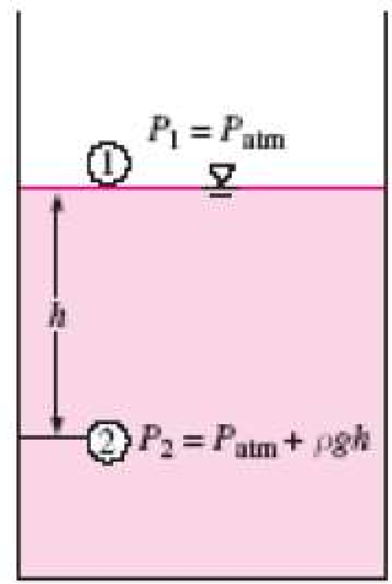
Static Consequence: In this consequence we have zero speed of flow, where the fluid is settle in its container. In this situation the kinetic energy density term leads to zero, and Bernoulli's equation becomes

$$p + \rho g y = constant$$

$$p_1 + \rho g y_1 = p_2 + \rho g y_2$$

$$p_2 = p_a + \rho g(y_1 - y_2) = p_a + \rho g h \qquad (5.13)$$

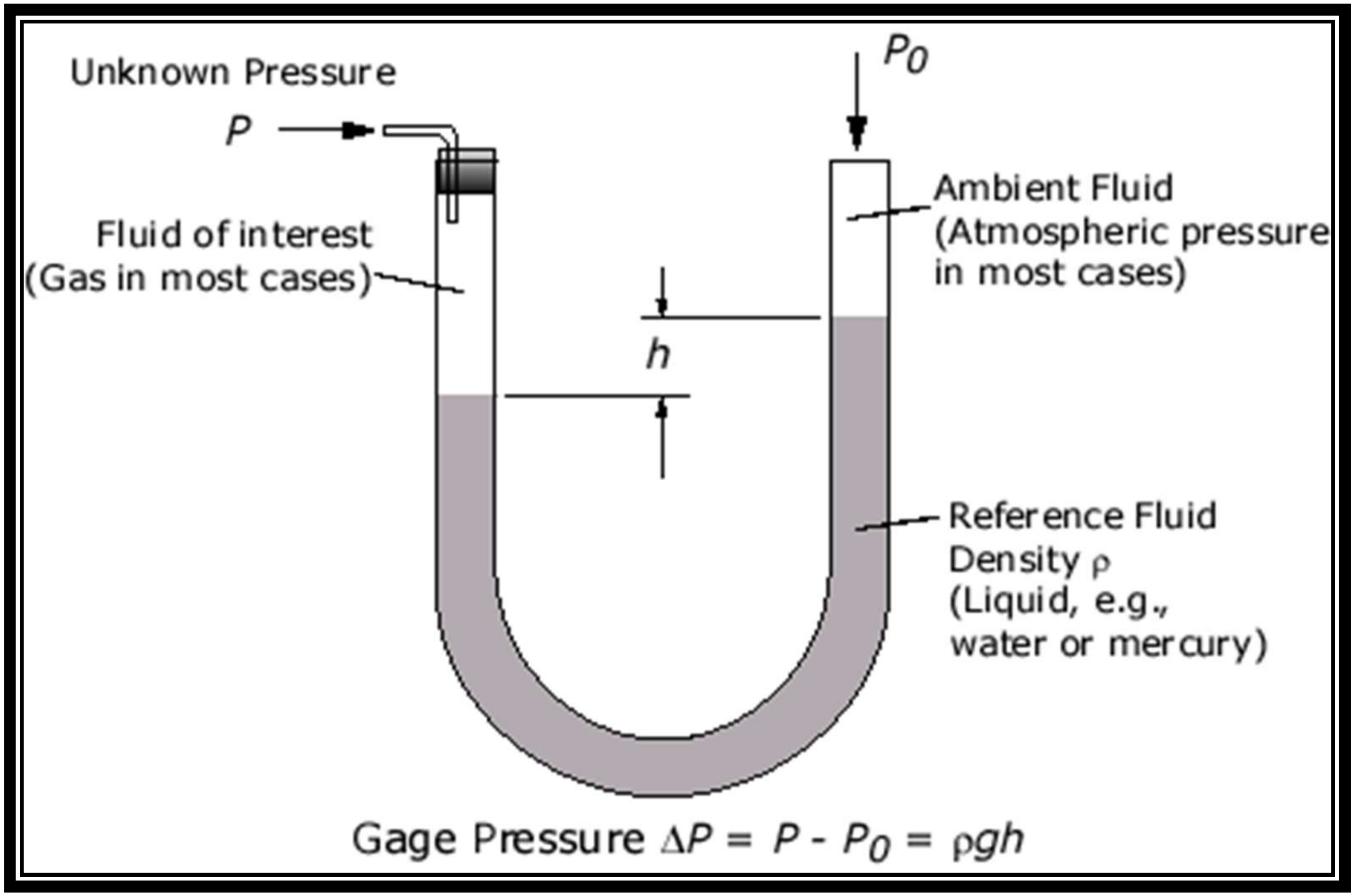
So the pressure inside any container is equal the pressure at the surface plus the potential energy density at that point.



Note: The difference between the absolute pressure at any point and the atmospheric pressure $(p - p_a)$ is called the gauge pressure.

Manometer

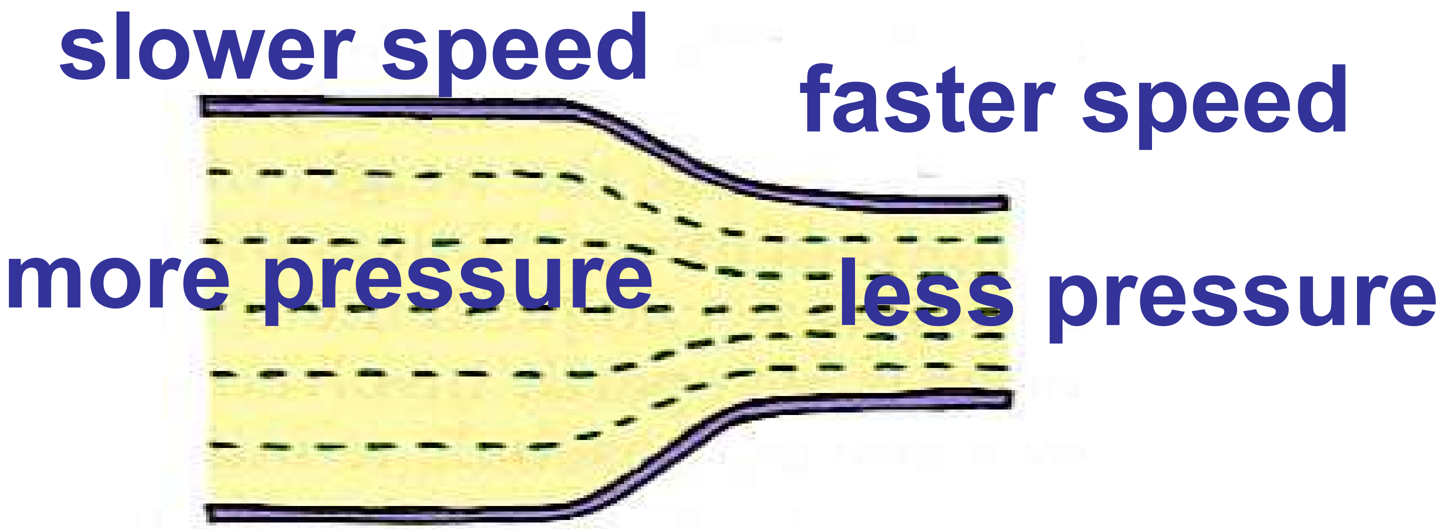
A manometer is a device for measuring fluid pressure consisting of a U tube containing one or more liquids of different densities. A known pressure (which may be atmospheric) is applied to one end of the manometer tube and the unknown pressure (to be determined) is applied to the other end. Differential pressure manometer measures only the difference between the two pressures.



Horizontal flow of fluids

$$P + \rho g y + \tfrac{1}{2}\rho v^2 = \text{constant}$$

If no change in height: $P + \tfrac{1}{2}\rho v^2 = \text{constant}$



$$p_1 + \frac{1}{2}\rho\,\overline{v_1^2} = p_2 + \frac{1}{2}\rho\,\overline{v_2^2}$$

As the <u>speed</u> of a fluid <u>increases</u>, the <u>pressure</u> in the fluid <u>decreases</u>.

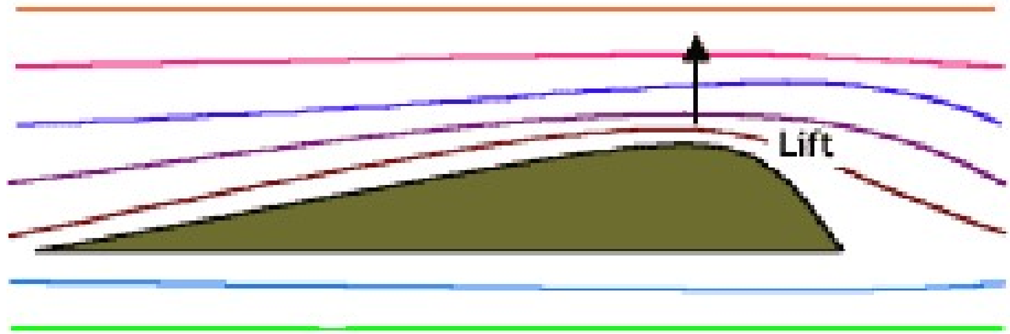
Bernoulli & Flight

Bernoulli's Principle is what allows birds and planes to fly.

The secret behind flight is 'under the wings

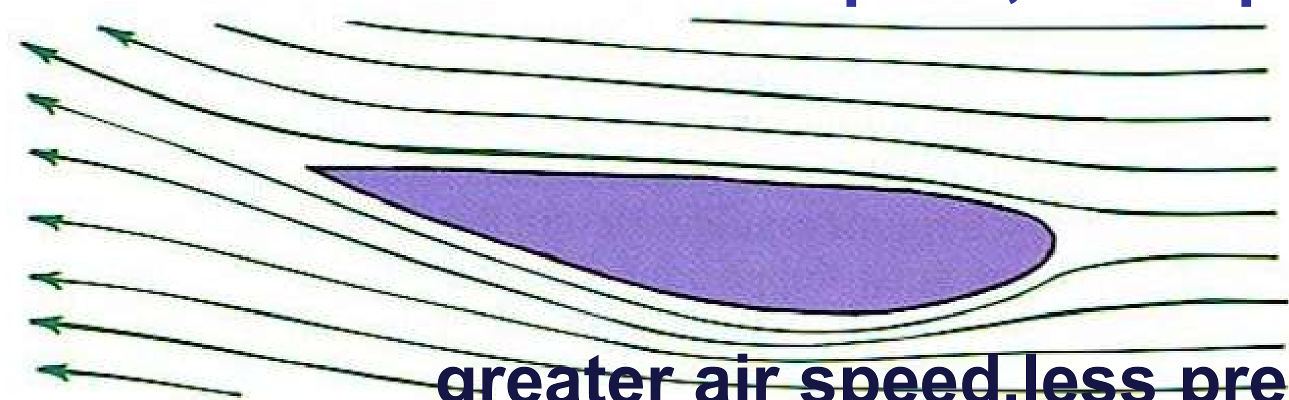
AIRFOIL

On top: greater air speed and less air pressure



On bottom: less air speed and more air pressure

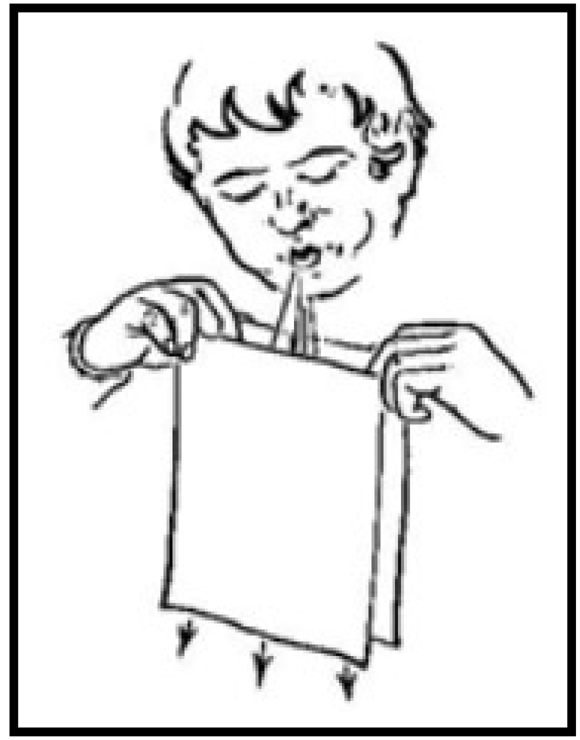
less air speed, more pressure



greater air speed,less pressure

net force: downward

Blowing air between two half sheets of paper



$$p_{out} + \frac{1}{2}\rho\,\overline{v_{out}^2} = p_{in} + \frac{1}{2}\rho\,\overline{v_{in}^2}$$

Rearrange the equation leads to the following

$$p_{out} - p_{in} = \frac{1}{2}\rho\,(\overline{v_{in}^2} - \overline{v_{out}^2})$$

When a person blows between the two sheets, so that the speed of air flowing inside will be larger than that outside ($v_{in} > v_{out}$): this means that the left hand side is positive and therefore $p_{out} > p_{in}$. This pressure difference results in the sheets moving closer toward one another. Thus, the pressure drops when the velocity of the flow increases for a fluid moving at a constant height.

Example 5.5

The diameter of a horizontal blood vessel is reduced from 12mm to 4 mm. What is the flow rate of blood in the vessel, if the pressure at the wide part is 8 kPa and 4 kPa at the narrow one. (Take the density of blood to be 1060 kgm^{-3}.)

the radius of the wider part $r_w = 6mm$,

the radius of the narrow part $r_n = 2mm$,

the pressure at the widew part $P_w = 8\ kPa$,

the pressure at the narrow part $P_n = 4\ kPa$

From the continuity equation, $\frac{v_w}{v_n} = \frac{r_n^2}{r_w^2} = \frac{1}{9}$, $\ggg v_n = 9v_w$

Apply BE

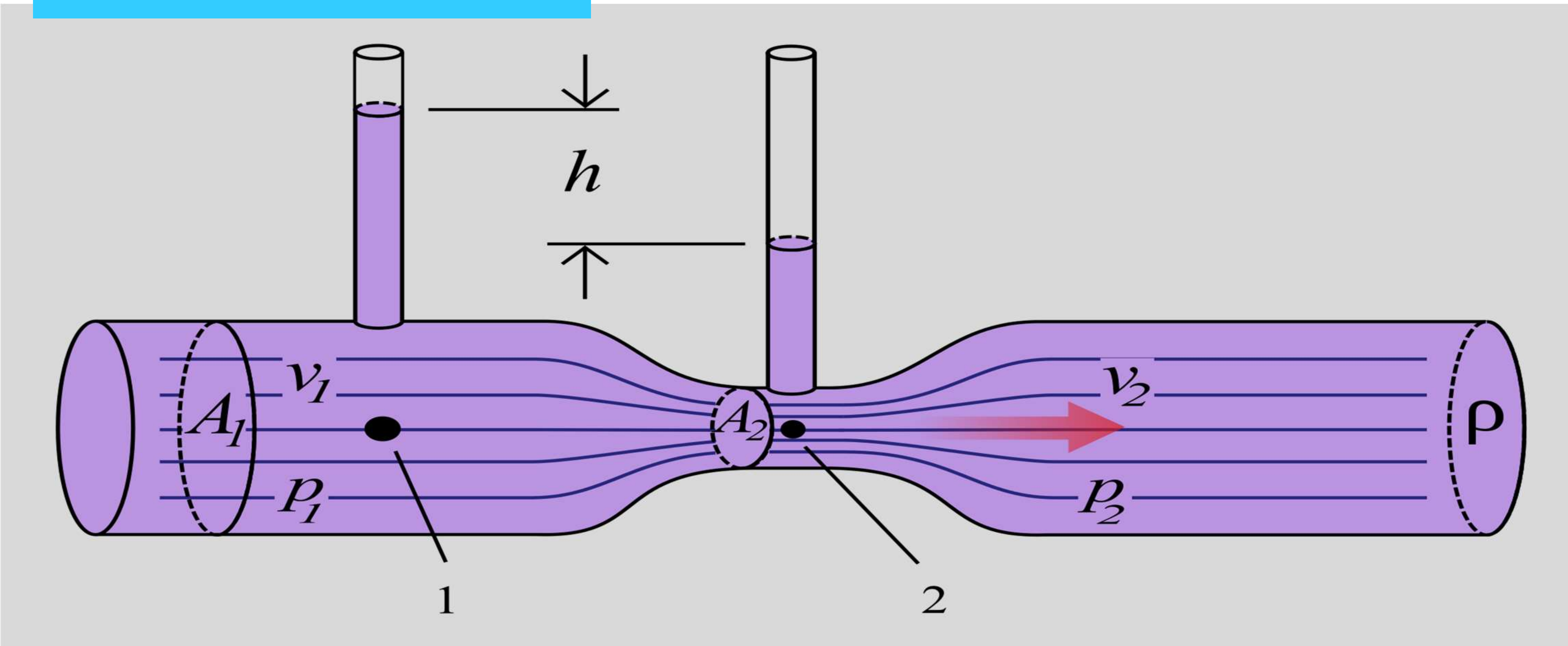
$$P_w - p_n = \frac{1}{2}\rho\left(v_n^2 - v_w^2\right)$$

$$4\times 10^3 = \frac{1}{2}\times 1060\times\left(80v_w^2\right)$$

$\longrightarrow$ $v_w = 0.307\ m/s$

Flow rate $Q = A_w v_w = \pi r_w^2 v_w = 3.14\times 36\times 10^{-6}\times 0.307$
$= 3.5\times 10^{-5} m^3/s$

Venturi Tube



$$p_1 - p_2 = \frac{1}{2}\rho\left(\overline{v_2^2} - \overline{v_1^2}\right)$$

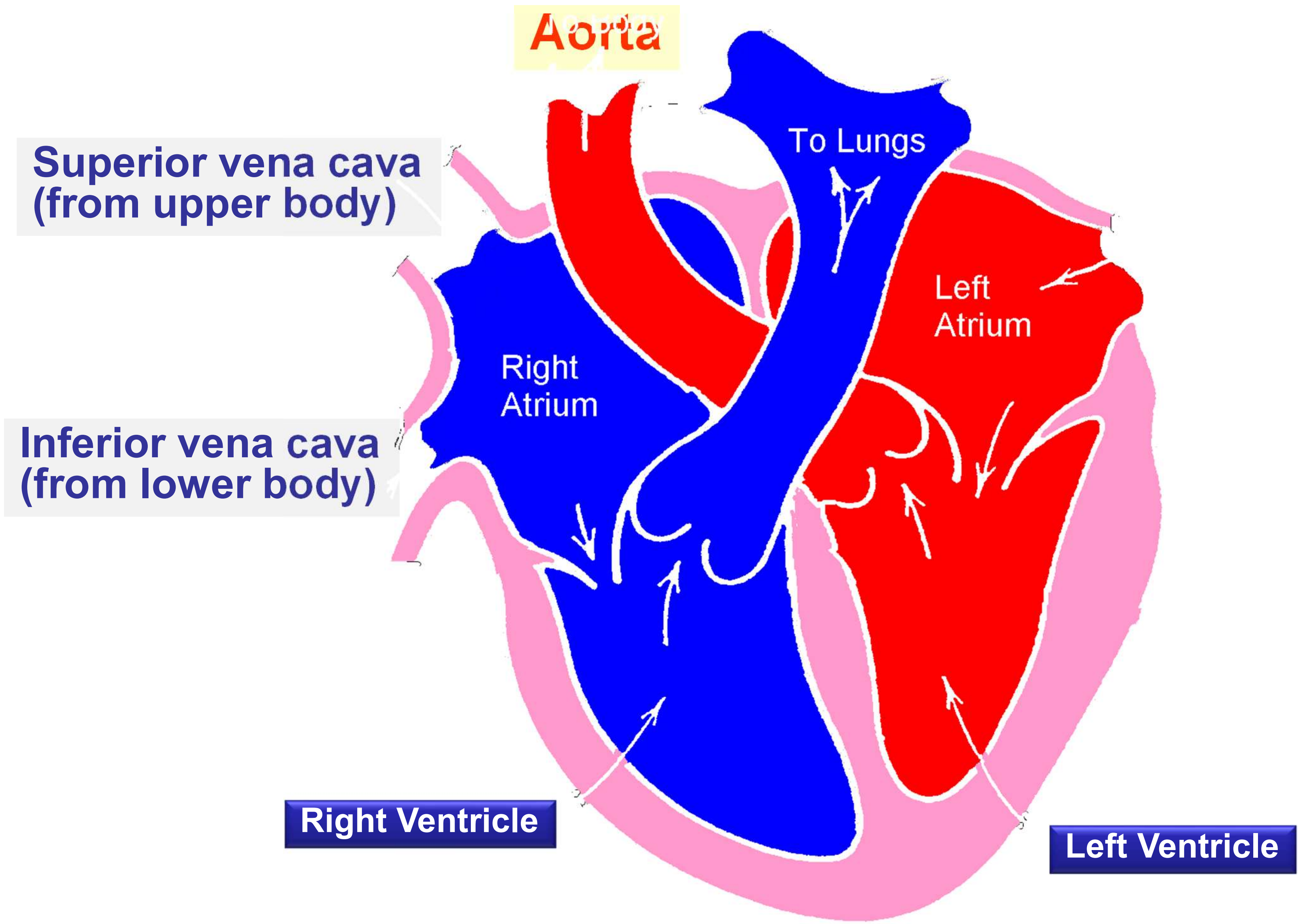
And from the continuity equation we have $v_2 = \frac{A_1}{A_2} v_1$, which can be substituted to get

$$v_1 = \sqrt{\frac{2\,(p_1 - p_2)}{\rho\left[\left({}^{A_1}/_{A_2}\right)^2 - 1\right]}}$$

Example 5.6

The flow of blood through a large artery in a dog is diverted through a Venturi flow meter. The wider part of the flow meter has an area of $0.08\ cm^2$, which equals the cross sectional area of the artery. The narrower part of the flow meter has an area of $0.04\ cm^2$. If the speed of the blood in the artery was $12.5\ \mathrm{cm\ s^{-1}}$, find the drop in pressure through the flow meter?

Blood Transport System



The role of gravity on blood circulation

We learned from Bernoulli's principle that the pressure of the fluid change according to its kinetic energy density and as well as it potential energy density. Because of that, the blood pressure in human organs is affected by its location from earth.

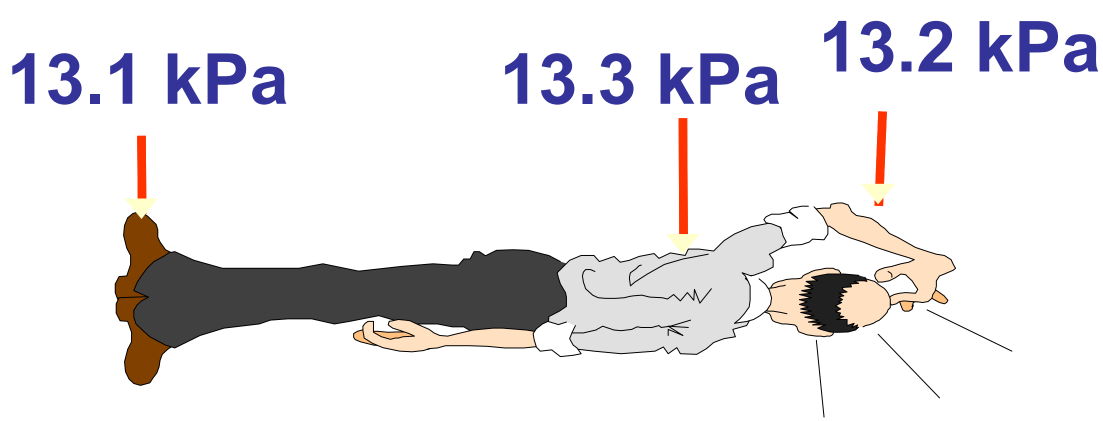
$$p_B + \tfrac{1}{2}\rho v_B^2 + \rho g y_B = p_H + \tfrac{1}{2}\rho v_H^2 + \rho g y_H = p_F + \tfrac{1}{2}\rho v_F^2 + \rho g y_F$$

Note that B, H, and F refer to the brain, heart and feet, respectively

Reclining State

The velocities in the three main arteries (Brain, heart, and feet) are small and roughly equal, so that the kinetic energy density term can be ignored

$$p_B \sim p_H \sim p_F$$



During the blood circulation, the venous system is used to return the blood from the lower extremities to the heart. It is expected to have a problem of lifting blood long distances to the heart against the force of gravity.

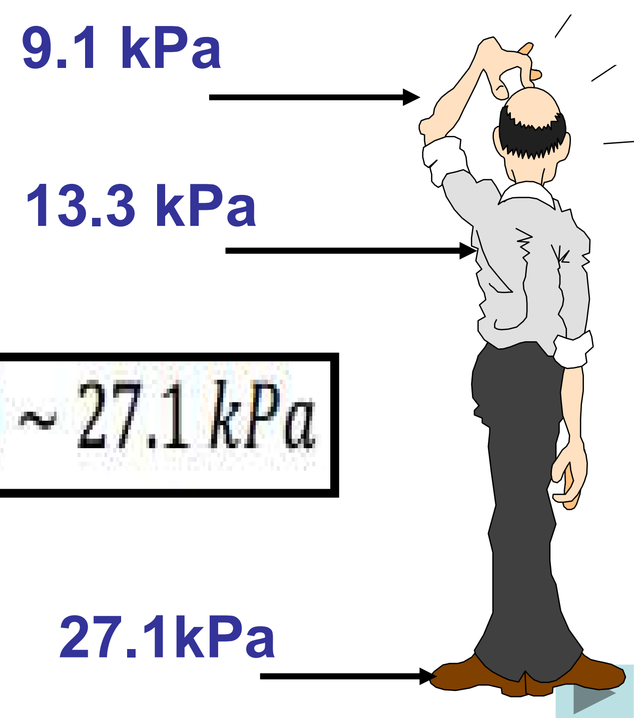
Standing Position

The kinetic energy density can be only ignored

$$p_B + \rho g y_B = p_H + \rho g y_H = p_F$$

Typical values for adults standing upward h_H = 1.3 m, h_B = 1.7 m. Typical value of the blood pressure at the heart is gä = 13.3 KPa , and take the blood density to be 1060 kg/ m^3, we find:

$$p_F = p_H + \rho g h_H = 13.3 \times 10^3 + (1060)(10)(1.3) \sim 27.1\, kPa$$



The blood pressure at brain

In a similar way, we find that:

$$p_B = p_H + \rho g(h_H - h_B) = 13.3 \times 10^3$$
$$+ (1060)(10)(-0.4) = 9.06\,kPa$$

❑This explains why the pressures in the lower and upper parts of the body are very different when the person is standing, although they are about equal in the reclining.

❑The high blood pressure at the foot explain the possibility of lifting blood uphill to the heart, and in addition the muscles surrounding the veins contract and cause constriction.

Example 5.7

When a $1.7\ m$ tall man stands, his brain is $0.5\ m$ above his heart. If he bends so that his brain is 0.4 m below his heart, by how much does the blood pressure in his brain changes?

from B.E.

$$P_B + \rho g h_B = P_H + \rho g h_H$$

$$P_B = P_H + \rho g (h_H - h_B)$$

by standing $h_H - h_B = -0.5\ m$

by Bending $h_H - h_B = 0.4\ m$

from B.E.

$$P_B + \rho g h_B = P_H + \rho g h_H$$

$$P_B = P_H + \rho g (h_H - h_B)$$

by standing $\quad h_H - h_B = -0.5 \text{ m}$

by Bending $\quad h_H - h_B = 0.4 \text{ m}$

$$(P_B)_{bending} - (P_B)_{standing} = \rho g (h_H - h_B)_{bending} - \rho g (h_H - h_B)_{stand}$$

$$\Delta P_B = \rho g \left[(h_H - h_B)_{bend.} - (h_H - h_B)_{standing} \right]$$

$$= 1060(10)[0.4 - (-0.5)]$$

$$= 10600 \times 0.9 = 9540 \text{ Pa}$$

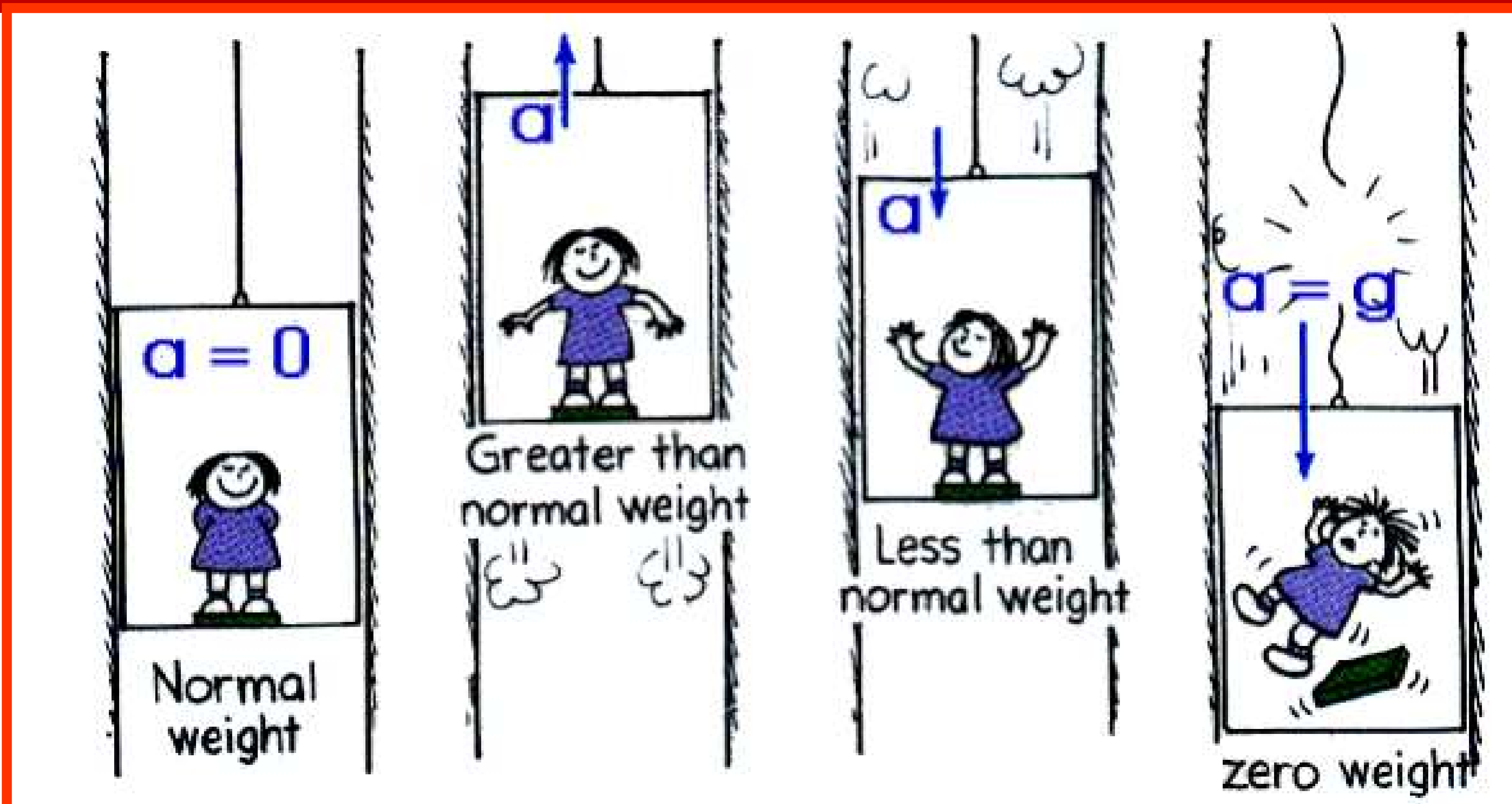
$$\Delta P_B = 9.54 \text{ kPa}.$$

Effect of acceleration on Blood pressure

The question arise:

Is the blood pressure at the organs affected when man under upward or downward acceleration.

When a person in an erect position experiences an upward or downward acceleration, his weight will be different.



Upward acceleration

If a man in an erect position experience upward acceleration a, then his effective weight becomes m(g+a).

Applying Bernoulli's equation to the foot, brain and heart with g replaced by g+a, so we have:

$$p_B = p_H + \rho\,(g + a)(h_H - h_B) = p_H - \rho\,(g + a)(h_B - h_H)$$

Fatal upward acceleration when $P_B = 0$

$$0 = p_H - \rho\,(g + a)(h_B - h_H)$$

$$(g + a) = p_H / \rho\,(h_B - h_H)$$

$$(g + a) = \frac{13.3 \times 10^3}{1060\,(0.4)} = 31.4\,ms^{-2} = 3.2\,g$$

So the value of the upward acceleration causing consciousness *is 3.2*

This factor should limit the speed with which a pilot can pull out of dive. A related experience is the feeling of light headache that sometimes occurs when one suddenly stands up

The blood pressure at the foot by the upward acceleration situation

$$p_F = p_H + \rho(g + a)h_H$$

So the blood pressure at the foot increases by increasing the upward acceleration

The blood pressure at the brain decrease by increasing the upward acceleration

Downword acceleration

In an erect position experience downward acceleration leads to change of his effective weight.
Applying Bernoulli's equation to the foot, brain and heart with *g* replaced by g – a, so we have

$$p_B = p_H + \rho\,(g - a)(h_H - h_B)$$

Or

$$p_B = p_H - \rho\,(g - a)(h_B - h_H)$$

Thus the blood pressure at the brain will increase even farther by increasing the downward acceleration. This increase should be controlled and observed, where at certain value of a the blood pressure at the brain may cause an explosion of the arteries in the brain, which is so dangerous. The same calculation for the blood pressure at the foot

$$p_F = p_H + \rho\,(g - a)h_H$$

Which decreases by increasing the downword acceleration

Example

If an elevator accelerates upward at 12 ms^{-2}, what is the average blood pressure in the brain? What is the average blood pressure in the feet? If the elevator accelerates downward with the same acceleration, what is the average blood pressure in the brain and feet? take g =10 ms^{-2}.

① accelerating upward

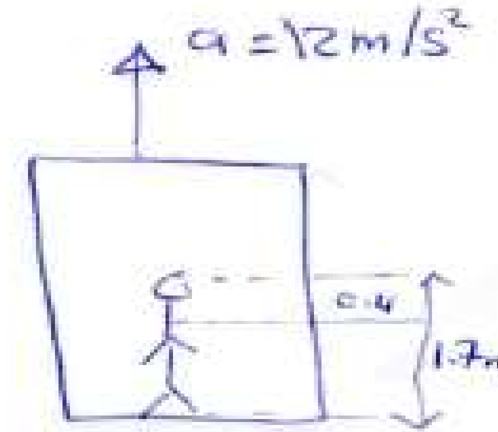
$$P_B = P_H + \rho(g+a)(h_H - h_B)$$

$$P_B = 13.6\times10^3 + 1060(10+12)(-0.4)$$

$$= 13.6\times10^3 + (-9328)$$

$$P_B\uparrow = \boxed{4272\ Pa}$$

نقصان

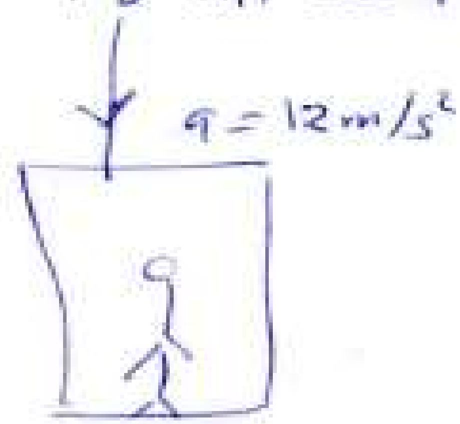


$h_H = 1.3m$

$h_B - h_H = 0.4\ m$

$$P_F\uparrow = P_H + \rho(g+a)h_H$$

② accelerating downward



$$\downarrow P_F = P_H + \rho(g-a)h_H$$

$$= 13.6\times10^3 + 1060(10-12)(1.3)$$

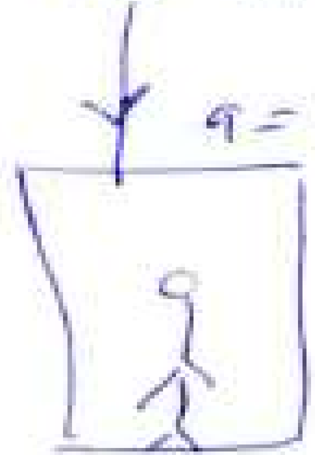
$P_B \uparrow = \boxed{42772 \ Pa}$ نقصان

$h_H = 1.3 m$

$h_B - h_H = 0.4 m$

$P_F \uparrow = P_H + \rho(g+a) h_H$

$a = 12 m/s^2$



② accelerating downward

$\downarrow P_F = P_H + \rho(g-a) h_H$

$= 13.6 \times 10^3 + 1060(10-12)(1.3)$

$= 13.6 \times 10^3 + (-2756)$

$\downarrow P_F = 10844 \ Pa \ \boxed{= 10.84 \ kPa}$ نقصان

$\downarrow P_B = P_H + \rho(g-a)(h_H - h_B)$

$= 13.6 \times 10^3 + 1060 (10-12)(-0.4)$

$= 13600 + 848$

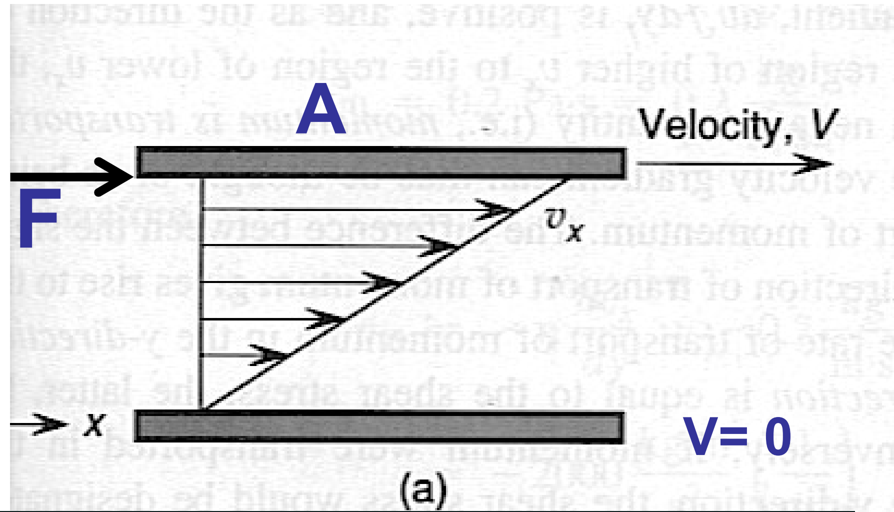
$\boxed{\downarrow P_B = 14.45 \ kPa}$ زيادة

Viscous Fluid Flow

Resistivity of flow
Frictional forces among the fluid layers and their containers

Viscosity in Gases are different that in Liquids

Viscosity can be defined using two parallel plates separated by a distance Δy and a fluid fills the space between the two plates, one stationary and the other moving at velocity v



It is found that the force required to move the upper plate at constant average speed is proportional directly with the speed gradient and the surface area of the plate and inversely proportional with the separation distance between the plates, which means that:

$$F \propto \Delta v \,,\; F \propto A \text{ and } F \propto \frac{1}{\Delta y} \,, \text{so } F \propto \frac{A\,\Delta v}{\Delta y}$$

$$F = \eta \frac{A\,\Delta v}{\Delta y}$$

The constant of proportion is called the viscosity coefficient, which is represented by the Greek symbol η,

$$\eta = \frac{F/A}{\Delta v/\Delta y}$$

Coefficient of Viscosity

$$\eta = \frac{F/A}{\Delta v/\Delta y}$$

The viscosity can be defines as the ratio between the shearing stress (F / A) to the rate of shearing strain or the gradient of velocity. The dimension of the viscosity coefficient can be deduced as:

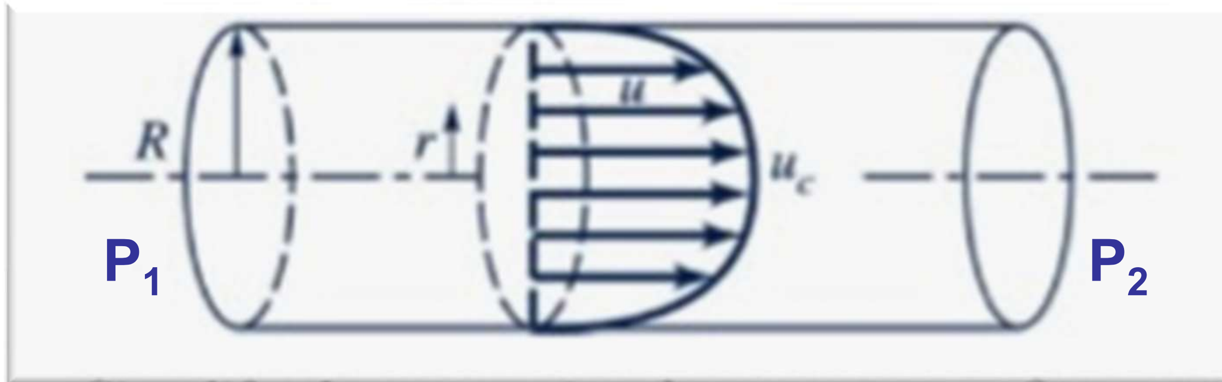
$$[\eta] = \left[\frac{F/A}{\Delta v/\Delta y}\right] = \left[\frac{MLT^{-2}/L^2}{LT^{-1}/L}\right] = ML^{-1}T^{-1}$$

The S.I. unit of the viscosity coefficient is Pascal second, where kg.m^{-1}s^{-1} = 1 Pas.s. The Pascal second is rarely used in technical and scientific publications. The common used unit is called poise, where 1 poise= 10^{-1} Pa.s, . l, so the abbreviation centipoises is equal 10^{-3} Pa.s

Temperature C	Castor Oil	Water	Air	Normal blood	Blood Plasma
0	5.3	1.792×10^{-3}	1.71×10^{-5}		
20	0.986	1.005×10^{-3}	1.81×10^{-5}	3.015×10^{-3}	1.81×10^{-3}
37	-	0.695×10^{-3}	1.87×10^{-5}	2.08×10^{-3}	1.257×10^{-3}
40	0.231	0.656×10^{-3}	1.9×10^{-5}		
60	0.08	0.469×10^{-3}	2.00×10^{-5}		
80	0.03	0.357×10^{-3}	2.09×10^{-5}		
100	0.017	0.284×10^{-3}	2.18×10^{-5}		

Laminar Flow in a Tube

Take a cylinder of radius R and length l



$F = \Delta P\ A = (P_1 - P_2)\ A$

Take half of the cylinder
Where the uper surface pass with the center, and the speed of the upper Layer is V_{max} , so that the cross sectional area of the half cylinder is $A = \pi(R/2)^2$ and its surface area $A_{surf} = 2\pi(R/2)l$

$F = \Delta P\ A = \eta\ A_{surf}\ v_{max} / R$

$\Delta P\ \pi(R/2)^2 = \eta\ 2\pi(R/2)l\ v_{max} / R$

So that

$$V_{max} = \frac{\Delta P\ R^2}{4\eta l}$$

The average speed is half of The maximum speed. So that

$$\bar{V} = \frac{\Delta P\ R^2}{8\eta l}$$

The flow rate is given by

$$Q = A\ \bar{V} = \pi R^2 \frac{\Delta P\ R^2}{8\eta l} = \frac{\pi \Delta P\ R^4}{8\eta l}$$

• Poiseulle's Equation

Volume flow rate, *Q*, related to pressure difference *ΔP*, length *L* and radius R by:

$$Q = \frac{\pi R^4}{8\eta L}\Delta p$$

- ❑The formula for Q is called ***Poiseuille's law*** **after the physician, Jean Luis Poiseuille.**
- **❑ High viscosity leads to low flow rate and speed of flow.**
- **❑ The flow rate is proportional to the 4th power of R, which is extremely dependent.**
- **❑This indicates that for blood vessel, any small change in the radius of the vessel results in a considerable change of the flow rate. For example, if the radius of an artery is halved, so the flow rate will be reduced to 1/16**

Example 5.9

What is the pressure drop in the blood as it passes through a capillary $5\,mm$ long and $3\mu m$ in radius if the speed of the blood at the center of the capillary is $0.6\ ms^{-1}$. Take the viscosity of blood to be $2.08\,cp$. If the radius of the capillary is reduced by 20 %, find the change in the flow rate?

The capillary radius R = $3 \times 10^{-6}\,m$, *and its length* $l = 5 \times 10^{-3}\,m$,

and the ***maximum speed of*** *the* ***blood inside it,*** $\quad v_{max} = 0.6\,\frac{m}{s} = \frac{\Delta P R^2}{4\eta l}$

$$\textit{so the pressure drop } \Delta p = \frac{4\eta l v_{max}}{R^2} = \frac{4\,(2.08\times10^{-3})5\times10^{-3}(0.6)}{9\times10^{-12}} = 2.7 \text{ MPa}$$

When the radius is reduced by 20%, so that $R_2 = 0.8\,R_1$

$$\frac{Q_2}{Q_1} = \frac{R_2^4}{R_1^4} = (0.8)^4 = 0.41$$

Power dissipation

The power dissipated during the flow of a fluid is the rate of energy required to maintain the flow. In general, the power is defined as the net force times the average speed,

$$\boxed{\mathbb{P} = F\,\bar{V} = \Delta P\,A\,\bar{V} = \Delta P\,Q}$$

Measured in Watt

Flow Resistance

The flow resistance can be defined in general as *the ratio of the pressure drop through a segment and the flow rate,* (notice the analogy to electric resistance, the pressure difference like the potential difference, and the flow rate like the electric current).

$$\mathcal{R}_f = \frac{\Delta P}{Q} = \frac{\Delta P}{\pi \Delta P R^4 / 8\eta l} = \frac{8\eta l}{\pi R^4}$$

The unit of flow Resistance is $\boxed{\mathcal{R}_f = \frac{Pa}{m^3/s} = \frac{Pa.s}{m^3}}$

Which is the viscosity per unit volume or viscosity density

Example 5.11

Compare the flow resistance in a capillary of 5 μm in radius and that in an artery of 5 cm in radius?

$$\frac{\mathcal{R}_f(capillary)}{\mathcal{R}_f(artery)} = \left(\frac{R_a}{R_c}\right)^4 = \left(\frac{5 \times 10^{-2}}{5 \times 10^{-6}}\right)^4 = 10^{16}$$

Example 5.12

A large artery has an inner radius of 4 mm. Blood flows through the artery at the rate of 1 cm^3s^{-1}. Find

a) The average and maximum speed of the blood in the artery

b) The pressure drop in a 10 cm long segment of the artery

c) The flow resistance of blood over the 10 cm segment

d) The power dissipated through the flow

The artery has radius R = $4 \times 10^{-3} m$,

$$the\ flow\ rate\ Q = 10^{-6}\ {m^3}/{s} = A\bar{v} = \pi R^2\bar{v},$$

$$so\ that\quad \bar{v} = \frac{Q}{\pi R^2} = \frac{10^{-6}}{3.14(16\times10^{-6})} = 0.02\ m/s\ ,then\ \ v_{max} = 0.04\ m/s$$

$$so\ the\ pressure\ drop\ \Delta p = \frac{4\eta l v_{max}}{R^2} = \frac{4\,(2.08\times10^{-3})10\times10^{-2}(0.04)}{16\times10^{-6}} = 2.08\ \text{Pa}$$

$$\mathcal{R}_f = \frac{\Delta P}{Q} = \frac{2.08}{10^{-6}} = 2.08\times10^{6}\ {Pa.s}/{m^3}$$

$$\wp = \Delta PQ = 2.08\times10^{-6} = 2.08\ \mu W$$

Turbulent Flow

When flow speed becomes high the streamlines of the flow start to intersect each other; and this flow is called turbulent flow. In such flow, the mechanical energy dissipated is much larger than that in the laminar flow.

Poiseuille's law is applicable for laminar flow.
There is a dimensionless quantity called Reynolds Number (N_R) used to distinguish the type of the flow. Consider a fluid of density ρ and viscosity coefficient η flows with an average velocity v through a tube of radius R, hence the Reynolds number is defined by

$$N_R = \frac{2\rho\bar{v}R}{\eta}$$

It is found experimentally that if

$$N_R < 2000 \qquad \textit{flow is laminar}$$

$$N_R > 3000 \qquad \textit{flow is turbulent}$$

$$2000 < N_R < 3000 \qquad \textit{flow is unstable}$$

Example 5.14

A small human capillary of 100 μm radius has a length of 2 cm, calculate

a) The blood flow resistance across this capillary
b) If the pressure drop across the capillary is 2.3 kPa, what is the flow rate
c) What is the maximum speed of blood through the capillary
d) The power dissipated across the capillary
e) Reynolds's number and then determine the type of flow.

If the total flow resistance of blood through three similar parallel arteries of length 30 cm is 4×10^{8} Pa.sm^{-3}.find

The diameter of each artery

If the power dissipated in pumping the blood in each artery is 0.4 mW, determine if the flow type of blood in the artery, laminar or turbulent? (explain) (take η_{blood} = 3.14 cp, ρ_{blood} = 1100 kgm^{-3}).