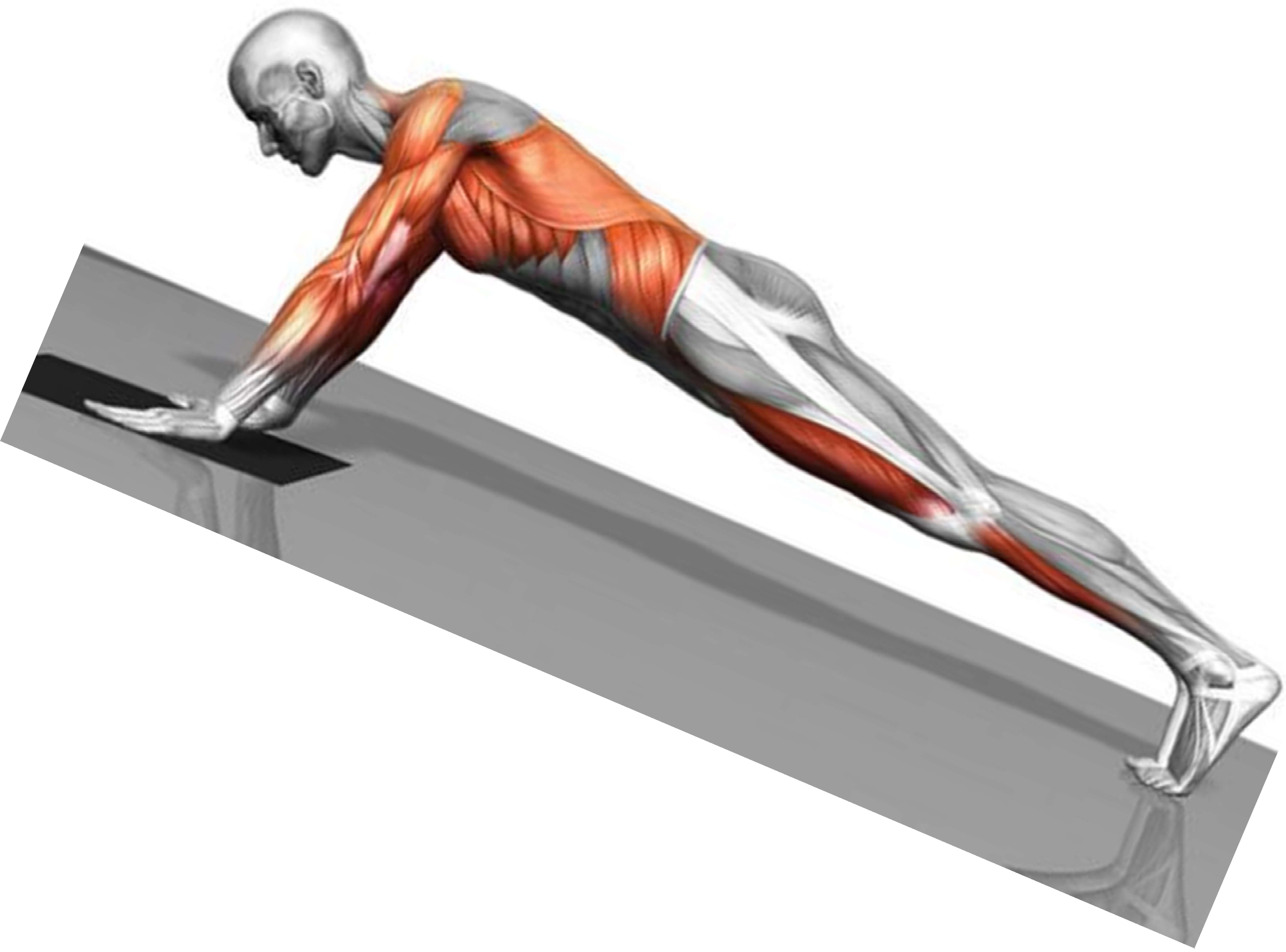
Chapter 2
Biostatic



MUSCULOSKELETAL SYSTEM:

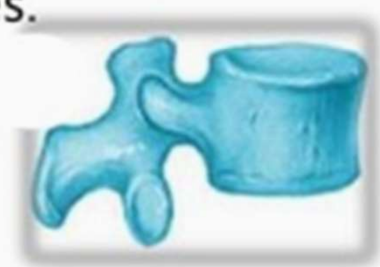
The combination of bones, joints, muscles and related connective tissues are known as the musculoskeletal system.

Joints:

Where the bones come together (and with the power of the muscles) give a variety of range of motion (R.O.M). Joints, (joint, classifications) consist of 360 joints.

Bones:

The framework of the body is the skeleton (Bones). Bones of the skeletal system consist of 206 bones.



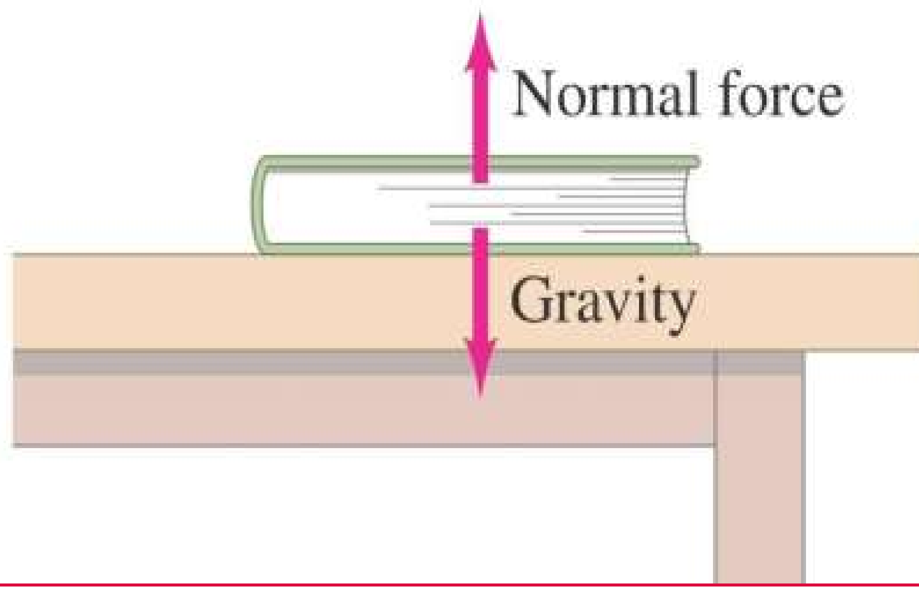
Muscles:

The body's motive power. (muscular system) consist of 640 muscles.

Equilibrim (Statics)

Object at rest, net force is zero

- Example: Book on a table, two forces acting on it
  - Gravitational force
  - Normal force
- Book at rest, acceleration zero, total force is zero
- Gravitational force and normal force balance each other
- The book is in equilibrium ("balance" in Latin)



Static Equilibrium (No Motion)

- Static equilibrium conditions

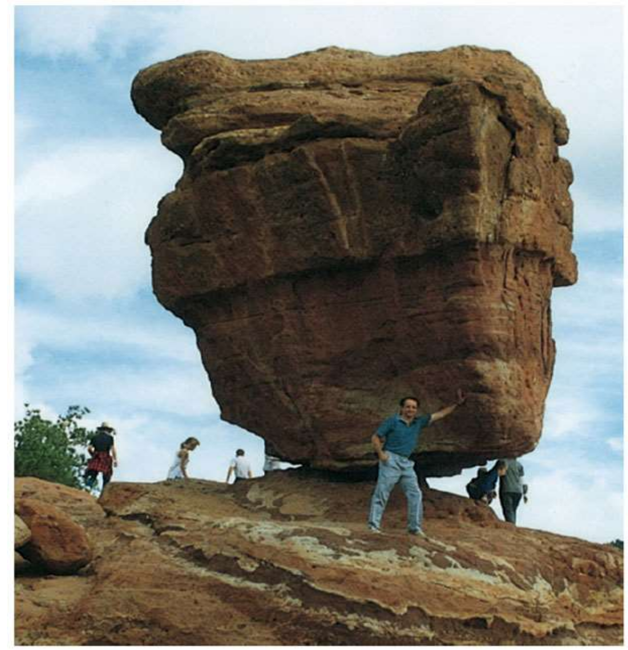
  1. No Translational Force

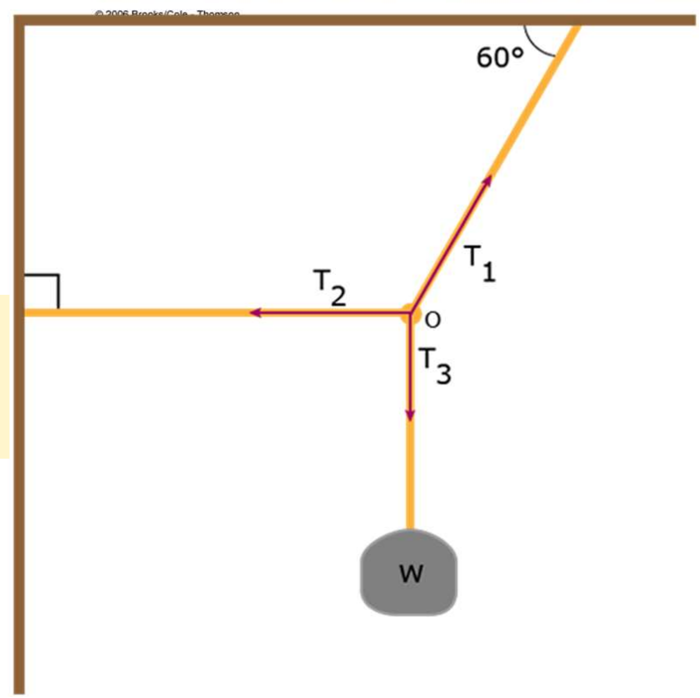
  Resultant external force must equal zero

$$\vec{F}_{net} = \sum \vec{F}_{ext} = 0$$

$$\sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum F_x = T_1 cos\theta - T_2 = 0$$
$$\sum F_y = T_1 sin\theta - T_3 = 0$$



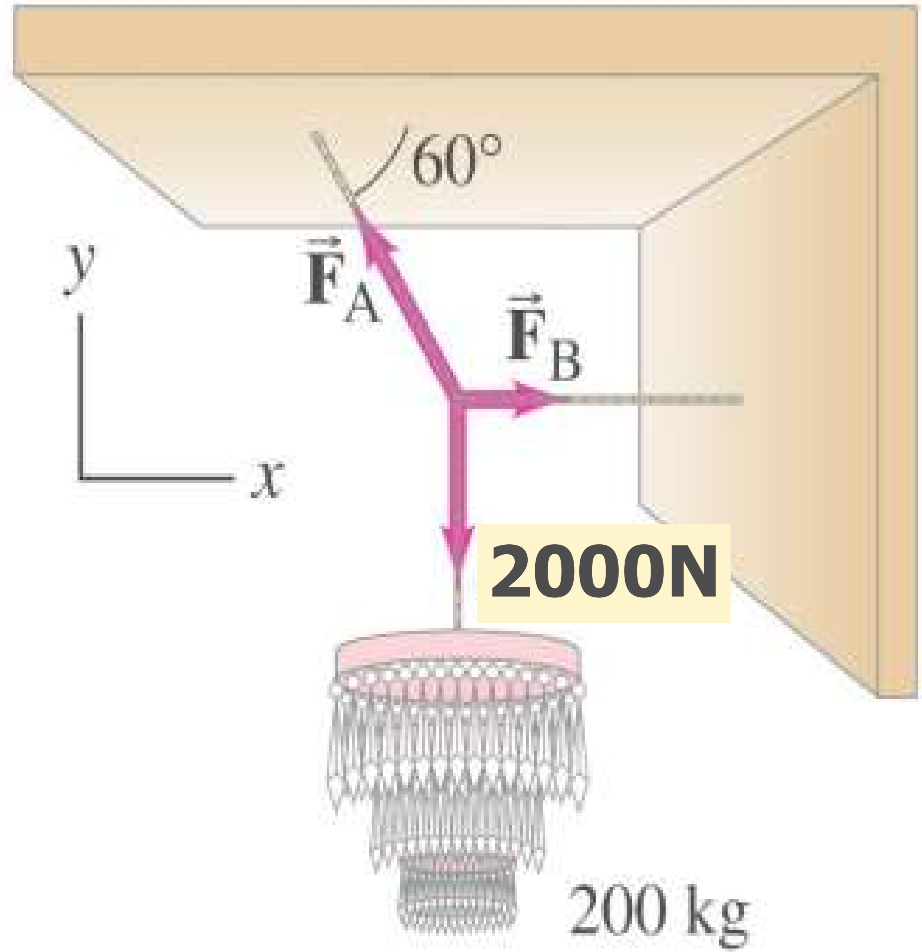


Upper Forces balance Down forces

Forces to the right balance Forces to the left

Example

Calculate the tensions $\vec{F}_A$ and $\vec{F}_B$ in the two cords that are connected to the vertical cord supporting the 200-kg chandelier shown. Ignore the mass of the cords.



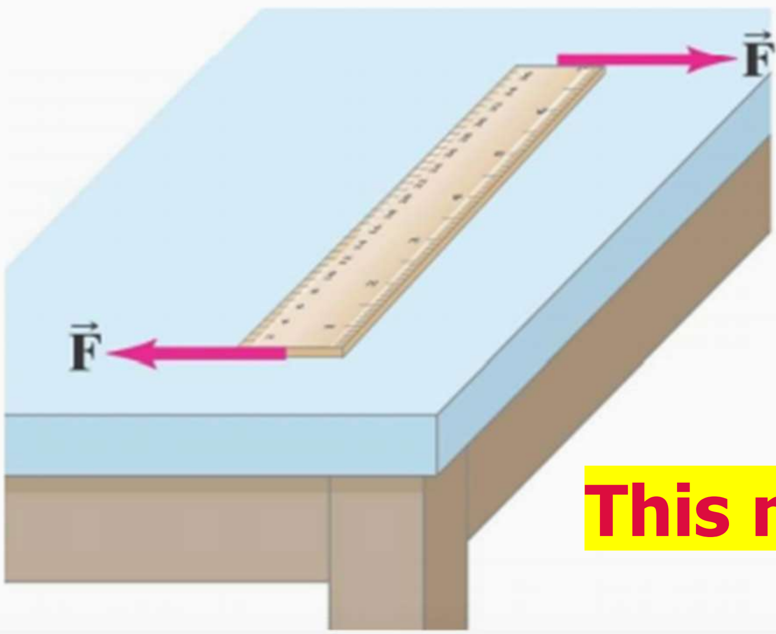
Conditions for Equilibrium

- For an extended object to be in equilibrium, a second condition must be satisfied

2. This second condition involves that No rotational motion of the extended object

The net external torque on the object must equal zero

The second condition of equilibrium:



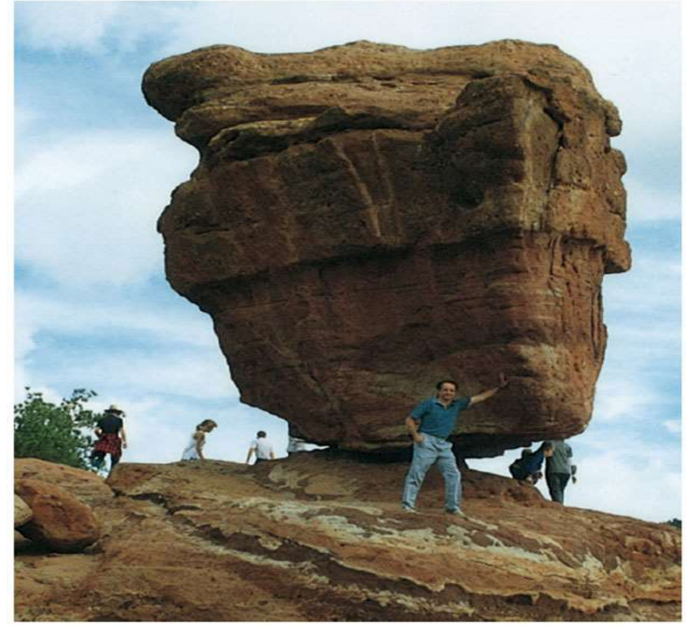
$$\Sigma\tau = 0$$

$$\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = I\vec{\alpha} = 0$$

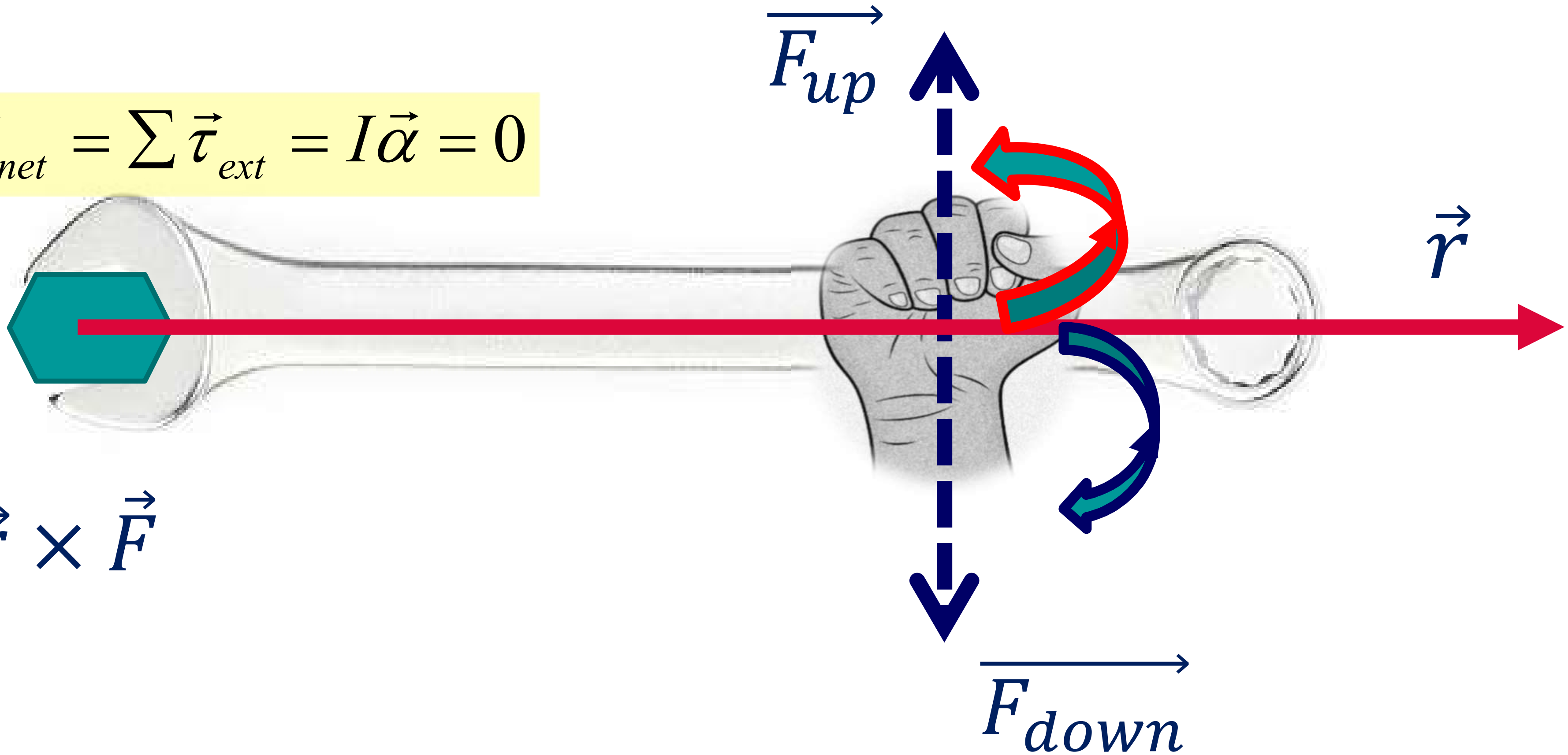
This must be true for any axis of rotation

$\overrightarrow{F_{up}}$ cause a counterclockwise rotation, then the torque is positive

$\overrightarrow{F_{down}}$ cause a clockwise rotation, then the torque is negative



$$\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = I\vec{\alpha} = 0$$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

2nd Conditions for Equilibrium

- The net torque equals zero
  - This means that the torque causes counterclockwise rotation balances with the torque that cause clockwise rotation

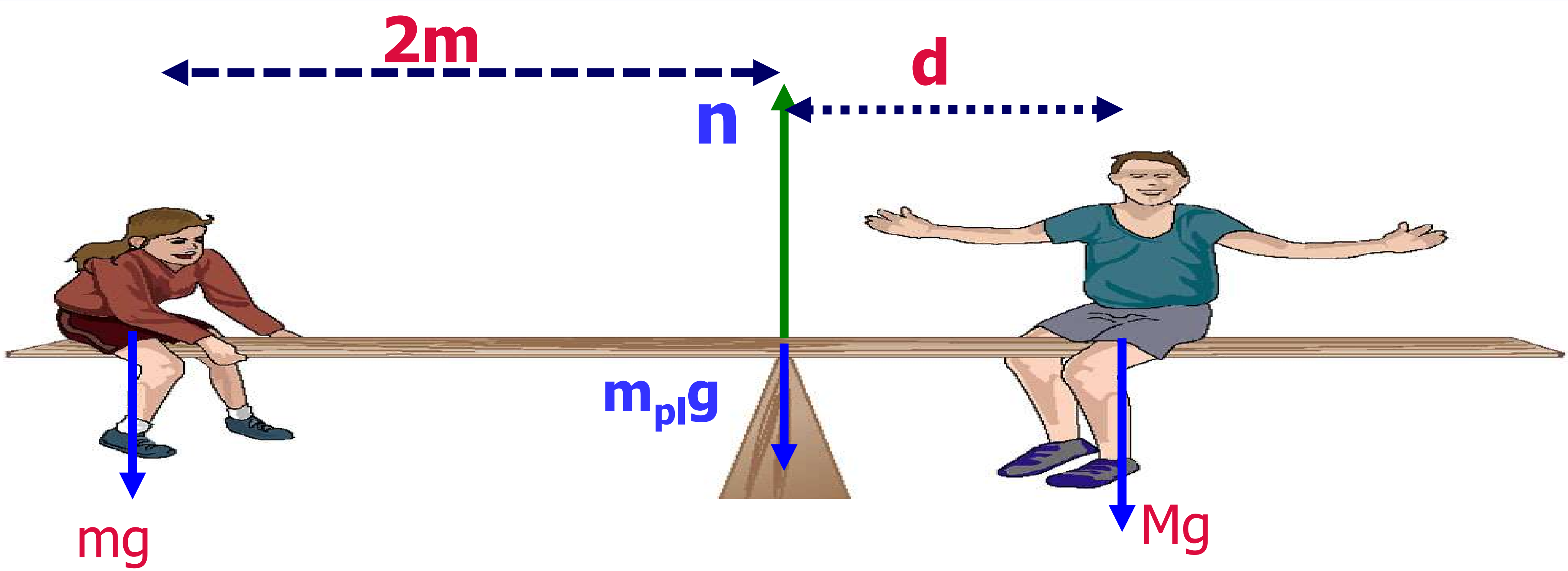
$$\sum \vec{\tau} = 0$$

$$\vec{\boldsymbol{\tau}}_{net} = \sum \vec{\boldsymbol{\tau}}_{ext} = \mathbf{0}$$

$$\boldsymbol{\tau}_{C.C.W} = \boldsymbol{\tau}_{C.W}$$

A seesaw consisting of a uniform board of mass m_{pl} and length L supports a father and daughter with masses M and m, respectively. The father is a distance d from the center, and the daughter is a distance 2 m from the center.

- A) Find the magnitude of the upward force n exerted by the support on the board.
- B) Find where the father should sit to balance the system at rest.



A) Find the magnitude of the upward force **n** exerted by the support on the board.

B) Find where the father should sit to balance the system at rest.

$$F_{net,y} = n - mg - Mg - m_{pl}g = 0$$

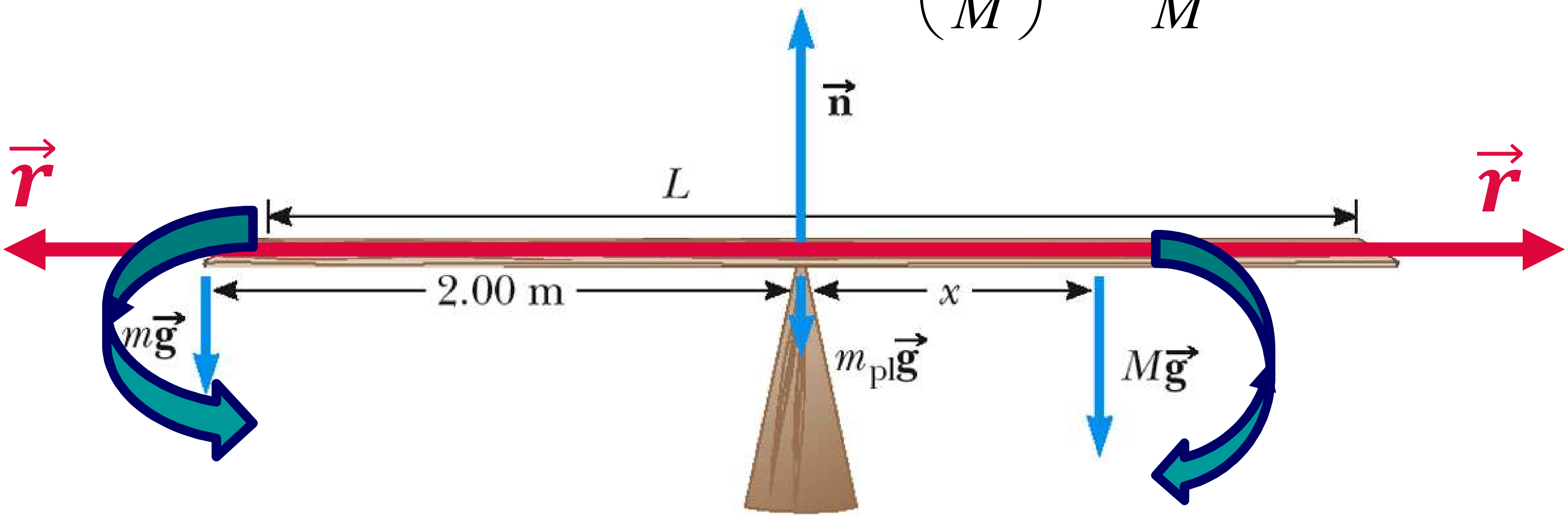
$$n = mg + Mg + m_{pl}g$$

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$

$$= mgd - Mgx + 0 + 0 = 0$$

$$mgd = Mgx$$

$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M} < 2.00\,\mathrm{m}$$



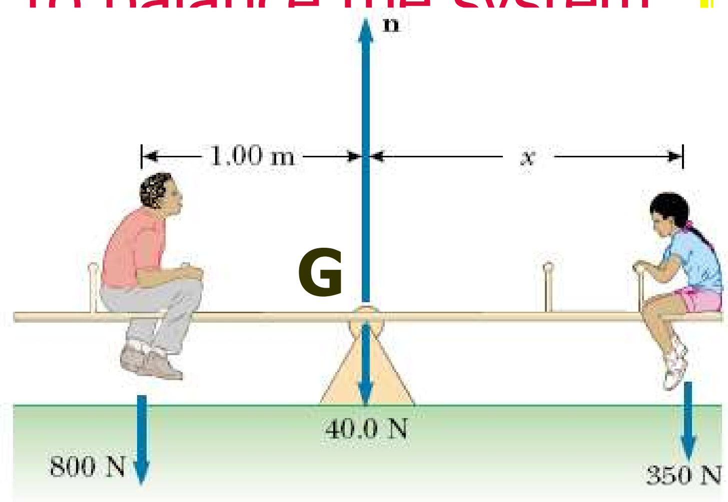
Axis of Rotation

- The net torque is about an axis through any point in the xy plane
- Does it matter which axis you choose for calculating torques?
- NO. The choice of an axis is arbitrary
- If an object is in translational equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis
- We should be smart to choose a rotation axis to simplify problems

EXAMPLE 1

A uniform 40.0-N board supports a father and daughter weighing 800 N and 350 N, respectively. If the support is under the center of gravity of the board and if the father is 1.00 m from the center,

(b) Determine where the child should sit to balance the system



(b) Take an axis perpendicular to the page through the center of gravity **G** as the axis for our torque

$\Sigma\tau = 0$ yields

$$n(1.00\text{ m}) - (40.0\text{ N})(1.00\text{ m}) - (350\text{ N})(1.00\text{ m} + x) = 0$$

$$\boxed{x = 2.29\text{ m.}}$$

Center of Gravity

- The torque due to the gravitational force on an object of mass M is the force Mg acting at the center of gravity of the object
- If g is uniform over the object, then the center of gravity of the object coincides with its center of mass
- If the object is homogeneous and symmetrical, the center of gravity coincides with its geometric center

Where is the Center of Mass ?

- Assume m_1 = 1 kg, m_2 = 3 kg, and x_1 = 1 m, x_2 = 5 m, where is the center of mass of these two objects?

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

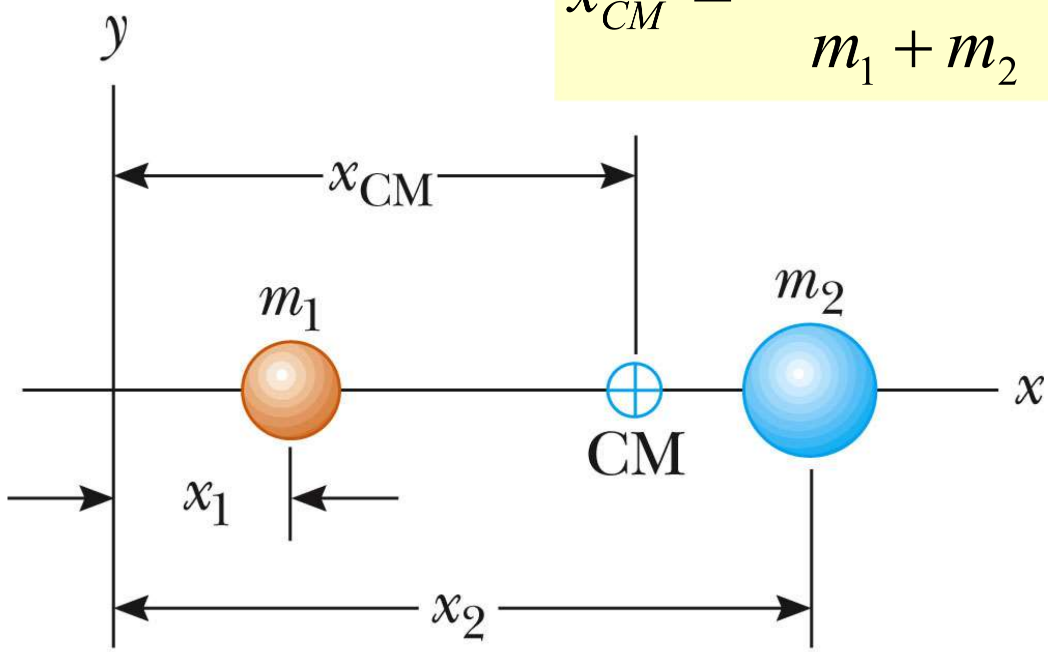
A) x_{CM} = 1 m

B) x_{CM} = 2 m

C) x_{CM} = 3 m

D) x_{CM} = 4 m

E) x_{CM} = 5 m



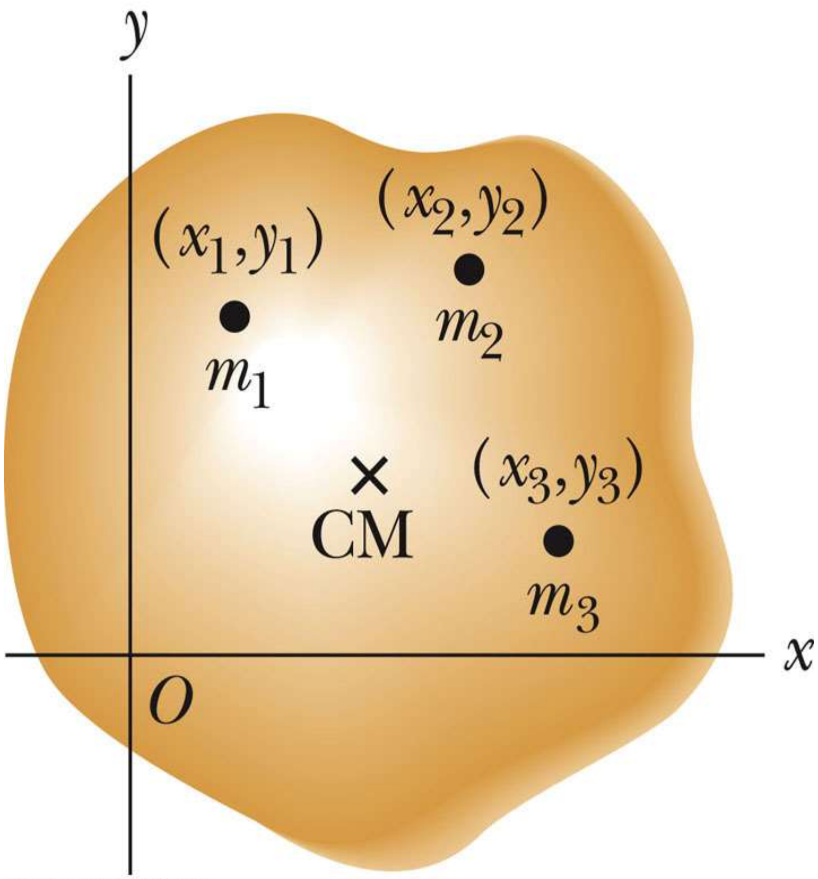
Center of Mass (CM)

- An object can be divided into many small particles
  - Each particle will have a specific mass and specific coordinates
- The x coordinate of the center of mass will be

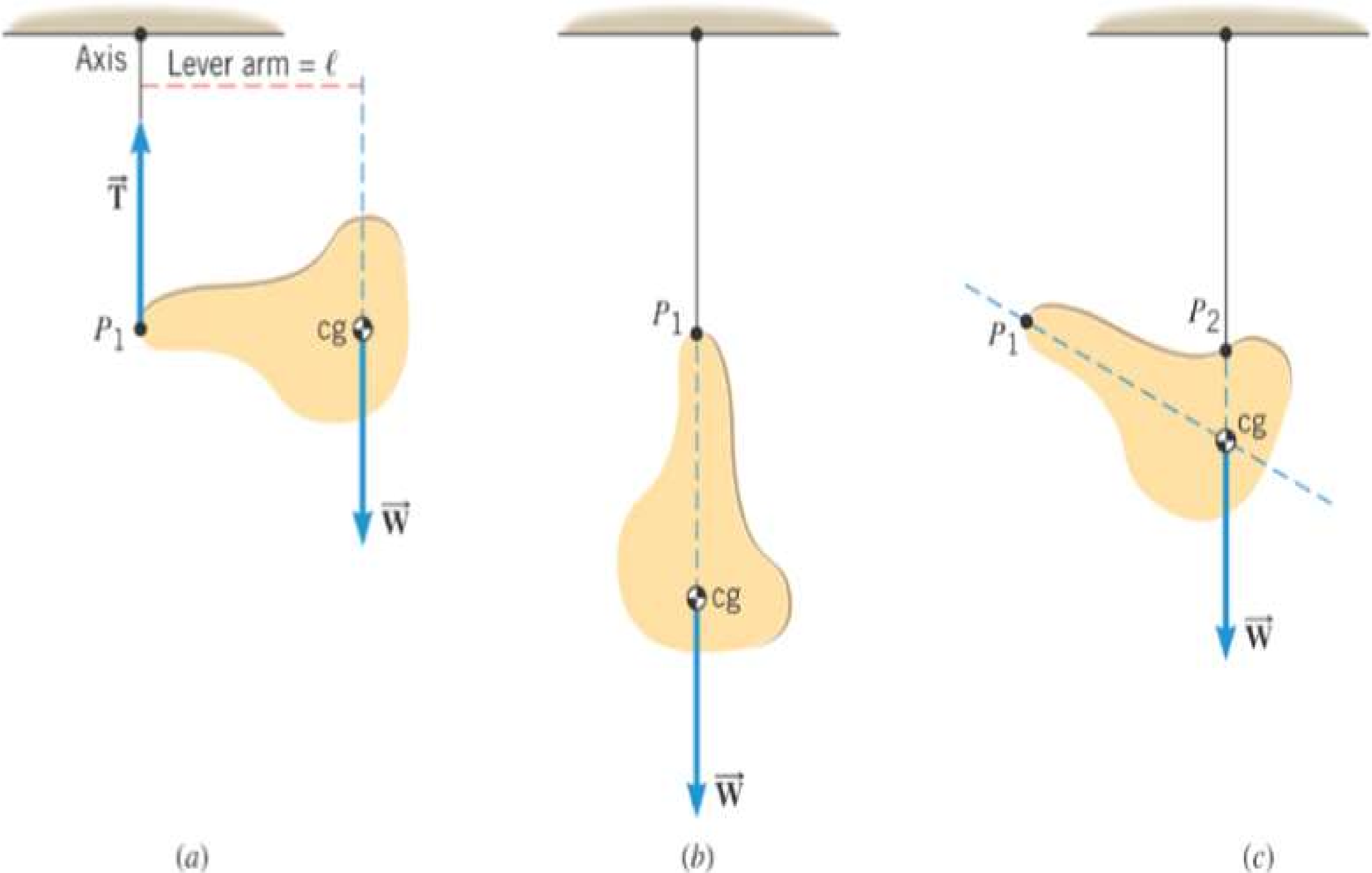
$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

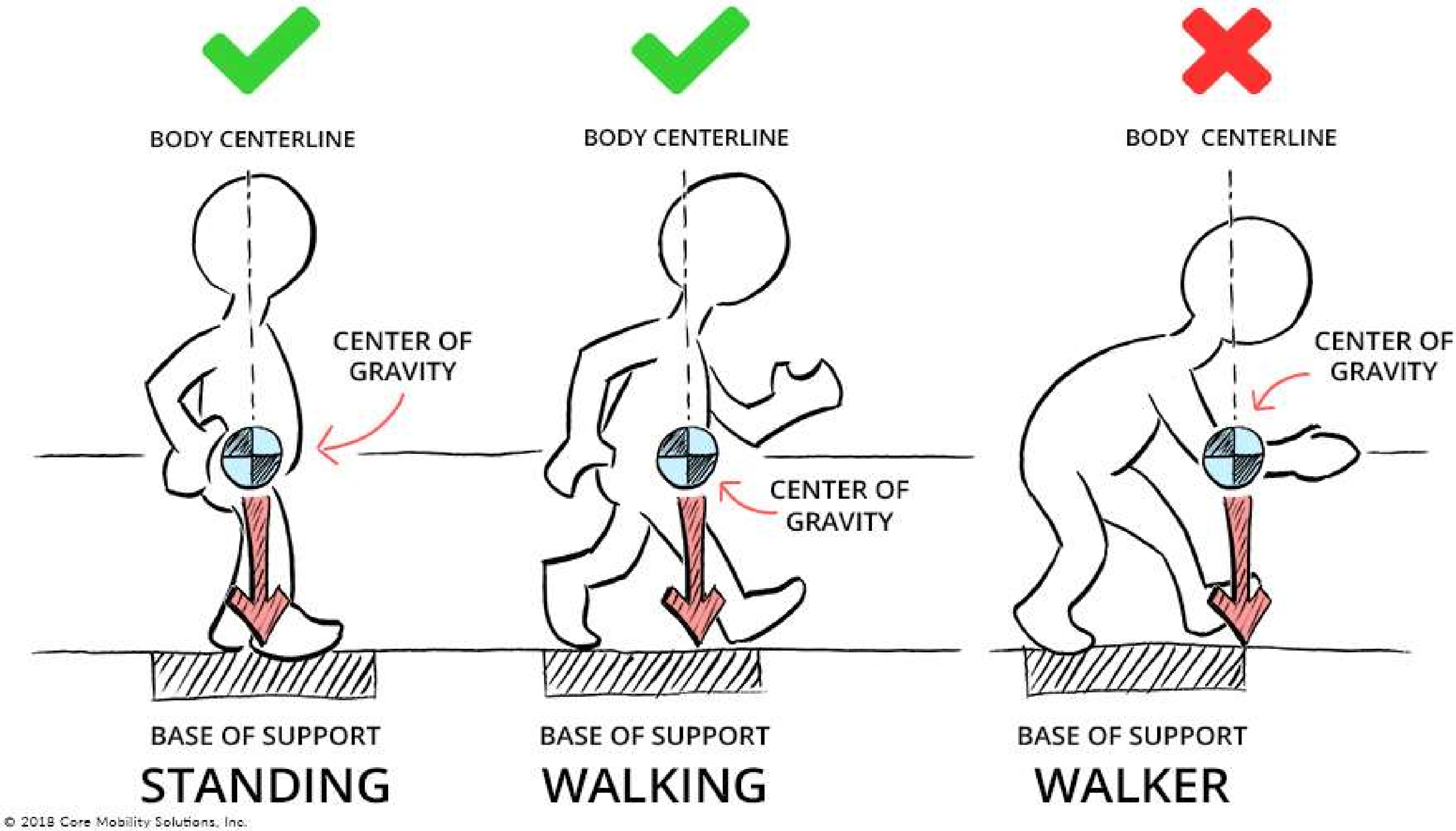
- Similar expressions can be found for the y coordinates

$$Y_{CM} = \frac{\sum_i m_i Y_i}{\sum_i m_i}$$



Center of Gravity for Irregular shape




BODY CENTERLINE
CENTER OF GRAVITY
BASE OF SUPPORT
STANDING
BODY CENTERLINE
CENTER OF GRAVITY
BASE OF SUPPORT
WALKING
BODY CENTERLINE
CENTER OF GRAVITY
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WALKER
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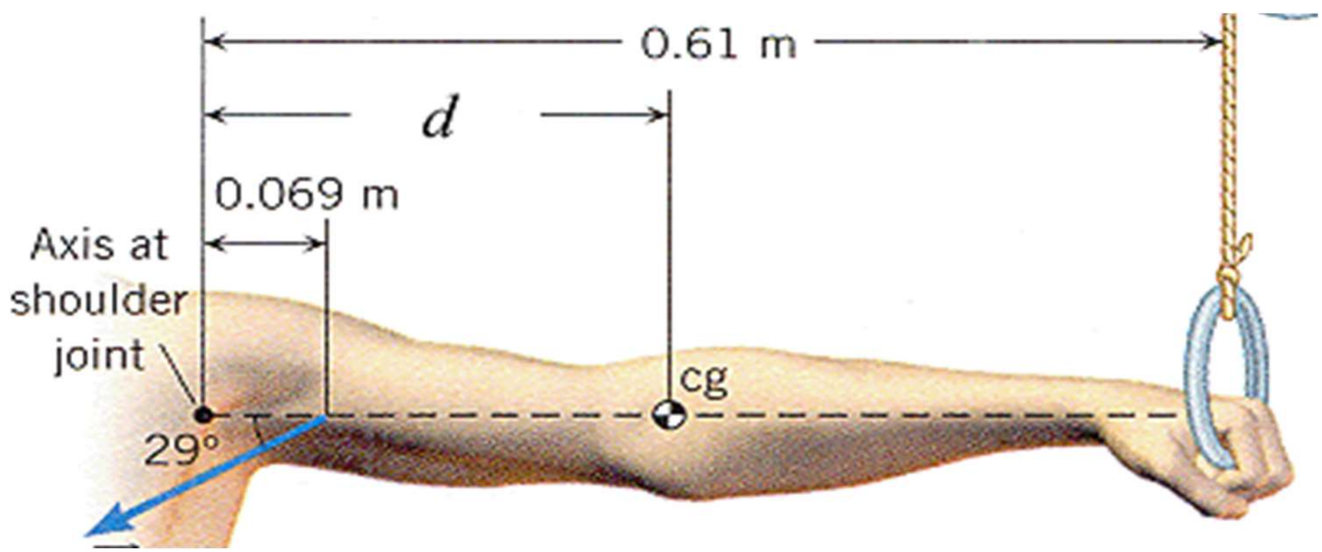

0.61 m
d
0.069 m
Axis at shoulder joint
cg
29°

Problem-Solving Strategy 1

- Draw sketch, decide what is in or out the system
- Draw a free body diagram (FBD)
- Show and label all external forces acting on the object
- Indicate the locations of all the forces
- Establish a convenient coordinate system
- Find the components of the forces along the two axes
- Apply the first condition for equilibrium
- Be careful of signs

$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

Problem-Solving Strategy 2

- Choose a convenient axis for calculating the net torque on the object
  - Remember the choice of the axis is arbitrary
- Choose an origin that simplifies the calculations as much as possible
  - A force that acts along a line passing through the origin produces a zero torque
- Be careful of sign with respect to rotational axis
  - positive if force tends to rotate object in CCW
  - negative if force tends to rotate object in CW
  - zero if force is on the rotational axis
- Apply the second condition for equilibrium $\tau_{net,z} = \sum \tau_{ext,z} = 0$

Problem-Solving Strategy 3

- The two conditions of equilibrium will give a system of equations
- Solve the equations simultaneously
- Make sure your results are consistent with your free body diagram
- If the solution gives a negative for a force, it is in the opposite direction to what you drew in the free body diagram
- Check your results to confirm

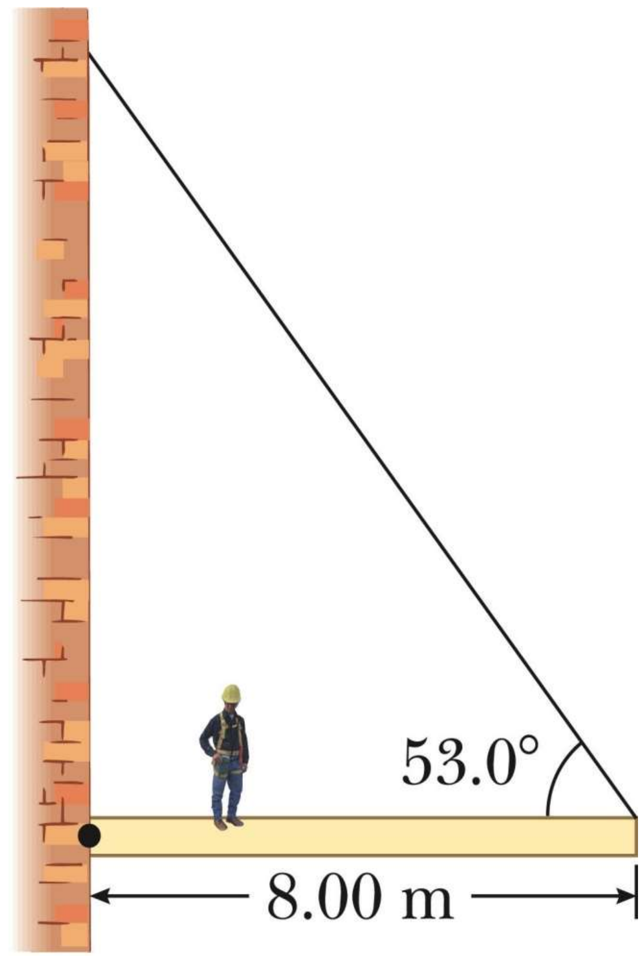
$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

Horizontal Beam Example

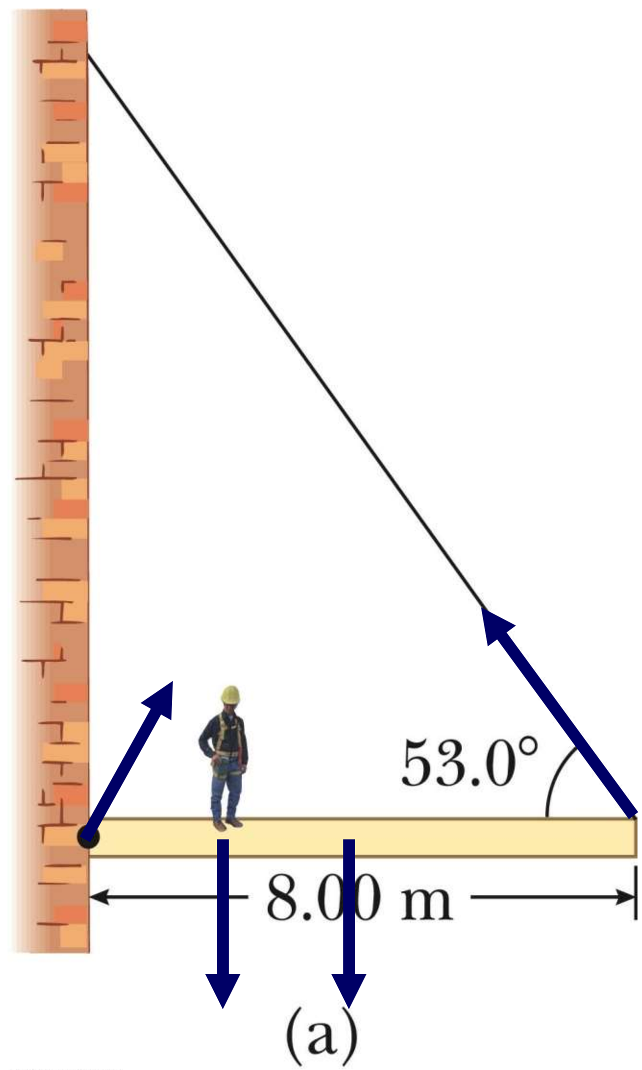
- A uniform horizontal beam with a length of l = 8.00 m and a weight of W_b = 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of $\phi = 53°$ with the beam. A person of weight W_p = 600 N stands a distance d = 2.00 m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



(a)

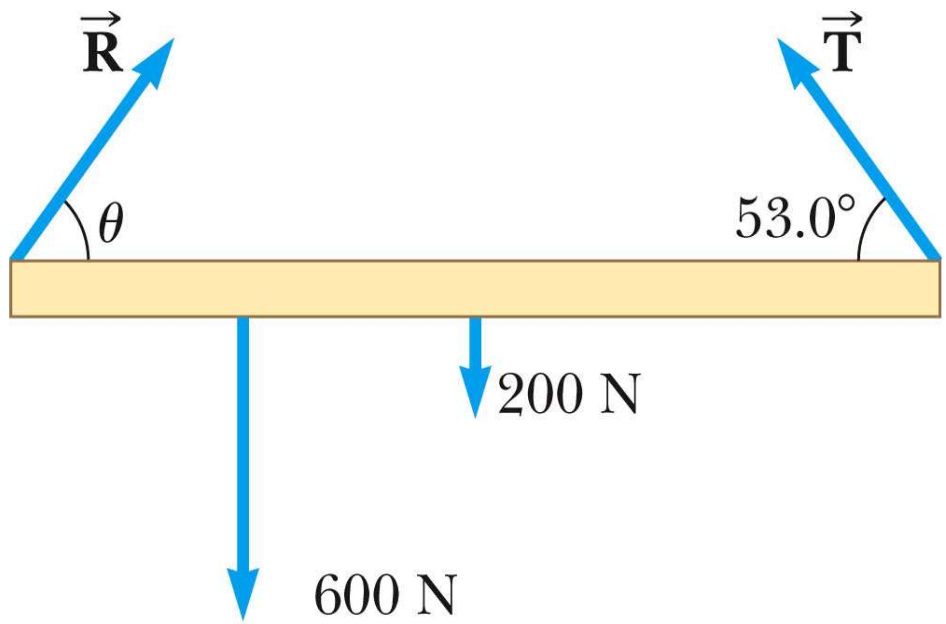
Horizontal Beam Example

- The beam is uniform
  - So the center of gravity is at the geometric center of the beam
- The person is standing on the beam
- What are the tension in the cable and the force exerted by the wall on the beam?



Horizontal Beam Example, 2

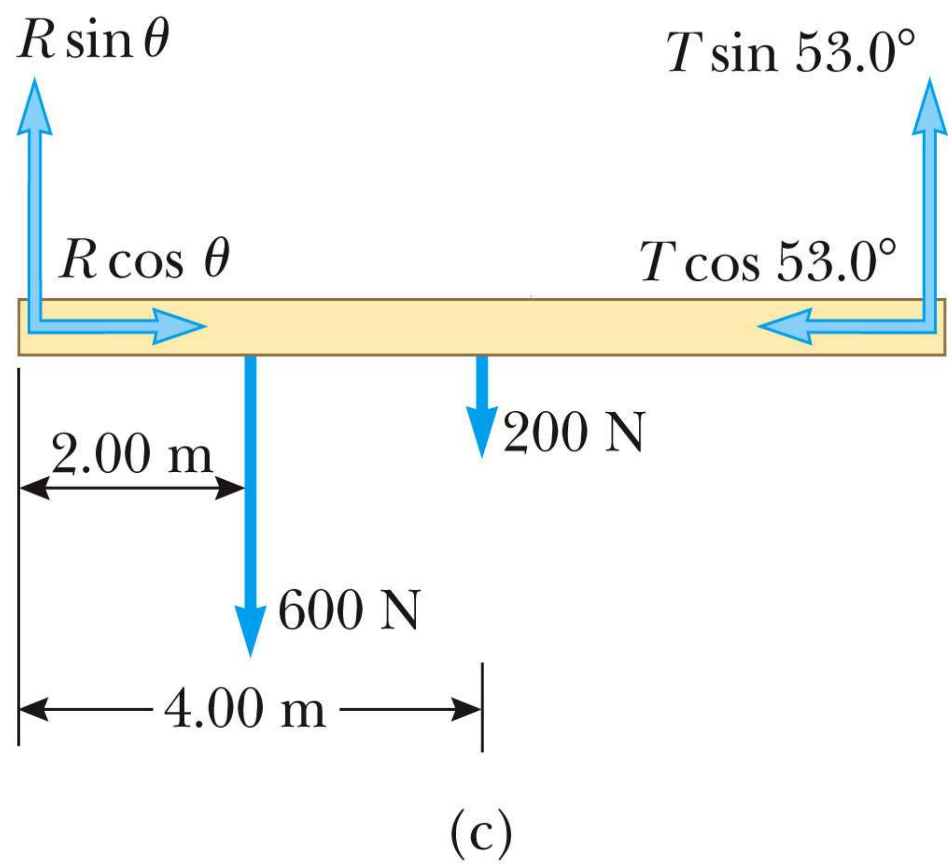
- Analyze
  - Draw a free body diagram
  - Use the pivot in the problem (at the wall) as the pivot
    - This will generally be easiest
  - Note there are three unknowns (T, R, θ)



(b)

Horizontal Beam Example, 3

- The forces can be resolved into components in the free body diagram
- Apply the two conditions of equilibrium to obtain three equations
- Solve for the unknowns



$$\sum \tau_z = (T\sin\phi)(l) - W_p d - W_b\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W_p d + W_b\left(\frac{l}{2}\right)}{l\sin\phi} = \frac{(600\,N)(2m) + (200\,N)(4m)}{(8m)\sin 53^\circ} = 313\,N$$

$$\sum F_x = Rx - T\cos 57 = 0$$

$$\sum F_y = Ry + T\sin 57 - 200 - 600 = 0$$

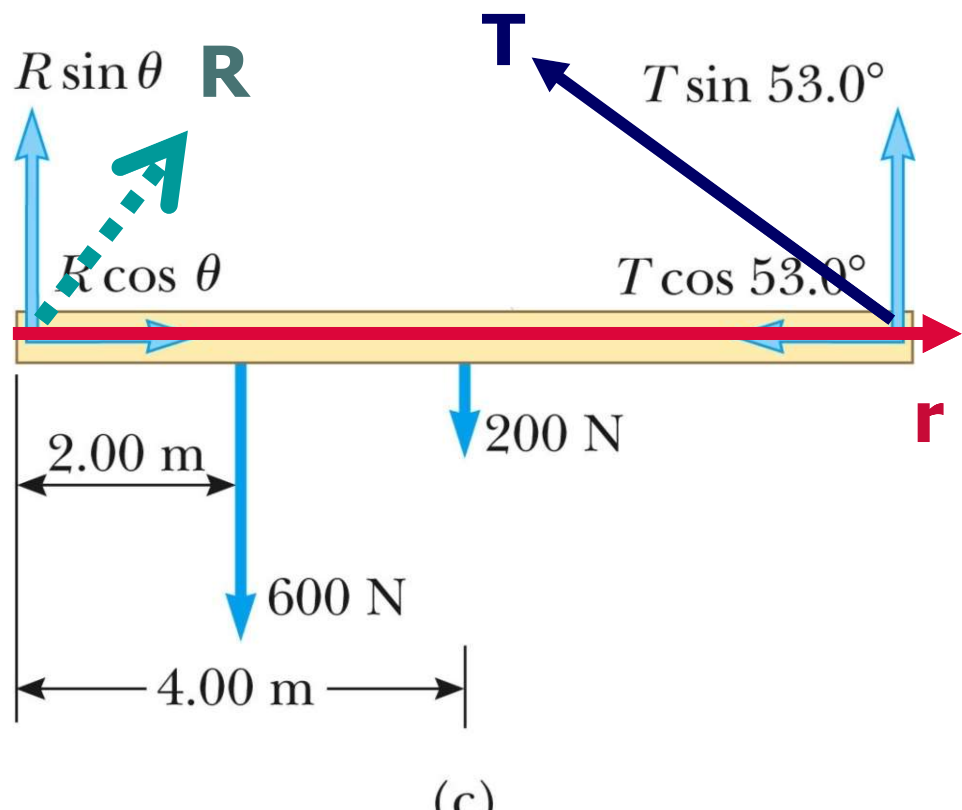
$$\frac{Ry}{Rx} = \tan\theta = \frac{800 - T\sin 57}{T\cos 57}$$

$$\theta = \tan^{-1}\left(\frac{663.4}{281.6}\right) = 71.7^\circ$$

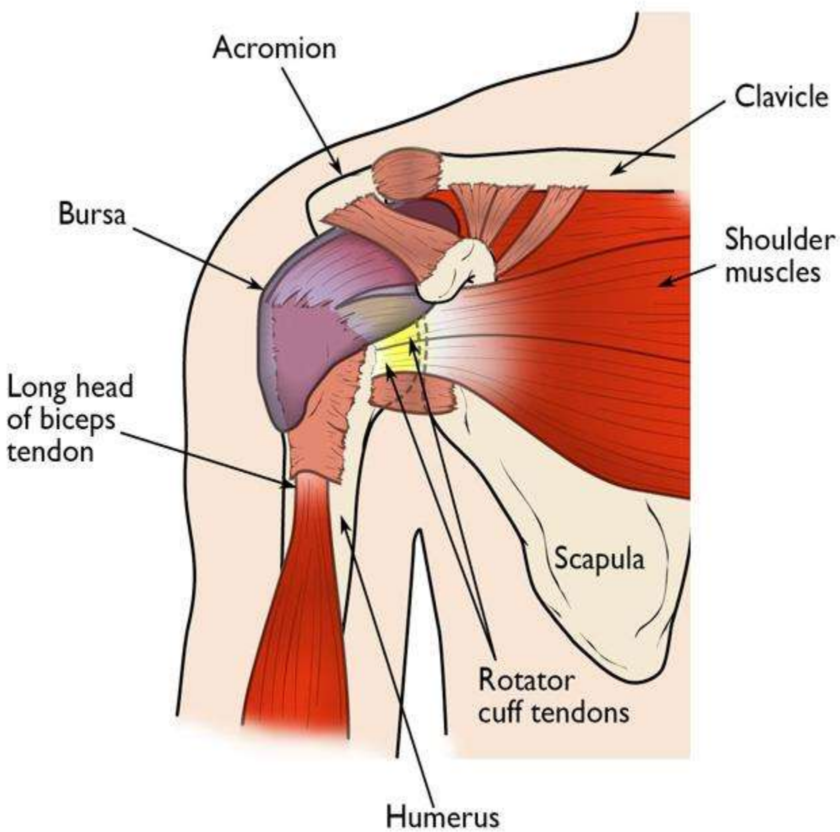
$$R = \frac{T\cos\varphi}{\cos\theta} = \frac{(313N)\cos 53^\circ}{\cos 71.7^\circ}$$

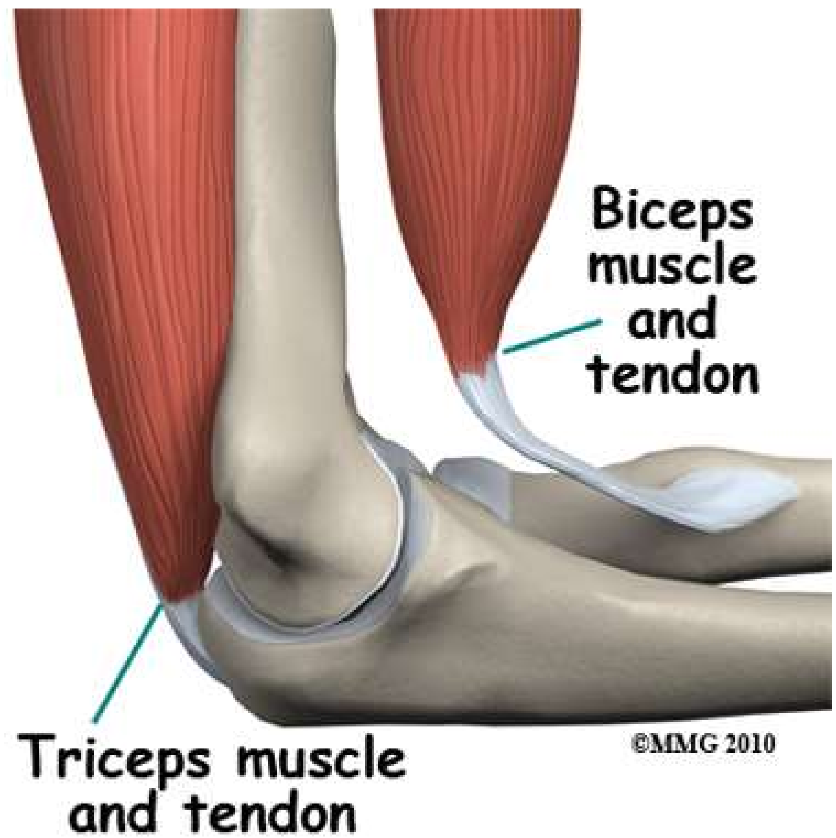
$$= 581N$$

$$R = \sqrt{R_x^2 + R_y^2}$$



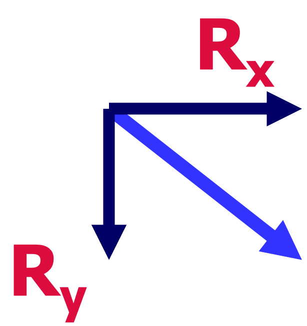
Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. Muscles apply torques against the torque exerted by the weight

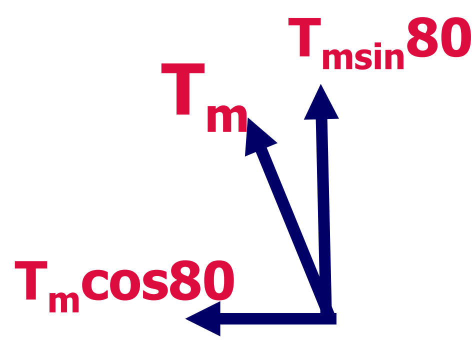


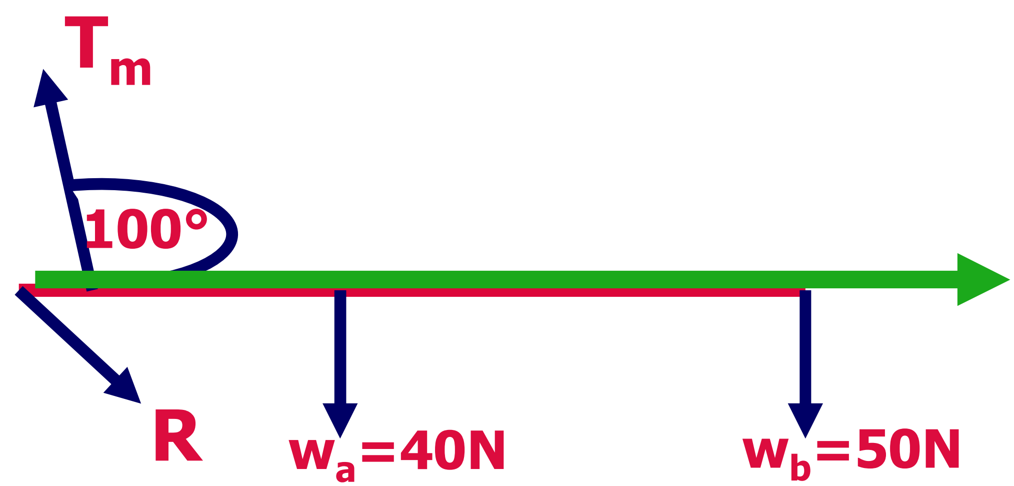


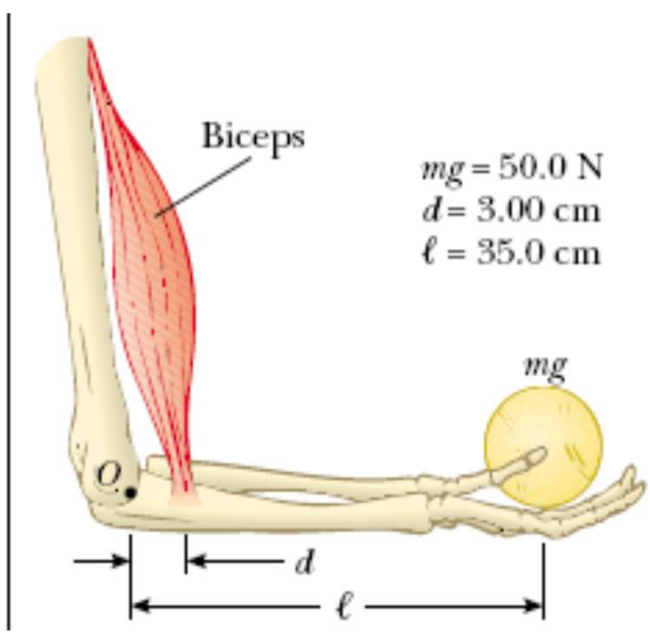
EXAMPLE

A person holds a 50 N sphere in his hand. The forearm is horizontal. The biceps muscle is attached 3 cm from the joint, and the sphere is 35 cm from the joint. Find the upward force exerted by the biceps on the forearm and the force exerted by the upper arm on the forearm and acting at the joint. Take the weight of the forearm is 40N acting at distance 15 cm from the joint.









$$\sum F_x = 0,\ R_x - T_m cos\theta = 0,\ R_x = T_m cos\theta \dots\dots (1)$$

$$\sum F_y = 0, T_m sin80 - R_y - w_a - w_b = 0\,,$$

$$R_y = T_m sin80 - 90 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

$$\sum \vec{\tau} = 0,$$

$$\boldsymbol{\tau_R + \tau_{T_m} + \tau_{w_a} + \tau_{w_b} = 0}$$

$0 + T_m(3)\sin 100 - 40(15)\sin 90 - 50(35)\sin 90 = 0$

$$T_m = \frac{2350}{3 sin 100} = 795\ N,$$

Substitute in equ. 1 and 2

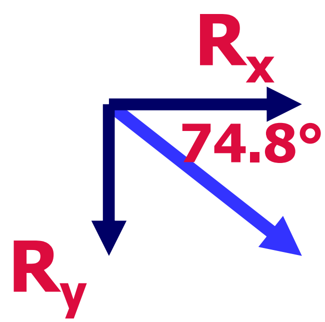
$$\boldsymbol{R_x = T_m cos80 = 138\ N}$$

$$\boldsymbol{R_y = T_m sin80 - 90 = 693\ N}$$

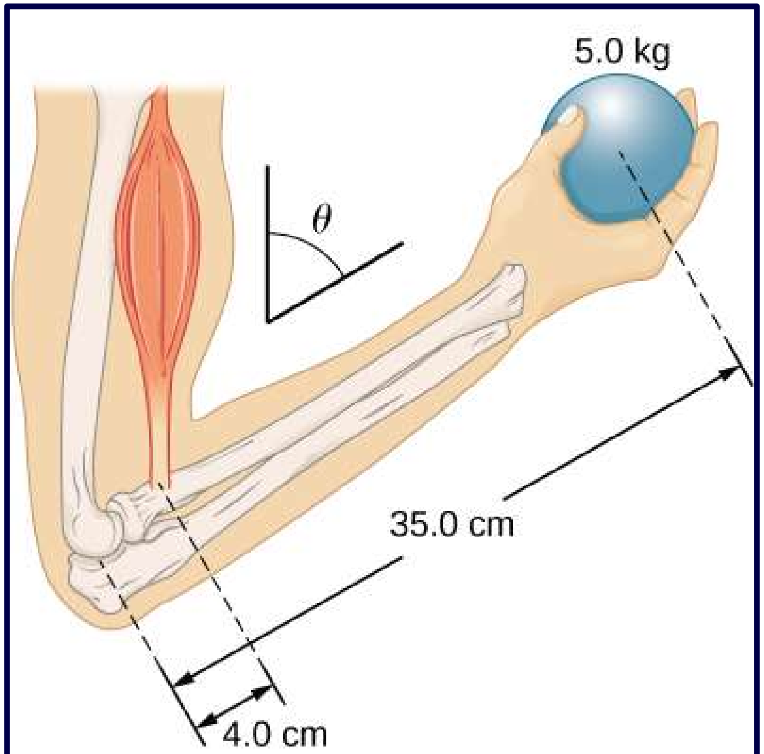
Then

$$\boldsymbol{R = \sqrt{R_x^2 + R_y^2} = 706\ N}$$

$$\boldsymbol{\theta = tan^{-1}\left(\frac{R_y}{R_x}\right) = tan^{-1}\left(\frac{168}{45.5}\right) = -78.7°}$$



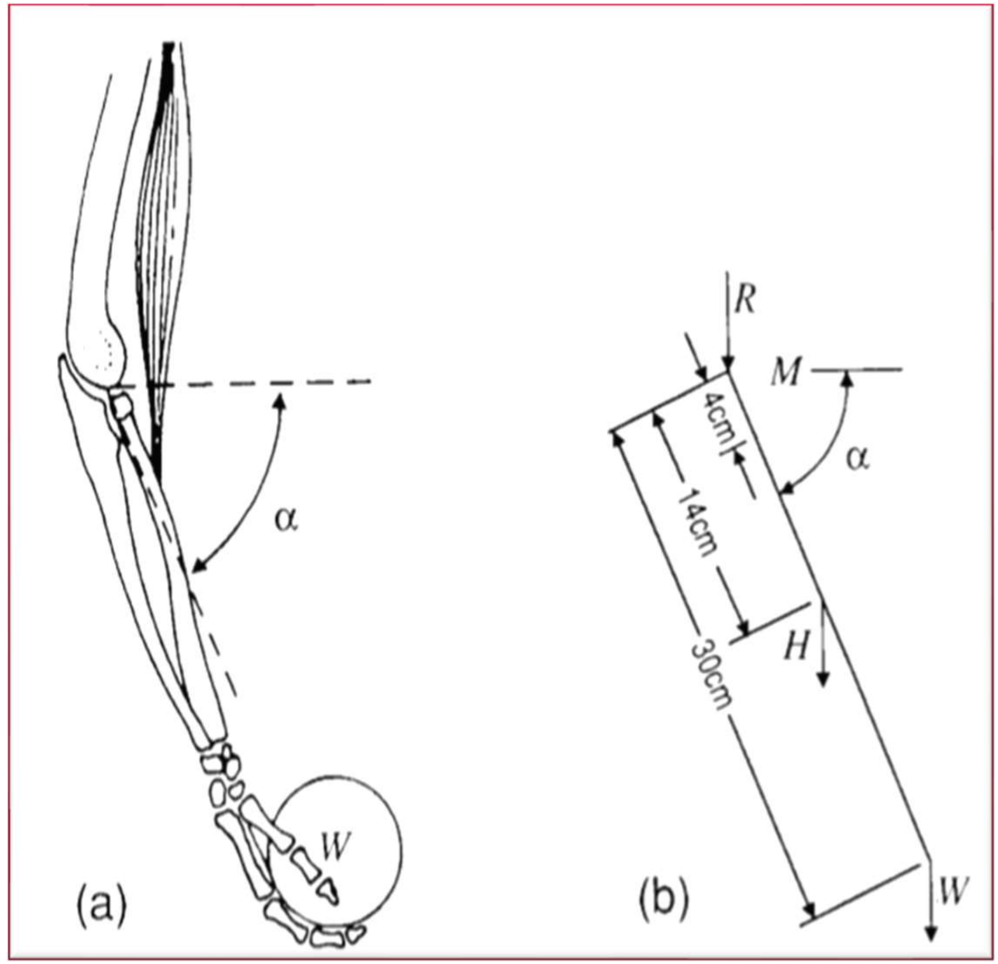
Find The Forcess T and R



In the shown free body diagram, Take the weight of the arm 3 kg affected

At distance 15 cm apart from the jointIf θ =45°, the angle that the arm

Makes with the hotizontal is 60°

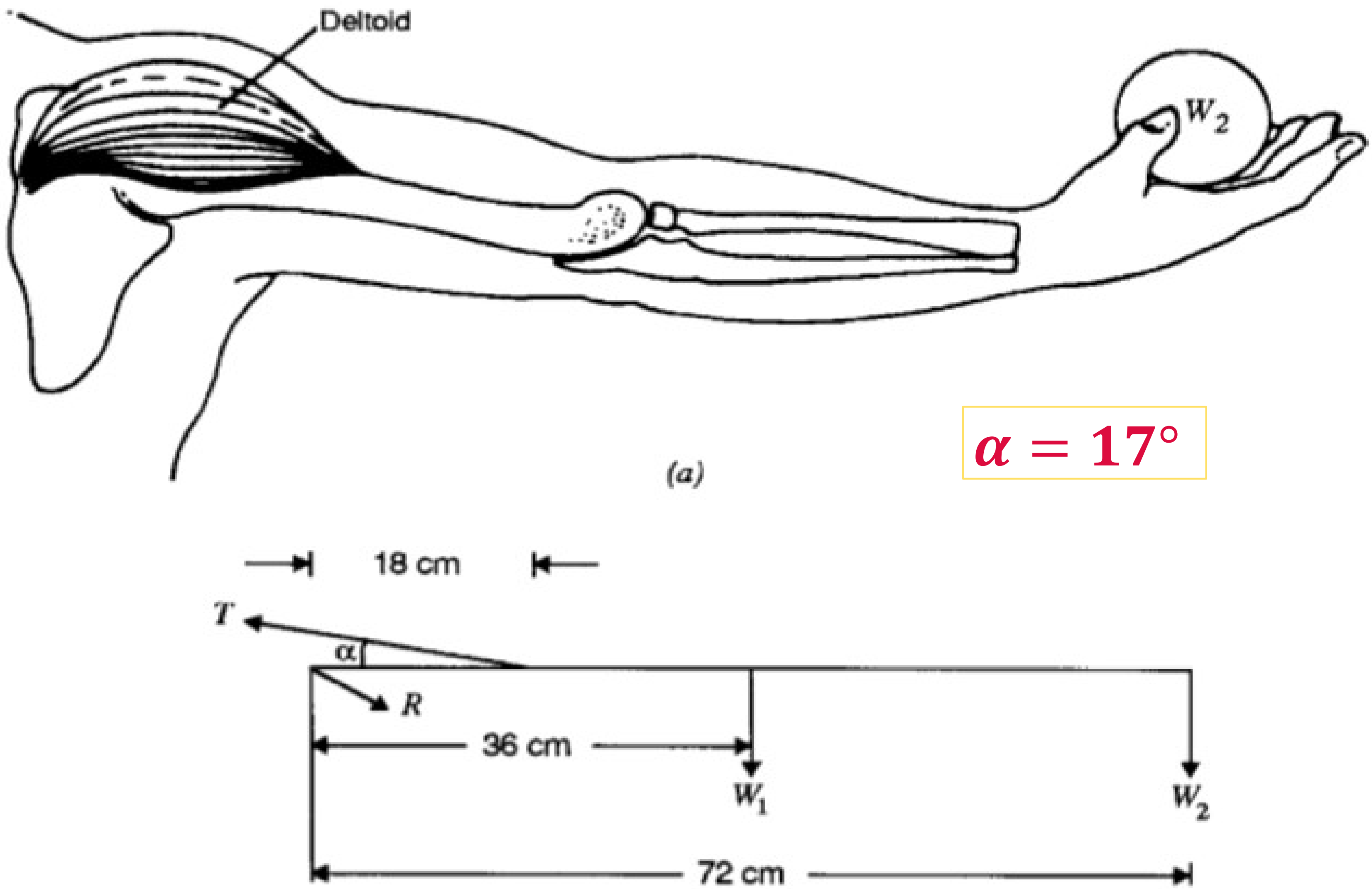


If $\alpha = 40° \; and \; the \; angle \; that \; the \; tension$

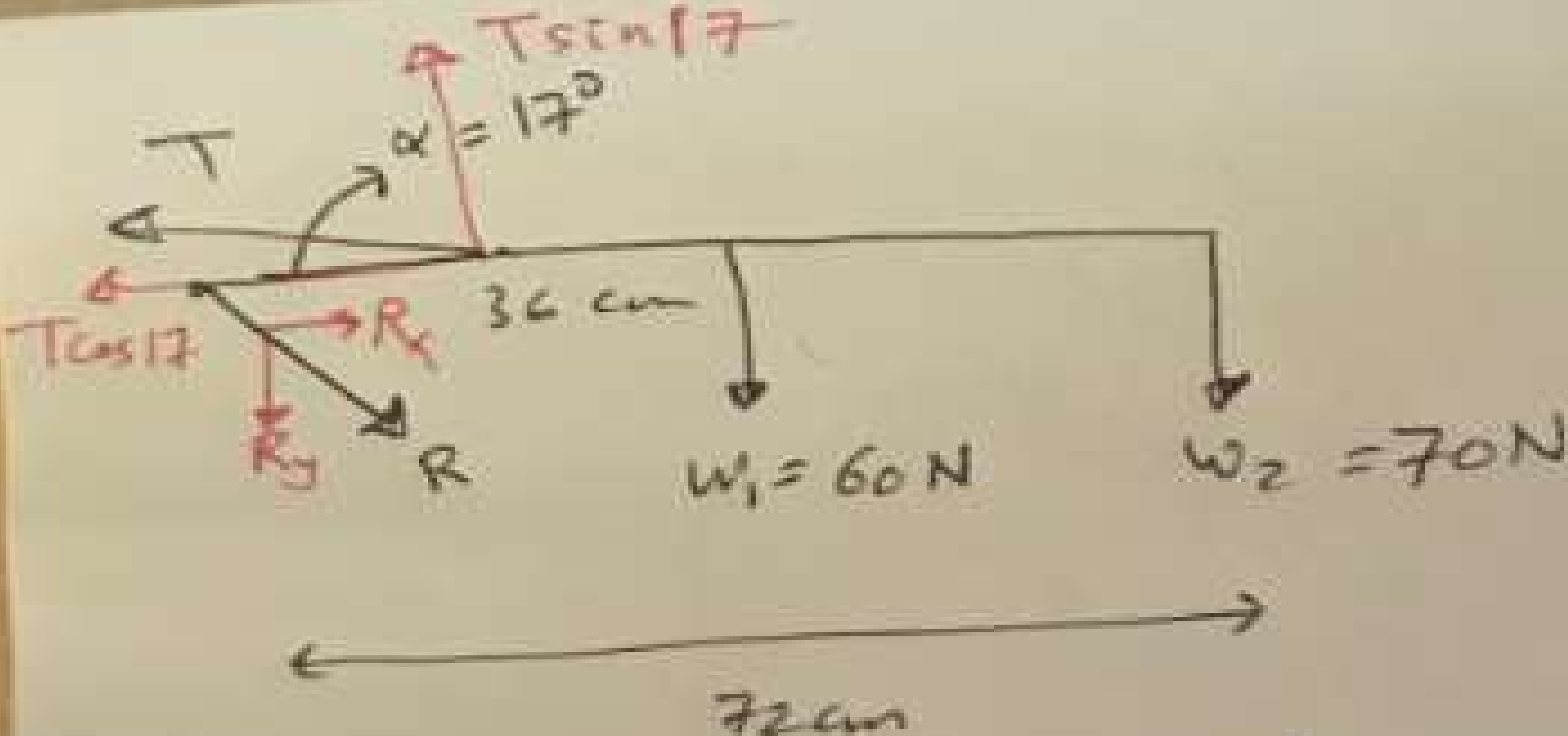
In the bicep muscle makes an angle 130° with the forearm

Deltoid Muscle

Find the tension in the deltoid muscle and the reaction force at the shoulder joint.



$\alpha = 17°$

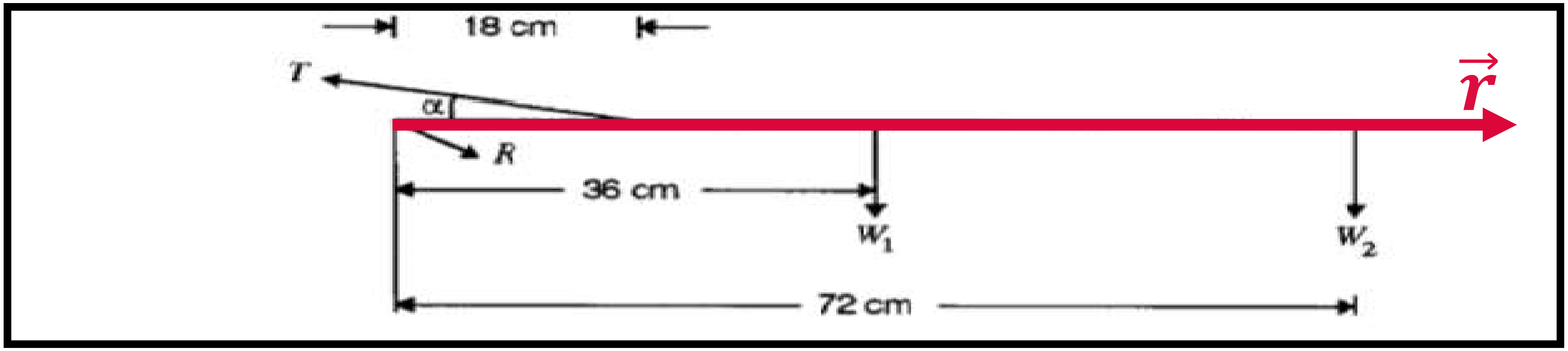


1) $\sum F_x = 0 \Rightarrow -T\cos 17 + R_x = 0$

$$R_x = T\cos 17 \quad \text{—} \ (1)$$

2) $\sum F_y = 0 \Rightarrow T\sin 17 - R_y - W_1 - W_2 = 0$

$$R_y = T\sin 17 - 130 \quad \text{—} \ (2)$$



3) $\sum \vec{\tau} = 0 \Rightarrow \tau_R + \tau_T + \tau_{W_1} + \tau_{W_2} = 0$

$\tau_R = 0 \quad , \quad \tau_T = +18\,T\sin(180-17) = 18\,T\sin 17$

$\tau_{W_1} = -36(60)\sin 90 = -2160\,N$

$\tau_{W_2} = -72(70)\sin 90 = -5040\,N$

$\therefore \quad 18\,T\sin 17 - 2160 - 5040 = 0$

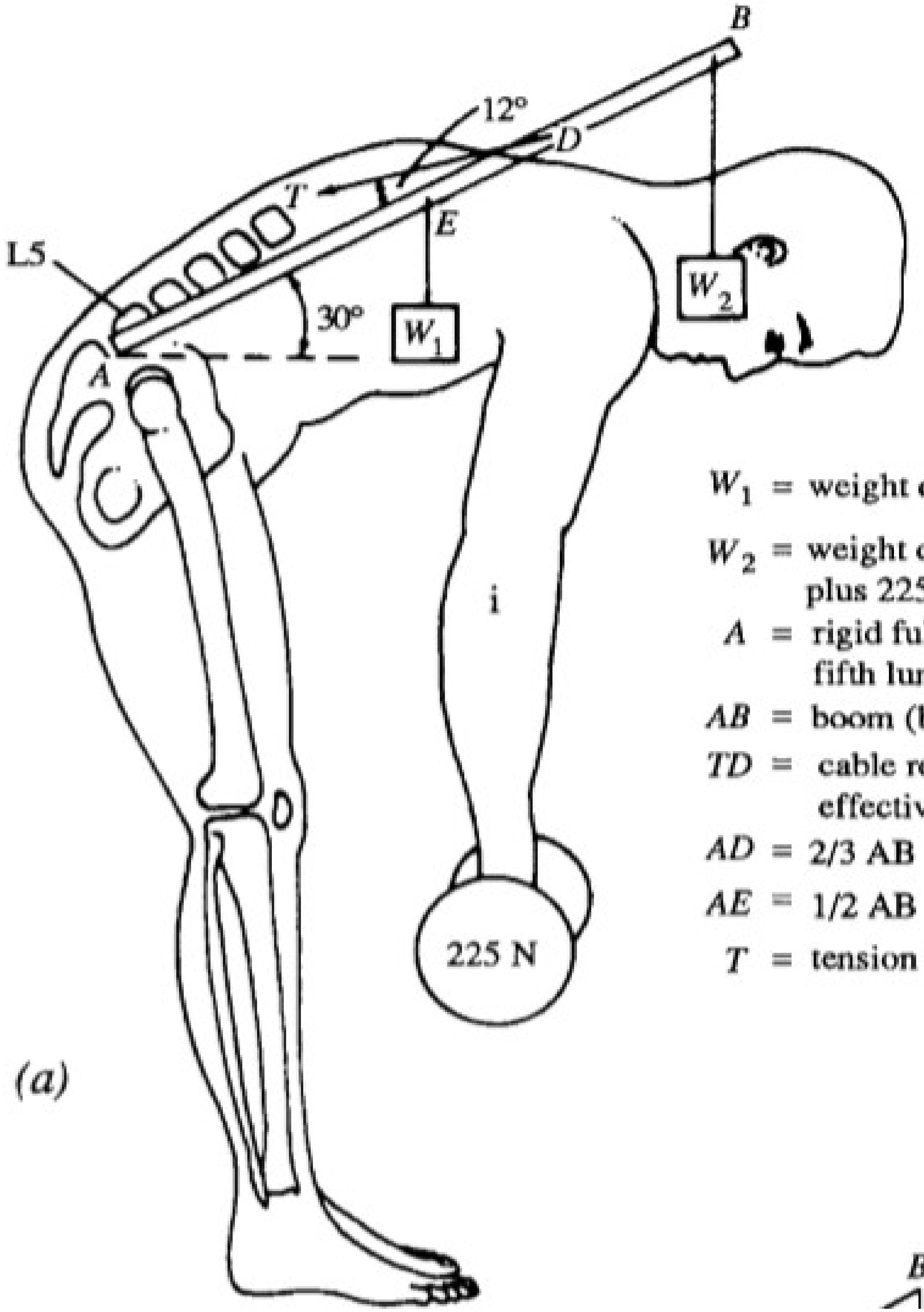
$18\,T\sin 17 = 7200 \Rightarrow T = \dfrac{7200}{18\sin 17} = 1368\,N$

$\therefore R_x = T\cos 17 = 1368\cos 17 = 1308\,N \quad , \quad R_y = T\sin 17 - 130 \simeq 270\,N$

$R = \sqrt{R_x^2 + R_y^2} \quad , \quad \theta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$

إذاً R اتجاهاً وكمياً ،

The Back and Spinal Column



W_1 = weight of body trunk (320 N)

W_2 = weight of arms and head plus 225 N (382 N)

A = rigid fulcrum at about fifth lumbar vertebra

AB = boom (body trunk)

TD = cable representing effective muscle

AD = 2/3 AB

AE = 1/2 AB

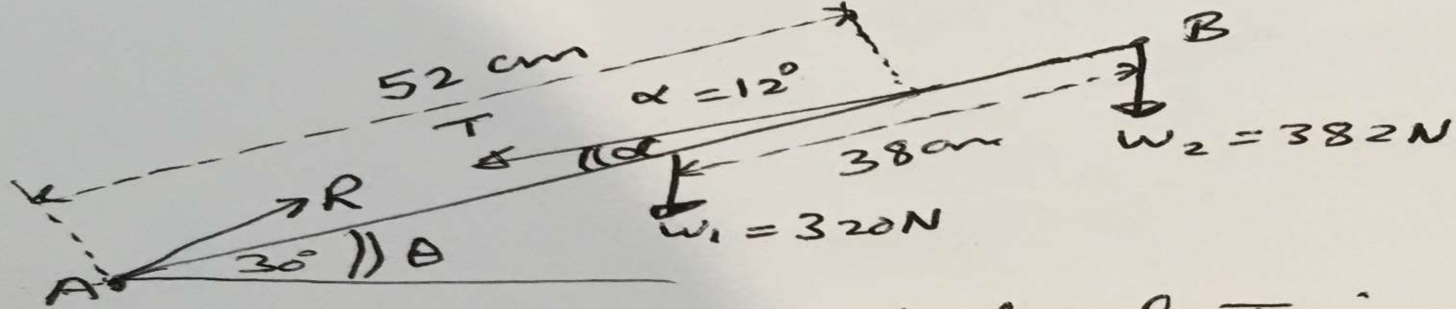
T = tension in the muscle

$\theta = 30^\circ$

$AB = 78\ cm$

$AD = \frac{2}{3} AB = 52\ cm$, $AE = 39\ cm$

We can see in the FBD



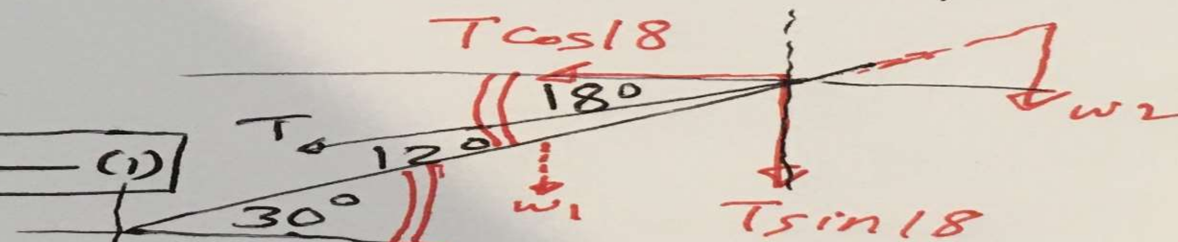
To find the magnitude of T in the back muscle and the joint reaction force ($\vec{R}$)

we analyze the T force into two components

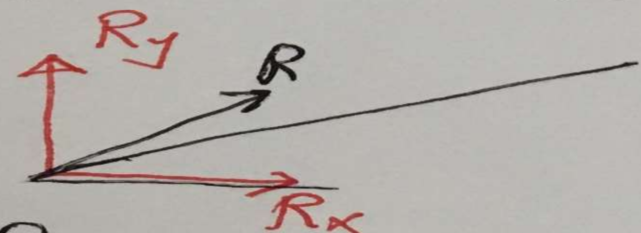
① $\Sigma F_x = 0$

$R_x - T\cos 18 = 0$

$\boxed{R_x = T\cos 18 \quad (1)}$



and analyze the force R



② $\Sigma F_y = 0$

$R_y - T\sin 18 - W_1 - W_2 = 0$

$\boxed{R_y = T\sin 18 + 702 \quad (2)}$

the $\Sigma \tau = 0$, take the torque around A

$$\tau_R + \tau_T + \tau_{W_1} + \tau_{W_2} = 0$$

$$\tau_R = 0 \quad , \quad \tau_T = +52\,T \sin(180-12) = \boxed{52\,T\sin 12}$$

$$\tau_{W_1} = -39\,W_1 \sin(180-60) = -39(320)\sin 60 = -\boxed{10531\ N}$$

$$\tau_{W_2} = -78\,W_2 \sin(180-60) = -78(382)\sin 60 = -\boxed{25840}$$

$$\therefore \quad 52\,T\sin 12 = 36335 \Rightarrow T = \frac{36335}{52\sin 12}$$

$$\boxed{T = 3361\ N}$$

from (1) $\Rightarrow R_x = T\cos 18 \approx \boxed{3196\ N}$

from (2) $\Rightarrow R_y = T\sin 18 + 702 \approx \boxed{1740\ N}$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(3196)^2 + (1740)^2}$$

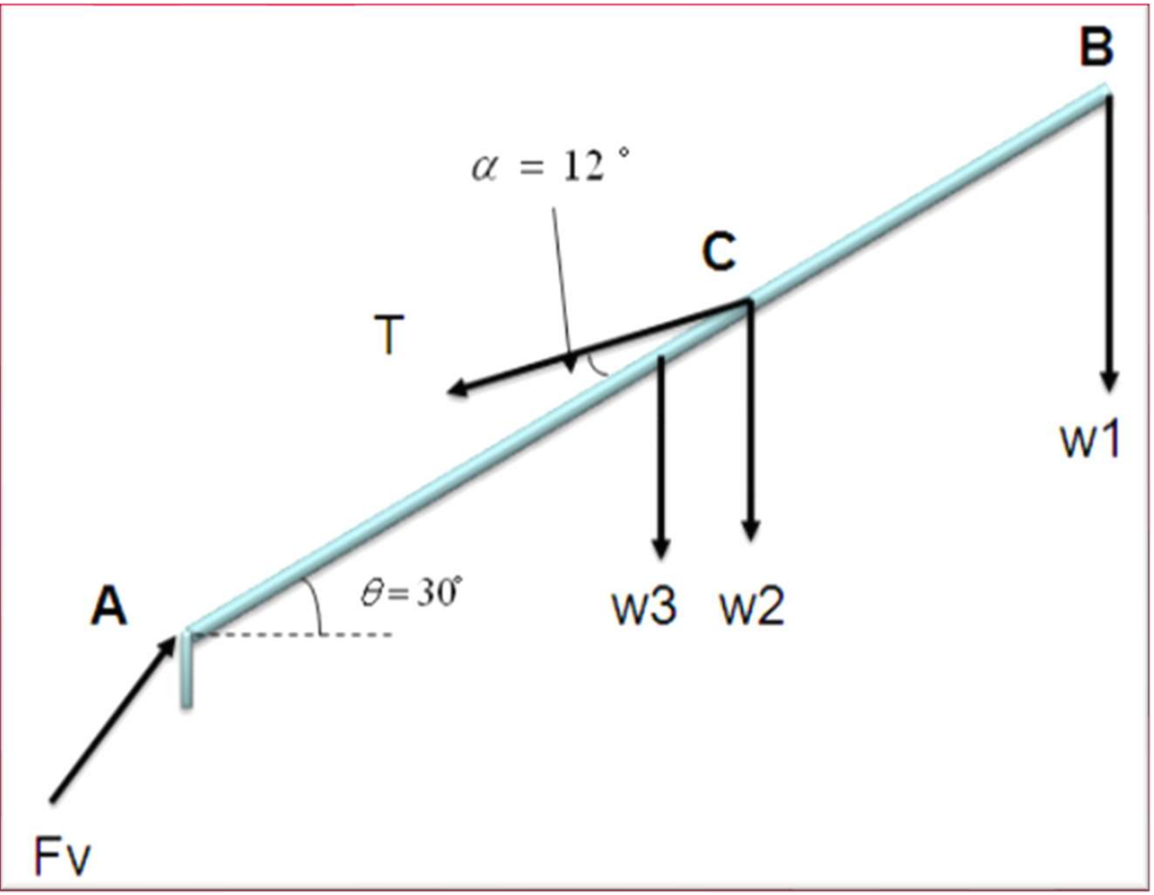
$$\boxed{R = 3639\ N}, \boxed{\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \approx 28.6^\circ}$$

$$\frac{R}{W_1 + W_2} \approx \frac{3639}{702} \approx 52 \text{ times}$$

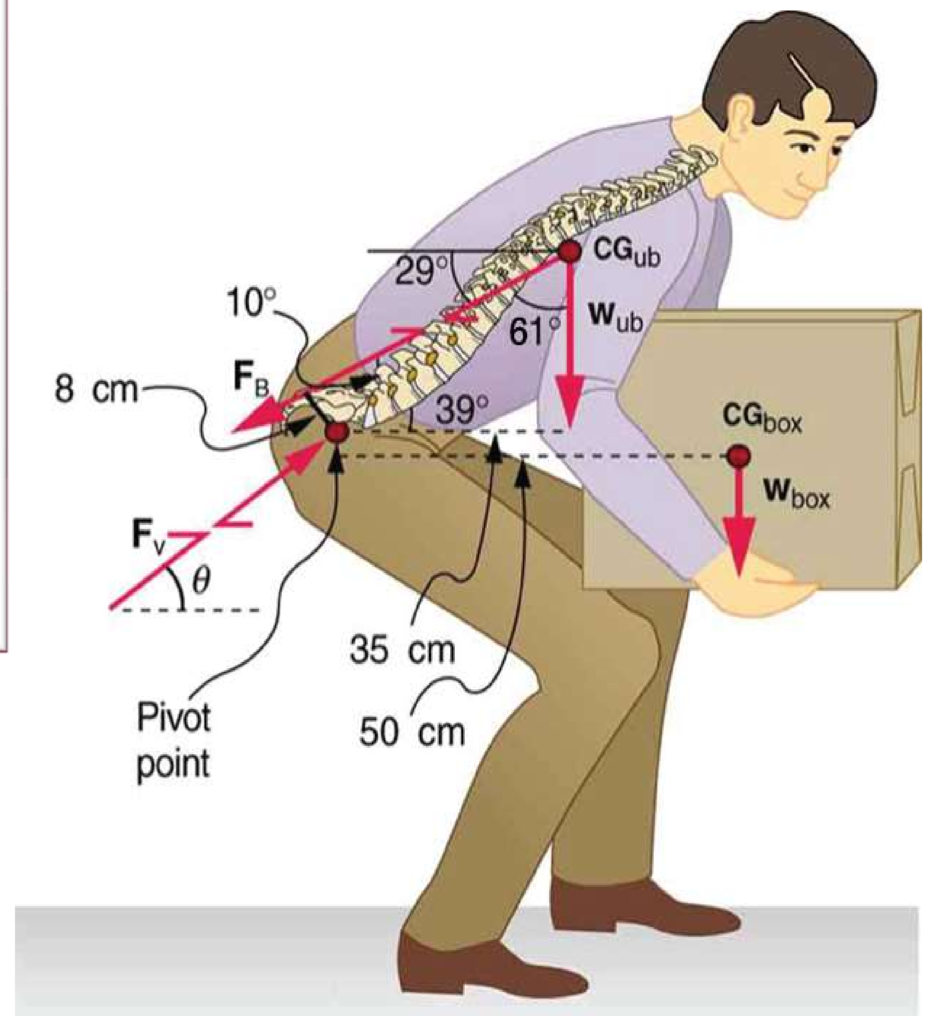
$$\frac{T}{W_1 + W_2} \approx 48 \text{ times}$$

... acting on the spinal Column

Back FBD



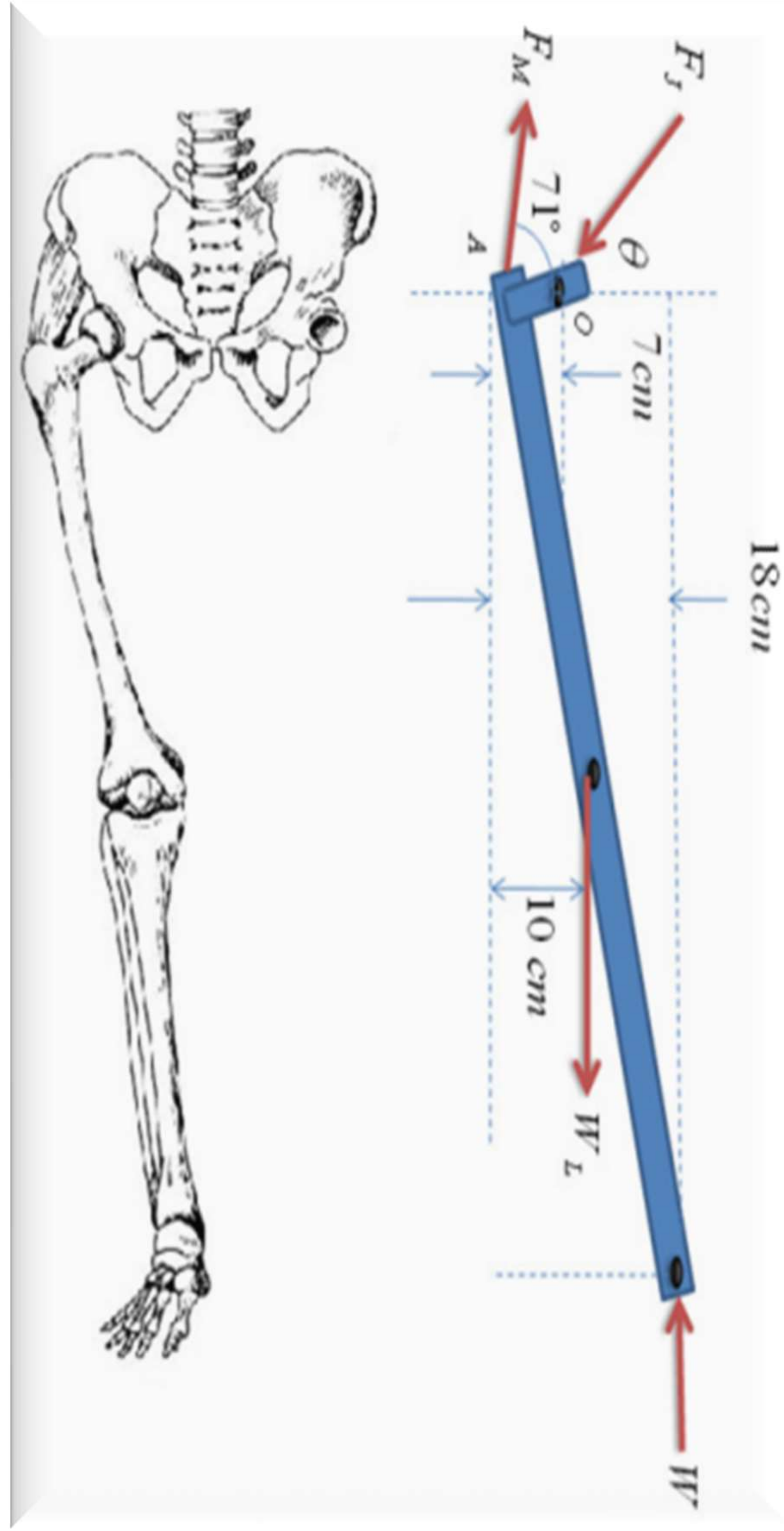
B
α = 12 °
C
T
w1
w3
w2
A
θ = 30°
Fv

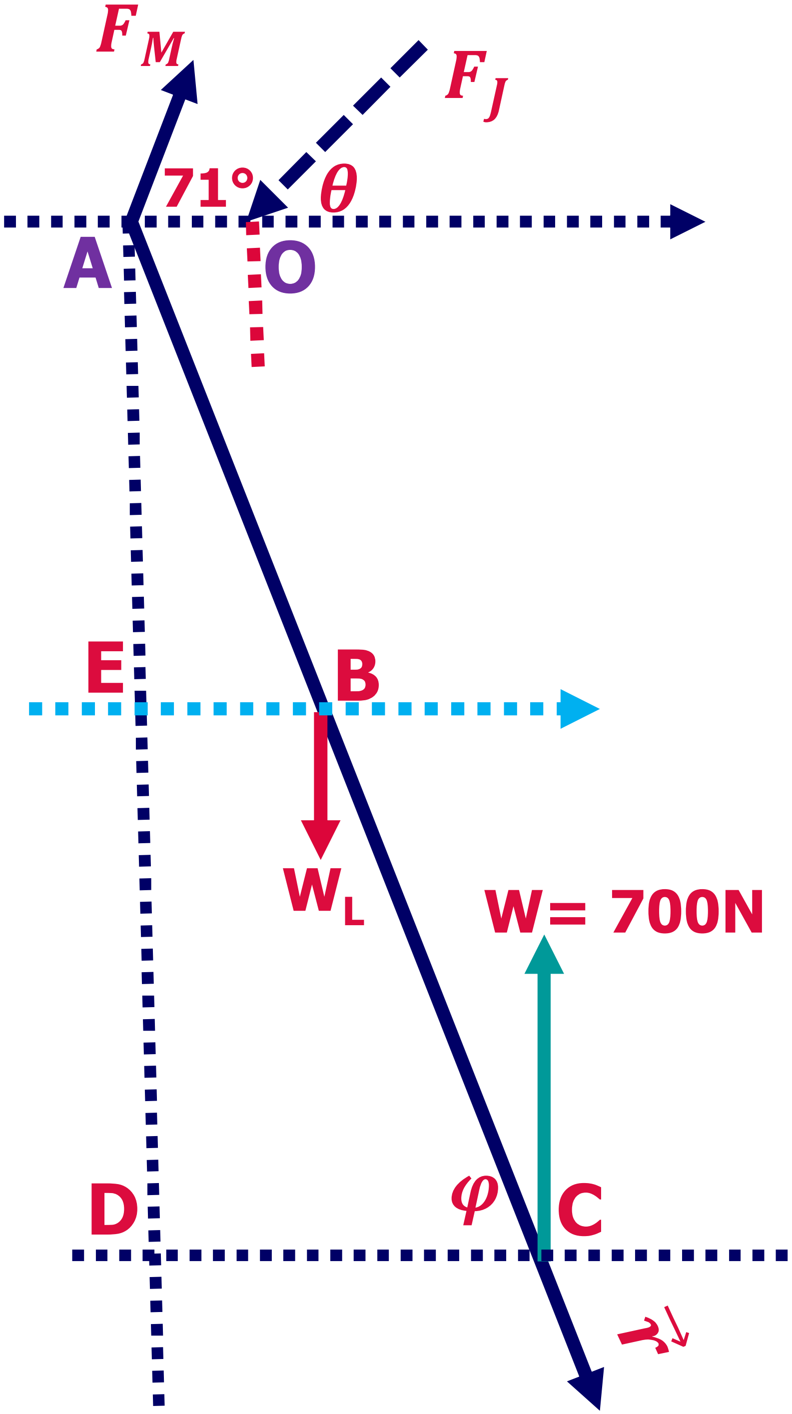


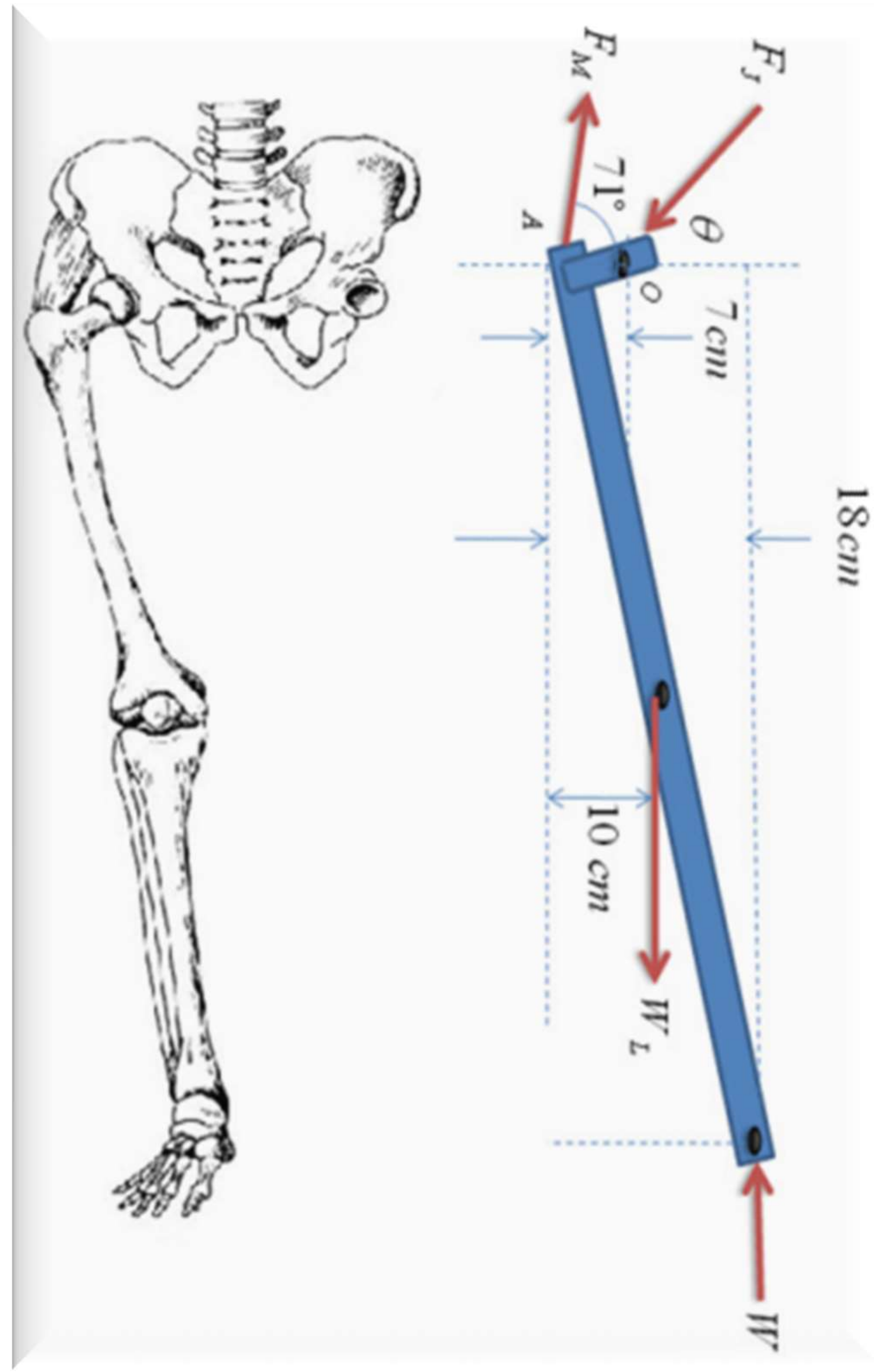
CGub
29°
10°
61°
wub
FB
8 cm
39°
CGbox
wbox
Fv
θ
35 cm
Pivot point
50 cm

The Hip Joint

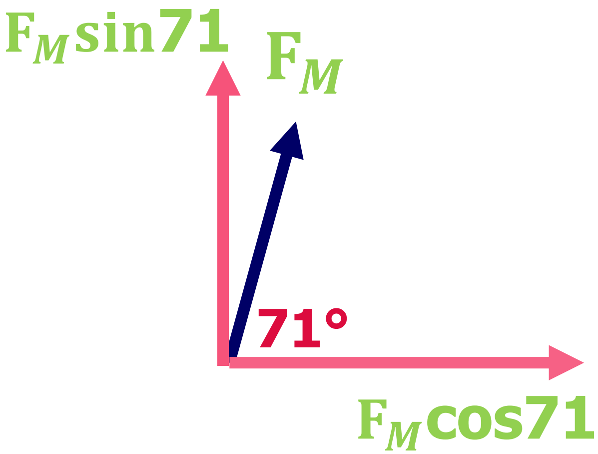
The hip joint and its simplified lever representation, with dimensions that are typical for a male body are shown in Figure 2.24. The hip is stabilized in its socket by a group of muscles, which is represented in Figure 2.24, (left) as a single resultant force F_M. When a person stands erect, the angle of this force is about 71° with respect to the horizon. W_L represents the combined weight of the leg, foot, and thigh. Typically, this weight is a fraction (0.185) of the total body weight W (i.e., $W_L = 0.185W$). The weight W_L is assumed to act vertically downward at the midpoint of the limb.





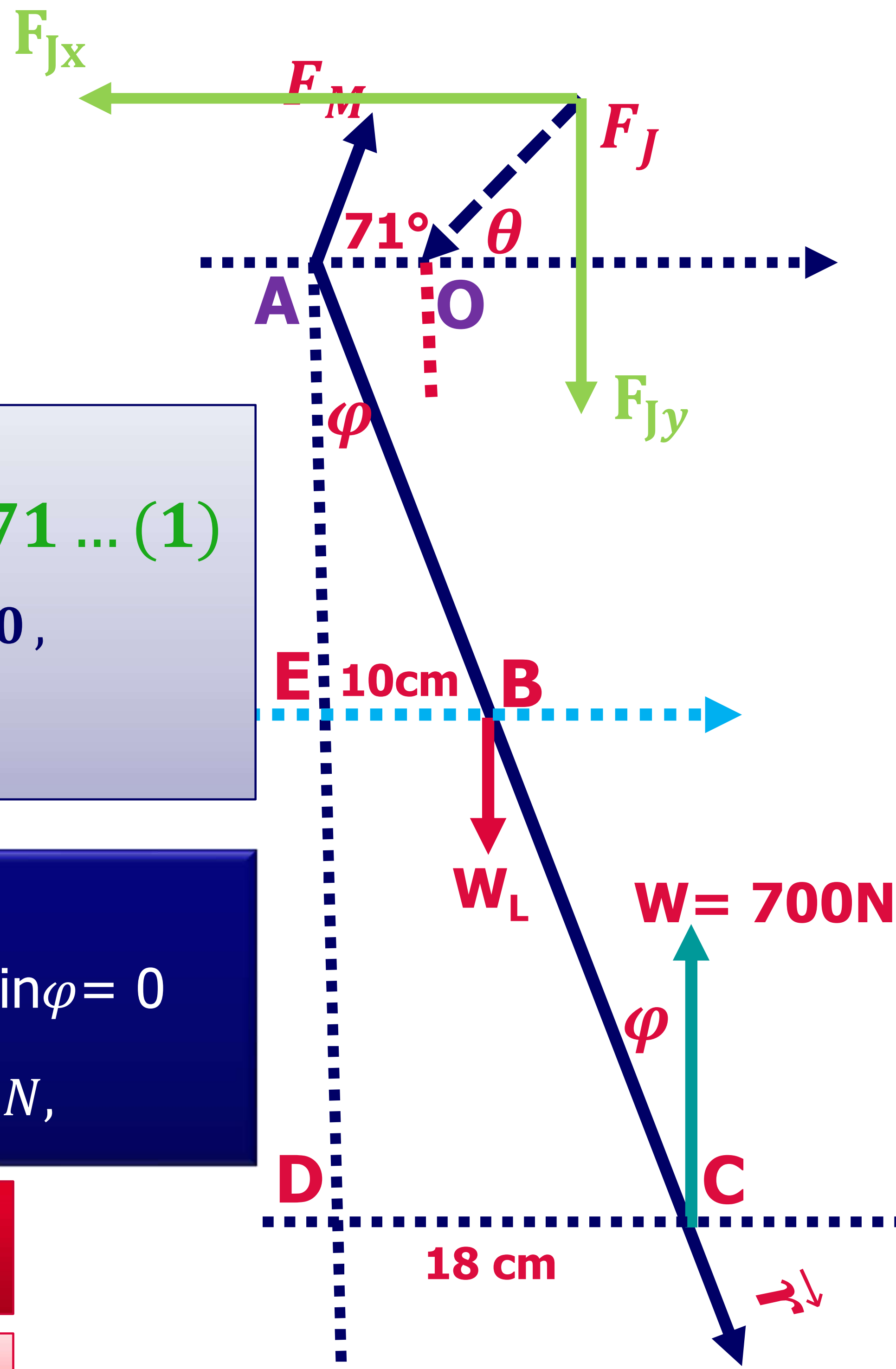


W_L=0.185W= 130N



$$\frac{EB}{AB} = \sin\varphi$$

$$\frac{DC}{AC} = \sin\varphi$$



$$\sum F_x = 0,$$

$$-F_{Jx} + F_M cos71 =, \quad F_{Jx} = F_M cos71 \ldots (1)$$

$$\sum F_y = 0, \quad F_M sin71 - F_{Jy} - w_L + W = 0,$$

$$F_{Jy} = F_M sin71 + 570 \ldots (2)$$

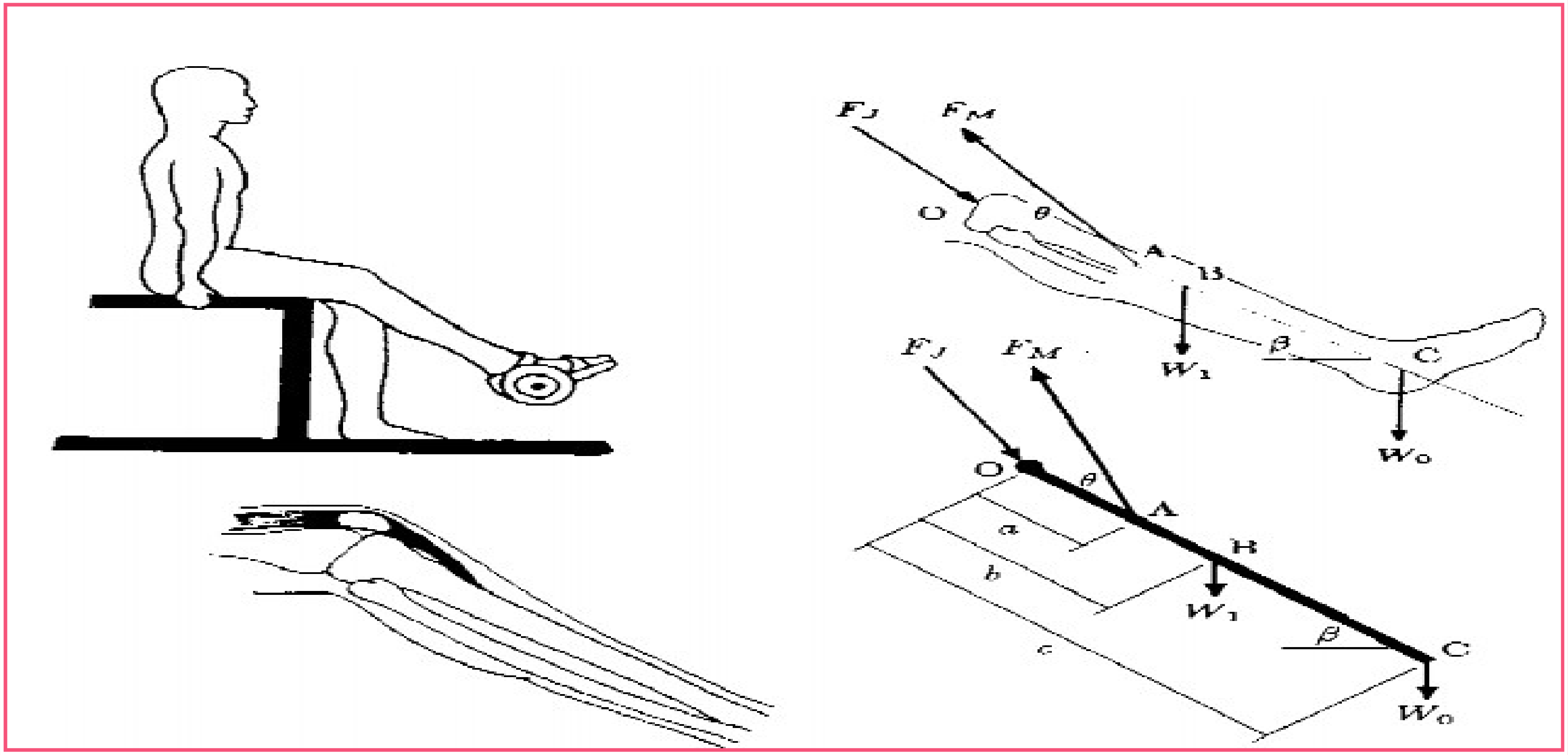
$$\sum \vec{\tau} = 0, \quad \tau_{F_M} + \tau_{F_J} + \tau_{w_L} + \tau_W = 0$$

$0 - F_J(AO)\sin\theta - 130(AB)\sin\varphi + 700(AC)\sin\varphi = 0$

$$F_{Jy} = \frac{700(18) - 130(10)}{7} = 1614\ N,$$

$$F_M = \frac{1614 - 570}{sin71} = 1104\ N$$

$F_{Jx} = F_M cos71$ =1104 cos71= 360 N

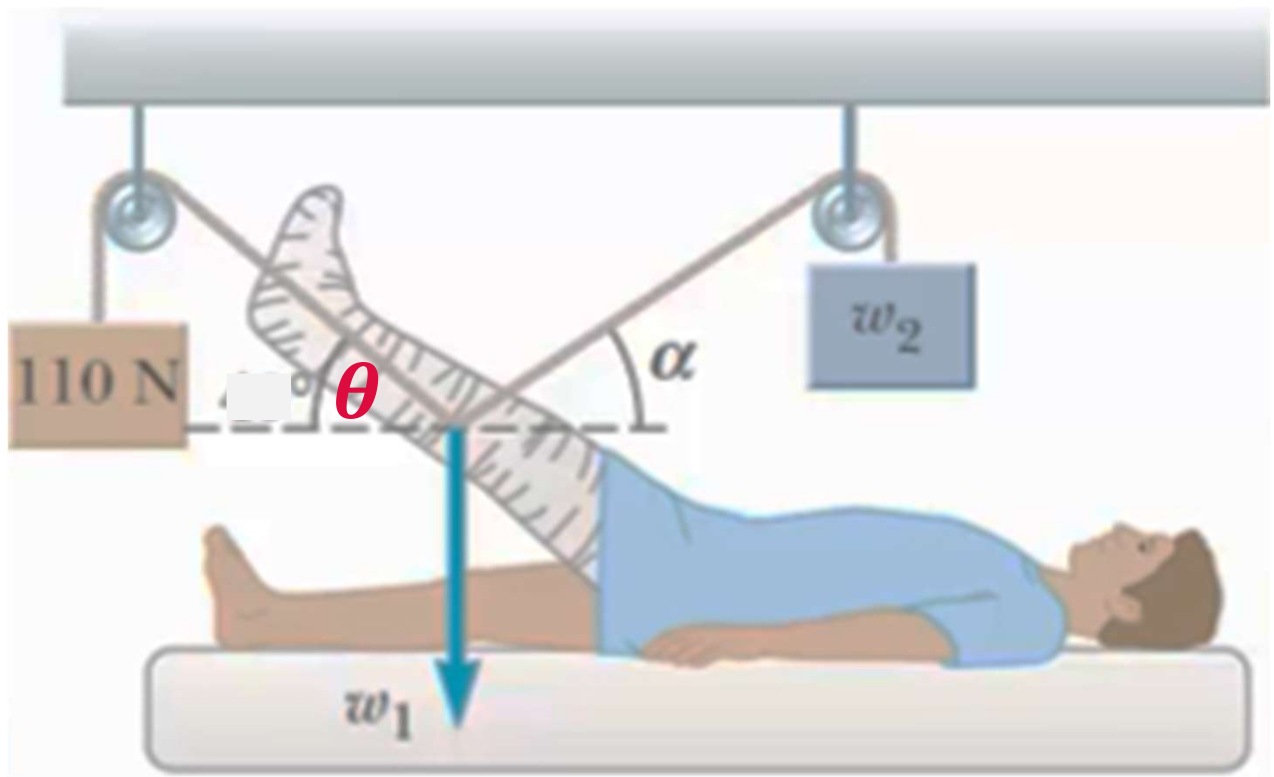


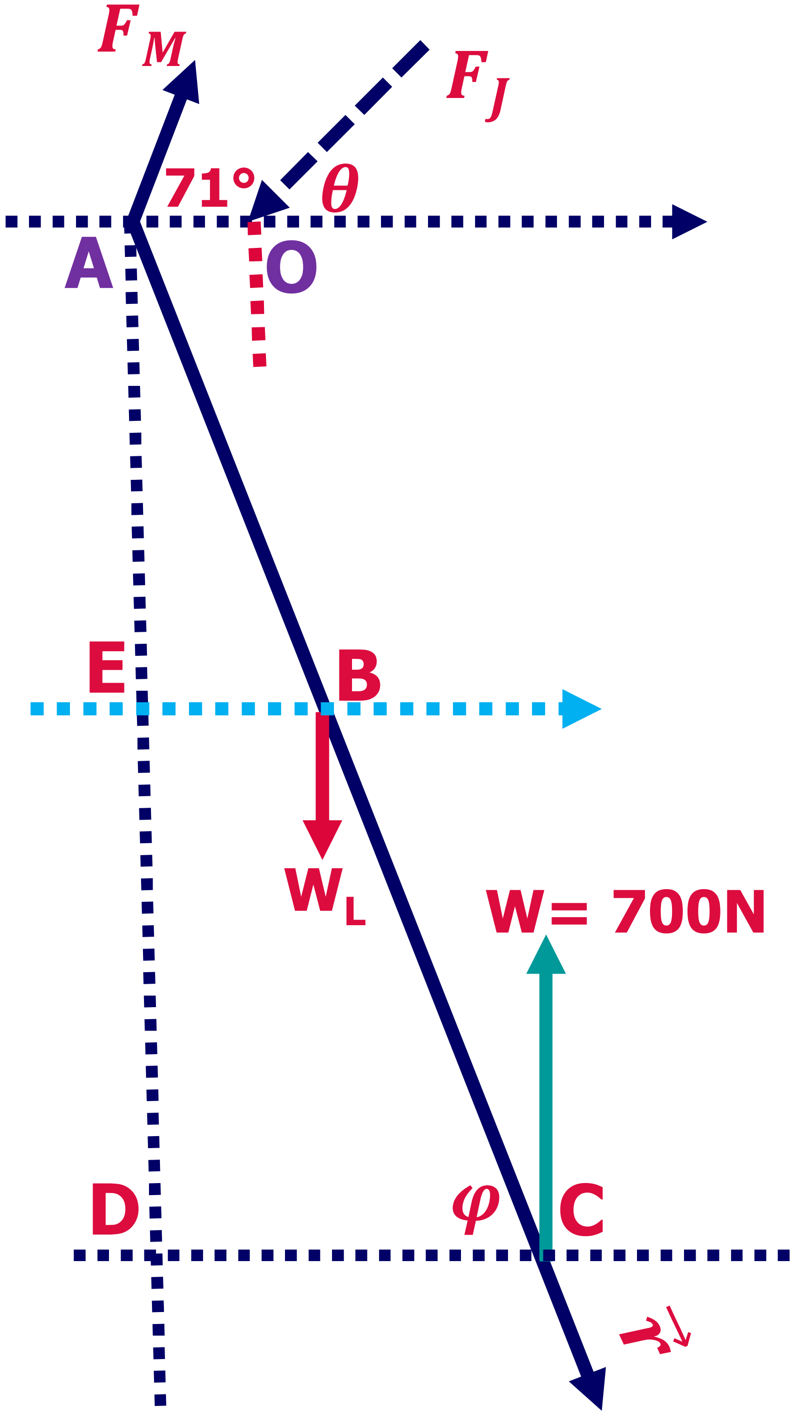
.The shown diagram illustrates the forces acting at knee and on tibia. The leg makes an angle of 30° with the horizontal. The weight of the leg W1=100N, and W0 = 75N. Find the bone-on-bone joint reaction force F_J and the force exerted by quadriceps muscle FM. Take $\theta = 32°$, $OA = 10cm$, $AB = 7$ cm and $BC = 15$ cm.

Example

The system of the shown leg and the cast is in equilibrium that no force is exerted on the hip joint by the leg and the cast , if w_1= 220N, and $\theta = 30°$, then the weight w_2 = ……………………

and the angle α =…………..





F_M
F_J
71°
θ
A
O
E
B
W_L
W= 700N
D
φ
C

The shown free body diagram represent the hip joint for a standing person, where the forces at the muscle, the joints and the weights affecting the system are shown in the figure.

Take AO= 7 cm, AE= 20 cm, ED= 24 cm, W_L= 130 N, and $\varphi = 45°$, find the magnitude of F_M and F_J